FREQUENCY OFFSET ESTIMATION FOR PCC-OFDM WITH SYMBOLS OVERLAPPED IN TIME DOMAIN

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Abstract

This paper presents a new algorithm for frequency offset estimation for Polynomial Cancellation Coded Orthogonal Frequency Division Multiplexing with symbols overlapped in the time domain (Overlap PCC-OFDM). The algorithm exploits the Subcarrier Pair Imbalance (SPI) caused by frequency offset. The estimation is performed in the frequency domain. No training symbols or pilot tones are required. Simulations show that this estimator is an approximately linear function of frequency offset. There are three ways to reduce the variance of the estimation: increasing the number of subcarrier pairs, using a two-dimensional Minimum Mean Square Error (MMSE) equalizer before the estimation and using PCC-OFDM pilot symbols with no overlapping.

1. Overlap PCC-OFDM

Polynomial Cancellation Coded Orthogonal Frequency Division Multiplexing (PCC-OFDM) was designed to reduce the Interchannel Interference (ICI) caused by frequency offset [1]. In PCC-OFDM, each data value to be transmitted is mapped onto a group of subcarriers. In this paper, the case where each data value to be transmitted is mapped onto pairs of subcarriers is considered. Despite its advantages, PCC-OFDM is not bandwidth efficient in its simplest form [2]. One way to overcome this drawback is to overlap PCC-OFDM symbols in the time domain [3].

Figure 1 shows the procedure of symbol overlapping. With overlapping, each overlapped symbol consists of three parts, the current PCC-OFDM symbol, the second half of the preceding PCC-OFDM symbol and the first half of the following PCC-OFDM symbol.

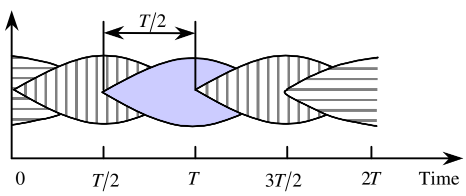

Fig. 1 PCC-OFDM symbols overlapped in the time domain

Figure 2 shows a simplified block diagram of an Overlap PCC-OFDM communication system. The high speed data stream is fed into a serial to parallel converter and converted into n lower speed parallel substreams. The ith vector to be transmitted is represented by $d_{0,i},\cdots,d_{n-1,i}$. They are mapped onto the values $a_{0,i}\cdots a_{N-1,i}$ that modulate the N subcarriers in the ith symbol period. For conventional OFDM, which has $n=N$ and $a_{k,i}=d_{k,i}$, there is a simple one-to-one mapping of data values onto the subcarriers. For PCC-OFDM, n and N are not equal. In this paper, we have $n=N/2$.

In the channel, the signal is filtered by the channel response $h(t)$ and additive noise $n(t)$ is injected. At the receiver the ith output vector of the DFT is $z_{0,i}\cdots z_{N-1,i}$. The demodulated subcarriers are then weighted and added to generate the data estimates $v_{0,i}\cdots v_{n-1,i}$. For the subcarrier pair $z_{2M,i}$ and $z_{2M+1,i}$, an estimate is calculated using $v_{M,i}=(z_{2M,i}-z_{2M+1,i})/2$. To recover the transmitted data values from the overlapped symbols, a two-dimensional Minimum Mean Square Error (MMSE) frequency domain equalizer can be used [3].

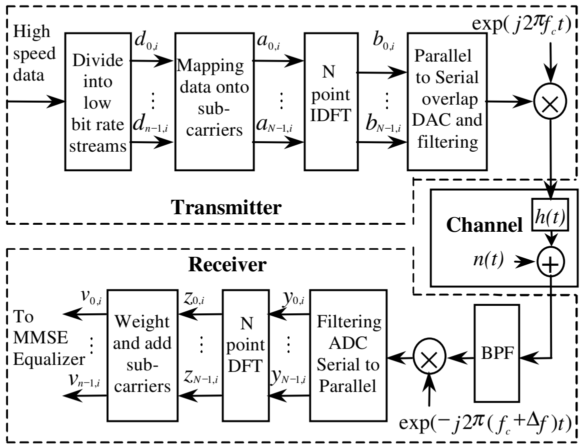

Fig. 2. Block diagram of a PCC-OFDM system

In [4], a blind frequency offset estimator has been presented for PCC-OFDM. As shown in [5], the frequency offset estimator for PCC-OFDM can also be used for PCC-OFDM with symbols overlapped in the time domain. Recently, a new MMSE frequency offset estimator has been proposed for PCC-OFDM [6]. In this paper it will be shown that the same form of the estimator can also be used for Overlap PCC-OFDM.

