# Design and analysis of Add-drop filter with circular cavity using FD-BPM 

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#### Abstract

Optical ring resonators are complex photonic devices that are used as channel dropping filters. Various numerical methods are developed for the analysis of ring resonators. Beam propagation method is one of the simplest and computationally efficient method that can be used for the analysis of complex optical waveguides. This paper presents the possible application of Explicit Finite Difference Beam Propagation Method[EFD-BPM] for the analysis of circular cavity resonator. Courant stability condition for selecting the time step along with Perfectly matched layer are applied to the computational window. Simulation results shows that for 1550 nm wavelength, maximum filtering performance is obtained for cavity with diameter 40 micrometer.


Keywords-FDTD; BPM; FD-BPM; PML.

## I. InTRODUCTION

With the ever increasing requirements for faster data rates, the need of wavelength dropping becomes an essential parameter in dense wavelength division multiplexing. Circular cavity ring resonators are successfully used as channel dropping filters in optical communication [10]. The guided wavelength through the primary waveguide gets coupled in to the circular cavity by the phenomenon of evanescence coupling. The coupled energy in the cavity builds up on multiple round trips. This energy gets coupled into the secondary waveguide and thus the wavelength dropping is done. The amount of energy dropped depends upon the size of the cavity, distance between the waveguides and the cavity and finally the coupling coefficients. Numerical methods like Finite difference time domain[FDTD][9], Method of lines [MoL] Finite Element analysis[FEM][6] etc. are used for analysis of complex waveguide structures. Most of these methods are computationally inefficient and inaccurate or they require large computational window, making then unsuitable for handling complex problems. Compared to these methods BPM[1]-[3], is easiest to implement, highly efficient \& accurate. The memory requirements are less and it can be used for the analysis of straight as well as bent waveguides. BPM has been used for modeling electro-optic modulators [11], star couplers[12][13], y-junctions[7], polarization splitters, multi-mode interference devices, waveguide polarizers etc.

## II. Finite Difference Beam Propagation Method

The E \& H field components of wave propagating in optical waveguide are given by Maxwell's equations.

$$
\begin{align*}
& \nabla \times E=-\mu_{0} \frac{\partial H}{\partial t}  \tag{1}\\
& \nabla \times H=\epsilon_{0} \epsilon_{r} \frac{\partial E}{\partial t} \tag{2}
\end{align*}
$$

These equations can be represented completely in terms of either magnetic field or electric field. The E field representation is given by taking curl of equation (1)

$$
\begin{equation*}
\nabla \times \nabla \times E=\nabla \times\left(-\mu_{0} \frac{\partial H}{\partial t}\right)=-\mu_{0} \frac{\partial}{\partial t}\{\nabla \times H\} \tag{3}
\end{equation*}
$$

Using vector identity for the double curl of E field vector.

$$
\begin{equation*}
\nabla \times(\nabla \times E)=\nabla(\nabla \cdot E)-\nabla^{2} E \tag{4}
\end{equation*}
$$

Substituting equation (2) \& using the above vector identity equation (3) becomes

$$
\begin{equation*}
\nabla(\nabla . E)-\nabla^{2} E=-\mu_{0} \frac{\partial}{\partial t}\left\{\epsilon_{0} \epsilon_{r} \frac{\partial E}{\partial t}\right\}=-\mu_{0} \epsilon_{0} \epsilon_{r} \frac{\partial^{2} E}{\partial t^{2}} \tag{5}
\end{equation*}
$$

For the source free field the divergence equation can be expressed as

$$
\begin{equation*}
\nabla \cdot D=\left(\epsilon_{r} E\right)=E \cdot \nabla \epsilon_{r}+\epsilon_{r} \nabla E=0 \tag{6}
\end{equation*}
$$

or

$$
\begin{equation*}
\nabla . E=-E \frac{\nabla \epsilon_{r}}{\epsilon_{r}} \tag{7}
\end{equation*}
$$

substituting above equation (7) into equation (5)

$$
\begin{equation*}
\nabla^{2} \cdot E+\nabla\left(E \cdot \frac{\nabla \epsilon_{r}}{\epsilon_{r}}\right)=\mu_{0} \epsilon_{0} \epsilon_{r} \frac{\partial^{2} E}{\partial t^{2}} \tag{8}
\end{equation*}
$$

Electric and magnetic fields components for a monochromatic field oscillating at angular frequency $\omega$,can be written as

$$
\left[\begin{array}{c}
E(r, t)  \tag{9}\\
H(r, t)
\end{array}\right]=\left[\begin{array}{c}
E(r, t) \\
H(r, t)
\end{array}\right] e^{(j \omega t)}
$$

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The second derivative in equation(8) with respect to $t$ can be replaced by $-\omega^{2}$, which after elimination of time dependence term $e^{j \omega t}$ results in vectorial equation

$$
\begin{equation*}
\nabla^{2} \cdot E+\nabla\left(E \cdot \frac{\nabla n^{2}}{n^{2}}\right)+k_{0}^{2} n^{2} E=0 \tag{10}
\end{equation*}
$$

In order to find the solution of equation(10), the electric filed component has to be solved in three directions $\mathrm{x}, \mathrm{y} \& \mathrm{z}$ simultaneously, which makes the problem very complex. For homogeneous medium the square terms in the equation can be neglected leading to

$$
\begin{equation*}
\nabla^{2} \cdot E+k_{0}^{2} n^{2} E=0 \tag{11}
\end{equation*}
$$

