

Math 2471 Calc III - Sample Test 3

1. Evaluate the following line integrals:

(i) $\int_C xy \, ds$ where C counter clockwise direction along the circle $x^2 + y^2 = 4$ from $(2, 0)$ to $(0, 2)$.

(ii) $\int_C (x + y)dx - 2xydy$ where C is the parabola $y = x^2$ from $(0, 0)$ to $(2, 4)$.

2. Is the following vector field conservative?

$$\vec{F} = \langle 2xy, x^2 + z^2, 2yz \rangle .$$

If so, determine the potential function f such that $\vec{F} = \vec{\nabla}f$ and use this to evaluate

$$\int_C 2xy \, dx + (x^2 + z^2) \, dy + 2yz \, dz$$

where C is any curve joining $(1, 1, 1)$ and $(2, 3, 4)$.

3. Evaluate the following line integral $\int_C 2y \, dx + x \, dy + dz$ where C is the line joining the points $P(0, 1, 2)$ to $Q(1, -2, 2)$.

4. Green's Theorem is

$$\int_C P \, dx + Q \, dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dA.$$

Verify Green's Theorem where $\vec{F} = \langle y^2, x^2 + 2xy \rangle$ where R is the region bound by the curves $y = x^2$, $y = 1$ and $x = 0$ in $Q1$.