2. Subcarrier imbalance in PCC-OFDM

In Overlap PCC-OFDM, each demodulated subcarrier contains overlapping components from adjacent PCC-OFDM symbols [4]. The subcarrier pair imbalance is still defined as the amplitude or power difference between two subcarriers in a demodulated subcarrier pair. That is

$$F(\Delta fT) = \left|z_{2M+1,i}\right|^2 - \left|z_{2M,i}\right|^2 \quad (1)$$

Each value of imbalance is a combination of the imbalance of the PCC-OFDM subcarrier pair and the imbalance of the overlapping components in the demodulated subcarrier pair.

In the absence of frequency offset, two demodulated subcarriers in an Overlap PCC-OFDM subcarrier pair are balanced in a particular way. The subcarrier pair imbalance depends not only on the frequency offset and transmitted data values in the current PCC-OFDM symbol, but also on the overlapping components from the preceding and following PCC-OFDM symbols. As for PCC-OFDM, the data dependency in Overlap PCC-OFDM can be removed by averaging over a number of subcarrier pairs. The average subcarrier pair imbalance depends on the frequency offset, therefore, we can use the average imbalance for frequency offset estimation.

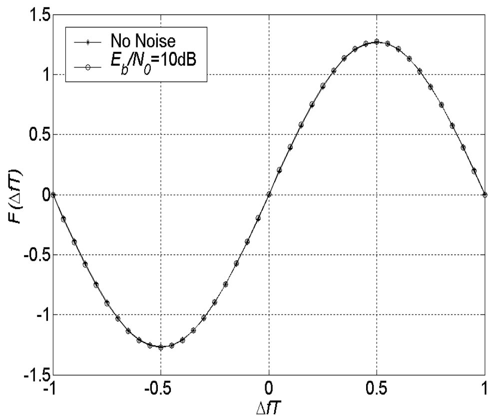

Fig. 3. Subcarrier pair imbalance as a function of frequency offset

Figure 3 shows the average SPI as a function of frequency offset for $N = 32$. The number of simulated subcarriers was 1,000 and all subcarrier pairs were used.

As for PCC-OFDM [6], The figure shows some interesting features:

- Crossing zero at zero frequency offset.
- For $|\Delta fT| < 0.5$, the imbalance increases monotonically as the frequency offset increases.
- Additive White Gaussian Noise (AWGN) does not affect the zero crossing position.
- The frequency estimation range can be extended to one subcarrier spacing.

3. Expression for an Overlap PCC-OFDM subcarrier

The kth sample in the ith time domain symbol in PCC-OFDM is given by [2]

$$b_{k,i} = \frac{1}{N}\sum_{l=0}^{N-1} a_{l,i} \exp\left(\frac{j2\pi kl}{N}\right) \quad (2)$$

where $a_{l,i}$ is the lth subcarrier in the ith frequency domain symbol. The overlapping components in the current symbol are introduced from the second half of the $(i-1)$th PCC-OFDM symbol and the first half of the $(i+1)$th PCC-OFDM symbol. The kth overlapping component is given by [4]

$$b'_{k,i} = \begin{cases} \frac{1}{N}\sum_{l=0}^{N-1}(-1)^l a_{l,i-1} \exp\left(\frac{j2\pi kl}{N}\right) & \text{for } 0 \le k \le N/2-1 \\ \frac{1}{N}\sum_{l=0}^{N-1}(-1)^l a_{l,i+1} \exp\left(\frac{j2\pi kl}{N}\right) & \text{for } N/2 \le k \le N-1 \end{cases} \quad (3)$$

