Math 2471 – Calc 3

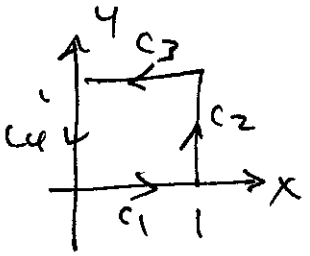
Green's Th^m

$$\oint_C P\,dx + Q\,dy = \iint_R (Q_x - P_y)\,dA$$

C – simple closed curve oriented CCW
piecewise smooth
enclosing a region R & P, Q cont^s deriv.

Ex Verify Green's Th^m when

$$\oint_C (5 - 2xy - y^2)\,dx + (x^2 - 2xy)\,dy$$



C is unit square

So there are 4 curves

C_1: $y = 0$ $\quad x: 0 \to 1$ $\quad dy = 0$

$$\int_0^1 5\,dx = 5x\Big|_0^1 = 5$$

C_2: $x=1,\ y: 0 \to 1 \qquad dx = 0$

$$\int_0^1 (1-2y)\,dy = y - y^2 \Big|_0^1 = 0$$

C_3 $y=1 \qquad x: 1 \to 0 \qquad dy = 0$

$$\int_1^0 (5 - 2x - 1)\,dx = 4x - x^2 \Big|_1^0 = 0 - (4-1) = -3$$

C_4 $x=0 \qquad dx = 0$

$$\int_{C_4} 0 = 0$$

$$5 + 0 - 3 + 0 = 2 \;//$$

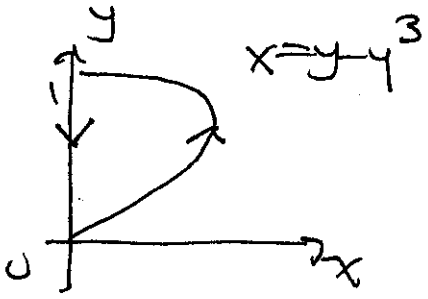
$$Q_x - P_y = 2x - 2y - (-2x - 2y)$$
$$= 4x$$

$$\int_0^1 \int_0^1 4x\,dy\,dx = \int_0^1 4xy \Big|_0^1 dx = \int_0^1 4x\,dx$$

$$= 2x^2 \Big|_0^1 = 2 \;//$$

Same!

Ex 2 $\int_C (1+y)dx + x^2 dy$



Directly

C_1 $x = y - y^3$, $dx = (1 - 3y^2)dy$

So $\int_0^1 (1-y)(1-3y^2)dy + (y-y^3)^2 dy$

$\int_0^1 (1 - y - 2y^2 + 3y^3 - 2y^4 + y^6)dy = \frac{137}{420}$

C_2 $x = 0$ $dx = 0$

$\int 0 = 0$

So $\int_C (1+y)dx + x^2 dy = \frac{137}{420}$

Green's Th^m

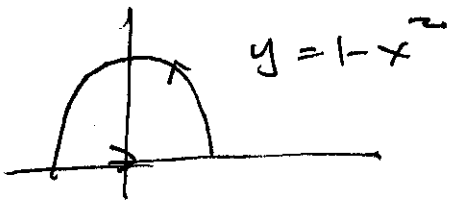
$Q_x - P_y = 2x - (-1) = 2x + 1$

$\int_{y=0}^{1} \int_{x=0}^{(y-y^3)} (2x+1)dx\,dy = \int_{y=0}^{1} x^2 + x \Big|_0^{y-y^3} dy$

$$\int_0^1 (y-y^3)^2 + (y-y^3)\,dy$$

$$\int_0^1 y + y^2 - y^3 - 2y^4 + y^6\,dy = \frac{137}{420}$$ saw

Ex3 $\oint_C (y-x)dx + (2x-y)dy$



Directly

C_1 $y = 1-x^2$ $dy = -2x\,dx$ $x: 1 \to -1$

$$\int_1^{-1} (1-x^2-x)dx + (2x-1+x^2)(-2x)dx$$

$$\int_{-1}^{1} (2x^3 + 5x^2 - x - 1)\,dx = 4/3$$

C_2 $y = 0$ $x: -1 \to 1$

$$\int_{-1}^{1} -x\,dx = -\frac{x^2}{2}\Big|_{-1}^{1} = 0$$

So $\oint_C (y-x)dx + (2x-y)dy = 4/3 + 0 = 4/3$

Green's Thm

$Q_x - P_y = 2 - 1 = 1$

$$\int_{-1}^{1} \int_{y=0}^{1-x^2} 1 \, dy \, dx = \int_{-1}^{1} y \Big|_0^{1-x^2} dx$$

$$= \int_{-1}^{1} (1-x^2) dx = x - \frac{x^3}{3} \Big|_{-1}^{1} = \left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)$$

$$= 2/3 + 2/3 = 4/3$$

Same