Which is scalar Helmholtz equation for propagation of wave in both directions of $z$. This equation can be transformed into one dimensional equation in Cartesian coordinate system as

$$
\begin{equation*}
\frac{\partial^{2} E}{\partial x^{2}}+\frac{\partial^{2} E}{\partial y^{2}}+\frac{\partial^{2} E}{\partial z^{2}}+k_{0}^{2} n^{2} E(x, y, z)=0 \tag{12}
\end{equation*}
$$

Replacing $E(x, y, z)=\Phi(x, y, z) e^{\left(-j k_{0} n_{0}\right)}$, rapidly changing field along the z -direction equation(12) becomes

$$
\begin{align*}
& \frac{\partial^{2} \Phi}{\partial x^{2}} e^{\left(-j k_{0} n_{0} z\right)}+\frac{\partial^{2} \Phi}{\partial y^{2}} e^{\left(-j k_{0} n_{0} z^{2}\right)} \\
& +\frac{\partial^{2} \Phi}{\partial z^{2}} e^{\left(-j k_{0} n_{0} z\right)}-2 j k_{0} n_{0} \frac{\partial \Phi}{\partial z} e^{\left(-j k_{0} n_{0} z\right)}  \tag{13}\\
& -k_{0}^{2} n_{0}^{2} \Phi e^{\left(-j k_{0} n_{0} z\right)}+k_{0}^{2} n^{2} \Phi e^{\left(-j k_{0} n_{0} z\right)}=0
\end{align*}
$$

Using slowly varying envelope approximation along propagation direction leads

$$
\begin{equation*}
\left|\frac{\partial^{2} \Phi}{\partial z^{2}}\right| \ll\left|2 k_{0} n_{0} \frac{\partial \Phi}{\partial z}\right| \tag{14}
\end{equation*}
$$

which can be simplified to give the well-known Fresnel's equation

$$
\begin{equation*}
2 j k_{0} n_{0} \frac{\partial \Phi}{\partial z}=\frac{\partial^{2} \Phi}{\partial x^{2}}+\frac{\partial^{2} \Phi}{\partial y^{2}}+k_{0}^{2}\left(n^{2}-n_{0}^{2}\right) \Phi \tag{15}
\end{equation*}
$$

It can be written as

$$
j \frac{\partial \Phi(x, y, z)}{\partial z}=G \Phi(x, y, z)
$$

For the small step size of $\Delta z$ along z direction the solution of the above equation can be written as

$$
\begin{equation*}
\Phi(x, y, z+\Delta z)=e^{(-j \Delta z G)} \Phi(x, y, z) \tag{16}
\end{equation*}
$$

The central difference approximation can be used to replace the partial derivatives given below which are accurate up to second order

$$
\begin{align*}
& \frac{\partial^{2} F(\gamma)}{\partial \gamma^{2}}=\frac{F(\gamma-\Delta \gamma)-2 F(\gamma)+F(\gamma+\Delta \gamma)}{\Delta \gamma^{2}}  \tag{17}\\
& \frac{\partial F(z)}{\partial z}=\frac{F(z+\Delta z)-F(z-\Delta z)}{\Delta z} \tag{18}
\end{align*}
$$

Where i \& m are discretized coordinates along x and y direction. equation (20) is known as explicit finite difference BPM[7][8][22]. The stability of this algorithm depends on the step size along z direction.

$$
\begin{equation*}
\Delta z<2 k_{0} n_{0}\left[\frac{4}{\Delta x^{2}}+\frac{4}{\Delta y^{2}}+k_{0}\left|n_{i, m}^{2}-n_{0}^{2}\right|_{\max }\right]^{-1} \tag{19}
\end{equation*}
$$

## III. RING RESONATOR WITH CIRCULAR CAVITY

A ring resonator is embedded between two parallel adjacent waveguides. A wavelength propagated through one of the waveguide gets coupled to the ring, if it satisfies the resonance condition given by $n_{e f f} L=m \lambda$.


Fig. 1 Power coupled in secondary wave guide at the time of resonance


Fig. 2 Power coupled in secondary wave guide at the time of Off-resonance

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## IV. Observations

In this analysis instead of cavity length 'L', observations are taken for different cavity inner diameter and the separation distance of cavity from the waveguide. While taking the observations, keeping the diameter constant, gap distance is varied from 5-10 $\mu m$ and its effect on \% power dropped into secondary waveguide is measured.

| Table1. Simulations Results |  |  |  |
| :---: | :---: | :---: | :---: |
| S. <br> No. | Ring <br> diameter <br> in $\mu m$ | Gap <br> distance $(\mu m)$ | Average <br> dropped <br> power(\% |
| 1 | 55 | 10 | 70 |
|  | 50 | 05 | 74 |
| 2 |  | 74 |  |
| 3 | 45 | 05 | 78 |
|  |  | 10 | 80 |
| 4 | 40 | 05 | 82 |
|  |  | 10 | 81 |
|  |  | 05 | 84 |

## V. CONCLUSION

Observations obtained from the simulations shows that, the power coupling in secondary waveguide increases as the diameter of cavity is reduced. However, as the cavity diameter is reduced, the coupling region between cavity and waveguide increases causing spurious field to escape thereby increasing losses which decreases the amount of power coupled in the drop port. It has been also observed that, with the increase in diameter of the cavity, energy buildup requires more round trips considered to the cavity with smaller diameter. In both the cases, energy coupling is maximum for smaller the gap distance $=5 \mu \mathrm{~m}$.

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