At the receiver, the kth value of the ith input data block to the FFT demodulator is given by

$$y_{k,i} = \exp\left(\frac{j2\pi k\varepsilon}{N}\right)\left(b'_{k,i} + b_{k,i}\right) + w_{k,i} \quad (4)$$

where ε is the frequency offset, $\varepsilon = \Delta fT$. $w_{k,i}$ is the Gaussian noise. The mth subcarrier in the ith demodulated symbol is then given by

$$\begin{aligned} z_{m,i} &= \sum_{k=0}^{N-1} y_{k,i} \exp\left(\frac{-j2\pi km}{N}\right) \\ &= \sum_{k=0}^{N/2-1} y_{k,i} \exp\left(\frac{-j2\pi km}{N}\right) + \sum_{k=N/2}^{N-1} y_{k,i} \exp\left(\frac{-j2\pi km}{N}\right) \end{aligned} \quad (5)$$

The $2M$th demodulated subcarrier is given by

$$\begin{aligned} z_{2M,i} = &\sum_{L=0}^{N/2-1}\left(CF_{2(L-M)} + CF_{2(L-M)+1}\right)d_{L,i-1} \\ &+ \sum_{L=0}^{N-1}\left(CS_{2(L-M)} + CS_{2(L-M)+1}\right)d_{L,i+1} \\ &+ \sum_{L=0}^{N-1}\left(c_{2(L-M)} - c_{2(L-M)+1}\right)d_{L,i} + W_{2M,i} \end{aligned} \quad (6)$$

where $W_{2M,i}$ is the FFT of $w_{k,i}$, c_{l-m} are complex coefficients given by [2]

$$c_{l-m} = \frac{\sin(\pi(l-m+\Delta fT))}{N\sin(\pi(l-m+\Delta fT)/N)} \times \exp(j\pi(N-1)(l-m+\Delta fT)/N) \quad (7)$$

It is shown in [7], $W_{2M,i}$ is Gaussian. The coefficients CF_{l-m} and CS_{l-m} are given by

$$CF_{l-m} = \frac{1}{N}\sum_{k=0}^{N/2-1}\exp\left(\frac{j2\pi k(l-m+\varepsilon)}{N}\right) \quad (8)$$

$$CS_{l-m} = \frac{1}{N}\sum_{k=N/2}^{N-1}\exp\left(\frac{j2\pi k(l-m+\varepsilon)}{N}\right) \quad (9)$$

Similarly, the $(2M+1)$th demodulated subcarrier can be obtained.

4. MMSE frequency offset estimator

It is shown in Appendix A that equation (1) for Overlap PCC-OFDM for 4QAM can be written as

$$F(\Delta fT) = K\sin(\pi\Delta fT) + e_M \quad (10)$$

where K is a constant given by [6]

$$K = \cos\left(\frac{\pi}{N}\right)\prod_{k=1}^{\log_2(N)-1}\cos\left(\frac{2^{k-1}\pi}{N}\right) \quad (11)$$

e_M is the error term. For small frequency offset, the error term can be approximated as additive noise with zero mean and finite variance. Applying MMSE techniques for equation (10) [8], we can obtain the frequency offset estimator

$$\hat{\Delta fT} = \frac{1}{\pi}\sin^{-1}\left(\frac{1}{KM_l}\sum_{t=1}^{M_l}F_t(\Delta fT)\right) \quad (12)$$

where M_l is number of subcarrier pairs to be used for the frequency offset estimation.

5. Simulation results

To evaluate the performance of the MMSE estimator, simulations were performed. In the following simulations, 4QAM is used for the modulation scheme with $M_l = 512$ and $N = 128$. Figure 4 shows the estimator as a function of the frequency offset for $E_b/N_0 = 10\text{dB}$. It is evident that the estimator has an approximately linear relationship with frequency offset.

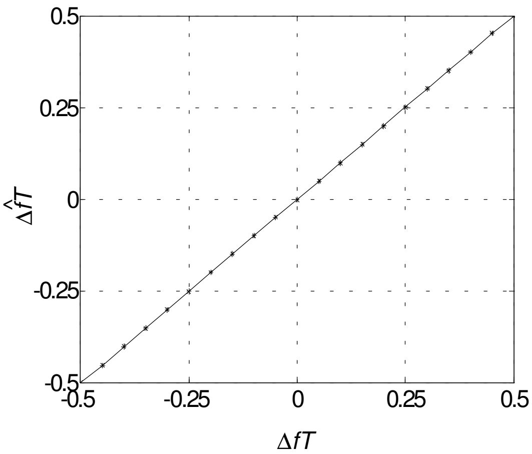

Fig. 4. Frequency offset estimator as a function of frequency offset

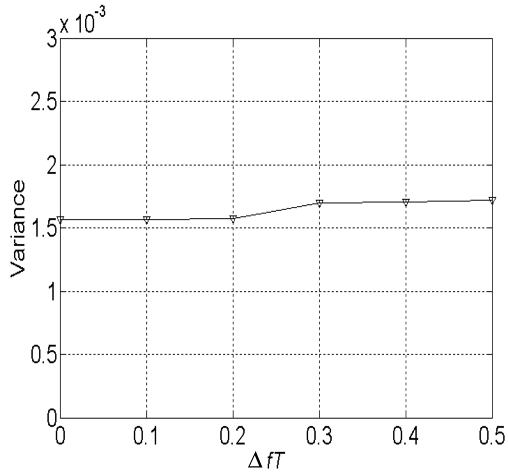

Fig. 5. Variance of the estimator as a frequency offset

Figure 5 shows the variance of the frequency offset estimator as a function of the frequency offset for an ideal channel. The variance does not change significantly as frequency offset increases. This means that the overlapping components in Overlap PCC-OFDM are the dominant factor for the variance. Lower variance can be obtained by increasing M_l or using a two-dimensional MMSE equalizer before the frequency offset estimation. Another way to reduce the variance is to use PCC-OFDM pilot symbols with no overlapping.

Figure 6 shows the variance of frequency offset estimator as a function of E_b/N_0. The frequency offset simulated was zero. The variance does not significantly change as the channel noise increases. The variance is dominated by the overlapping components.

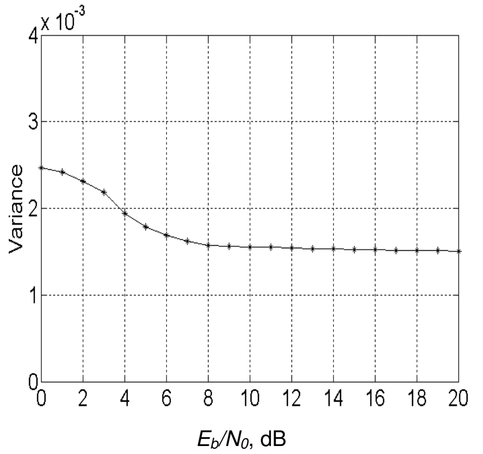

Fig. 6. Variance of the estimator as a function of E_b/N_0

6. Conclusion

An MMSE frequency offset estimator for Overlap PCC-OFDM has been presented. A frequency offset estimate is obtained by using the average SPI at the output of the receiver FFT. No training symbols or pilot tones are required. The MMSE estimator has an approximately linear relationship with frequency offset. There are three measurements to reduce the variance, one is to increase the number of subcarrier pairs, the second is to use a two-dimensional MMSE equalizer before the estimation and the third is to use PCC-OFDM pilot symbol with no overlapping components.

Appendix A

From (6), the power of the $2M$th subcarrier is given by

$$\begin{aligned}|z_{2M,i}|^2 &= \sum_{L=0}^{N/2-1}\left|\left(CF_{2(L-M)}+CF_{2(L-M)+1}\right)\right|^2|d_{L,i-1}|^2\\ &+\sum_{L=0}^{N/2-1}\left|\left(CS_{2(L-M)}+CS_{2(L-M)+1}\right)\right|^2|d_{L,i+1}|^2 \quad (A.1)\\ &+\sum_{L=0}^{N/2-1}\left|\left(c_{2(L-M)}-c_{2(L-M)+1}\right)\right|^2|d_{L,i}|^2+e_{2M,i}\end{aligned}$$

Equation (A.1) indicates that the power of each demodulated subcarrier can be represented in terms of the individual power components from the preceding, current and following PCC-OFDM symbols. Note for 4QAM with each subcarrier normalized to unity, $|d_{L,i}|^2=1$. The error term of (A.1) is then given by

$$\begin{aligned}&e_{2M,i}=\\ &\sum_{\substack{L=0\\L\neq K}}^{N/2-1}\sum_{K=0}^{N/2-1}\left(c_{2(L-M)}-c_{2(L-M)+1}\right)\left(c_{2(K-M)}-c_{2(K-M)+1}\right)^* d_{L,i}d^*_{K,i}\\ &+\sum_{\substack{L=0\\L\neq K}}^{N/2-1}\sum_{K=0}^{N/2-1}\left(CF_{2(L-M)}+CF_{2(L-M)+1}\right)\left(CF_{2(K-M)}+CF_{2(K-M)+1}\right)^* d_{L,i-1}d^*_{K,i-1}\\ &+\sum_{\substack{L=0\\L\neq K}}^{N/2-1}\sum_{K=0}^{N/2-1}\left(CS_{2(L-M)}+CS_{2(L-M)+1}\right)\left(CS_{2(K-M)}+CS_{2(K-M)+1}\right)^* d_{L,i+1}d^*_{K,i+1}\\ &+\sum_{L=0}^{N/2-1}\sum_{K=0}^{N/2-1}\left(c_{2(L-M)}-c_{2(L-M)+1}\right)\left(CS_{2(K-M)}+CS_{2(K-M)+1}\right)^* d_{L,i}d^*_{K,i+1}\\ &+\sum_{L=0}^{N/2-1}\sum_{K=0}^{N/2-1}\left(CF_{2(L-M)}+CF_{2(L-M)+1}\right)\left(c_{2(K-M)}+c_{2(K-M)+1}\right)^* d_{L,i-1}d^*_{K,i}\\ &+\sum_{L=0}^{N/2-1}\sum_{K=0}^{N/2-1}\left(c_{2(L-M)}-c_{2(L-M)+1}\right)^*\left(CS_{2(K-M)}+CS_{2(K-M)+1}\right) d^*_{L,i}d_{K,i+1}\\ &+\sum_{L=0}^{N/2-1}\sum_{K=0}^{N/2-1}\left(CF_{2(L-M)}+CF_{2(L-M)+1}\right)^*\left(c_{2(K-M)}+c_{2(K-M)+1}\right) d^*_{L,i-1}d_{K,i}\\ &+2\mathrm{Re}\left\{W_{2M,i}\sum_{K=0}^{N/2-1}\left(CF_{2(K-M)}+CF_{2(K-M)+1}\right)^* d^*_{K,i-1}\right\}\\ &+2\mathrm{Re}\left\{W_{2M,i}\sum_{K=0}^{N/2-1}\left(CS_{2(K-M)-1}+CS_{2(K-M)}\right)^* d^*_{K,i+1}\right\}\\ &+2\mathrm{Re}\left\{W_{2M,i}\sum_{K=0}^{N/2-1}\left(c_{2(K-M)}-c_{2(K-M)+1}\right)^* d^*_{K,i}\right\}+|W_{2M,i}|^2\end{aligned} \quad (A.2)$$

Because the expected value of all cross terms is zero, the expected value of the error term is equal to the variance of the noise. Similarly, we can obtain the power for the $(2M+1)$th subcarrier. Using $|d_{L,i}|^2=1$ for 4QAM, the subcarrier pair imbalance is then given by

$$\begin{aligned}&|z_{2M+1,i}|^2-|z_{2M,i}|^2=\\ &\sum_{L=0}^{N/2-1}\left|\left(CF_{2(L-M)-1}+CF_{2(L-M)}\right)\right|^2+\sum_{L=0}^{N/2-1}\left|\left(CS_{2(L-M)-1}+CS_{2(L-M)}\right)\right|^2\\ &-\sum_{L=0}^{N/2-1}\left|\left(CF_{2(L-M)}+CF_{2(L-M)+1}\right)\right|^2-\sum_{L=0}^{N/2-1}\left|\left(CS_{2(L-M)}+CS_{2(L-M)+1}\right)\right|^2\\ &+\sum_{L=0}^{N/2-1}\left|\left(c_{2(L-M)-1}-c_{2(L-M)}\right)\right|^2-\sum_{L=0}^{N/2-1}\left|\left(c_{2(L-M)}-c_{2(L-M)+1}\right)\right|^2\\ &+e_{2M+1,i}-e_{2M,i}\end{aligned} \quad (A.3)$$

It is shown in Appendix B that the combination of the first four summations in (A.3) equals zero. Thus we can obtain the subcarrier pair imbalance

$$F(\Delta fT)=|z_{2M+1,i}|^2-|z_{2M,i}|^2=S(\Delta fT)+e'_{M,i} \quad (A.4)$$

where $S(\Delta fT)$ is given by [6]

$$S(\Delta fT)=K\sin(\pi\Delta fT) \quad (A.5)$$

K is a factor defined in (11). $e'_{M,i}$ is the Mth total error, $e'_{M,i}=e_{2M+1,i}-e_{2M,i}$. In Overlap PCC-OFDM, the error term is more complicated than in PCC-OFDM. However, the dominant distribution in the error term is still Gaussian. As for PCC-OFDM, $e'_{M,i}$ can be approximated as AWGN with zero mean. The variance is larger than that of PCC-OFDM because of the overlapping components. Appendix C shows the variance of the coupling-crossing terms for $e'_{M,i}$. Thus, under the condition of a small frequency offset, the overall approximate variance is given by

$$e'_{M,i} \sim N(0, 8+8\sigma_n^2+2\sigma_n^4) \qquad \text{(A.6)}$$

where "~" means distributed as. Note that we have used $\sigma_s^2 = 1$ for 4QAM in (A.6). The SPI estimator is therefore given by

$$\Delta \hat{f}T = \frac{1}{\pi}\sin^{-1}\left(\frac{1}{KM_I}\sum_{i=1}^{M_I} F_i(\Delta fT)\right) \qquad \text{(A.7)}$$

Appendix B

The element of the first summation of (A.3) is given by

$$\left|\left(CF_{2(L-M)-1}+CF_{2(L-M)}\right)\right|^2 = \frac{\sin^2\left(\frac{\pi\varepsilon}{2}\right)}{N^2\sin^2\left(\frac{\pi(2L+\varepsilon)}{N}\right)} + \frac{\cos^2\left(\frac{\pi\varepsilon}{2}\right)}{N^2\sin^2\left(\frac{\pi(2L-1+\varepsilon)}{N}\right)} - \frac{\sin(\pi\varepsilon)}{N^2\sin\left(\frac{\pi(2L+\varepsilon)}{N}\right)\sin\left(\frac{\pi(2L-1+\varepsilon)}{N}\right)}\sin\left(\frac{\pi}{\mathrm{N}}\right) \qquad \text{(B.1)}$$

The element of the second summation of (A.3) is given by

$$\left|\left(CS_{2(L-M)-1}+CS_{2(L-M)}\right)\right|^2 = \frac{\sin^2\left(\frac{\pi\varepsilon}{2}\right)}{N^2\sin^2\left(\frac{\pi(2L+\varepsilon)}{N}\right)} + \frac{\cos^2\left(\frac{\pi\varepsilon}{2}\right)}{N^2\sin^2\left(\frac{\pi(2L-1+\varepsilon)}{N}\right)} + \frac{\sin(\pi\varepsilon)}{N^2\sin\left(\frac{\pi(2L+\varepsilon)}{N}\right)\sin\left(\frac{\pi(2L-1+\varepsilon)}{N}\right)}\sin\left(\frac{\pi}{\mathrm{N}}\right) \qquad \text{(B.2)}$$

Similarly the third and fourth terms can be derived.

Substituting the elements derived to the combination of the first four summations of (A.3) gives

$$\begin{aligned}&\sum_{L=0}^{N/2-1}\left|\left(CF_{2(L-M)-1}+CF_{2(L-M)}\right)\right|^2+\sum_{L=0}^{N/2-1}\left|\left(CS_{2(L-M)-1}+CS_{2(L-M)}\right)\right|^2\\&-\sum_{L=0}^{N/2-1}\left|\left(CF_{2(L-M)}+CF_{2(L-M)+1}\right)\right|^2-\sum_{L=0}^{N/2-1}\left|\left(CS_{2(L-M)}+CS_{2(L-M)+1}\right)\right|^2\\&=2\sum_{L=0}^{N/2-1}\left\{\frac{\cos^2\left(\frac{\pi\varepsilon}{2}\right)}{N^2\sin^2\left(\frac{\pi(2L+\varepsilon-1)}{N}\right)}-\frac{\cos^2\left(\frac{\pi\varepsilon}{2}\right)}{N^2\sin^2\left(\frac{\pi(2L+1+\varepsilon)}{N}\right)}\right\}\\&=0\end{aligned} \qquad \text{(B.3)}$$

Appendix C

The coupling-cross terms $CC_{2M,i}$ in (A.2) is given by

$$\begin{aligned}CC_{2M,i}=&\\&\sum_{L=0}^{N/2-1}\sum_{K=0}^{N/2-1}\left(c_{2(L-M)}-c_{2(L-M)+1}\right)\left(CS_{2(K-M)}+CS_{2(K-M)+1}\right)^* d_{L,i}d_{K,i+1}^*\\&+\sum_{L=0}^{N/2-1}\sum_{K=0}^{N/2-1}\left(CF_{2(L-M)}+CF_{2(L-M)+1}\right)\left(c_{2(K-M)}-c_{2(K-M)+1}\right)^* d_{L,i-1}d_{K,i}^*\\&+\sum_{L=0}^{N/2-1}\sum_{K=0}^{N/2-1}\left(c_{2(L-M)}-c_{2(L-M)+1}\right)^*\left(CS_{2(K-M)}+CS_{2(K-M)+1}\right)d_{L,i}^*d_{K,i+1}\\&+\sum_{L=0}^{N/2-1}\sum_{K=0}^{N/2-1}\left(CF_{2(L-M)}+CF_{2(L-M)+1}\right)^*\left(c_{2(K-M)}-c_{2(K-M)+1}\right)d_{L,i-1}^*d_{K,i}\end{aligned} \qquad \text{(C.1)}$$

Similarly, the coupling-cross terms $CC_{2M+1,i}$ for the $(2M+1)$th error term can also be obtained. Note the first term and the third term are mutually conjugated, the second term and the fourth term are mutually conjugated in (C.1). Thus, $CC_{2M+1,i} - CC_{2M,i}$ can be written as

$$\begin{aligned}&CC_{2M+1,i}-CC_{2M,i}=\\&-2\mathrm{Re}\left\{d_{M,i}\sum_{K=0}^{N/2-1}\left(CS_{2(K-M)-1}+2CS_{2(K-M)}+CS_{2(K-M)+1}\right)^* d_{K,i+1}^*\right\}\\&-2\mathrm{Re}\left\{d_{M,i}^*\sum_{L=0}^{N/2-1}\left(CF_{2(L-M)-1}+2CF_{2(L-M)}+CF_{2(L-M)+1}\right)d_{L,i-1}\right\}\end{aligned} \qquad \text{(C.2)}$$

where $\mathrm{Re}(\bullet)$ represents the real part of a complex number. For the first summation in (C.2), considering the most significant complex coefficients CS_0, we get

$$\begin{aligned}&d_{M,i}\sum_{K=0}^{N/2-1}\left(CS_{2(K-M)-1}+2CS_{2(K-M)}+CS_{2(K-M)+1}\right)^* d_{K,i+1}^*\\&\approx 2d_{M,i}CS_0^*d_{M,i+1}^*\end{aligned} \qquad \text{(C.3)}$$

Similarly, we can also simplify the second summation in (C.2). In addition, in the absence of frequency offset, the value of the most significant complex coefficient can be obtained from (7) and (8), that is $CS_0 = CF_0 = 0.5$. Thus (C.2) can be written as

$$CC_{2M+1,i} - CC_{2M,i} \approx -2\mathrm{Re}\{d_{M,i} d^*_{M,i+1}\} - 2\mathrm{Re}\{d^*_{M,i} d_{M,i-1}\} \tag{C.4}$$

The variance of the left side of (C.4) is given by

$$\begin{aligned} &E\left[\left|CC_{2M+1,i} - CC_{2M,i}\right|\right]^2 \approx \\ &E\left[4\left(\mathrm{Re}\left(d_{M,i} d^*_{M,i+1}\right)\right)^2 + 4\left(\mathrm{Re}\left(d^*_{M,i} d_{M,i-1}\right)\right)^2 \right. \\ &\left. + 4\left(\mathrm{Re}\left(d_{M,i} d^*_{M,i+1}\right)\right)\left(\mathrm{Re}\left(d^*_{M,i} d_{M,i-1}\right)\right)\right] \\ &= 8\sigma_s^4 \end{aligned} \tag{C.5}$$

When all complex coefficients are considered, the variance will be slightly larger than above result. Please note that this result is obtained under the assumption of no frequency offset. In the presence of a small frequency offset, the variance could be slightly larger.

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