

ex. Is  $f(x) = \begin{cases} x^2 & x < 0 \\ x & x \geq 0 \end{cases}$

Differentiable at  $x=0$   $f(0)=0$

Der. from left  $\lim_{x \rightarrow 0^-} \frac{x^2 - 0}{x - 0} = \lim_{x \rightarrow 0^-} x = 0$

from right  $\lim_{x \rightarrow 0^+} \frac{x - 0}{x - 0} = \lim_{x \rightarrow 0^+} 1 = 1 \neq 0$

Since these are different then  $f'$  DNE exist at  $x=0$

ex2 Is  $f(x) = \begin{cases} 5-2x & x \geq 1 \\ 4-x^2 & x < 1 \end{cases}$  7-8  
 Cont<sup>s</sup> and Diff  
 at  $x=1$

Cont<sup>s</sup>

①  $\lim_{x \rightarrow 1^-} 4-x^2 = 3$  so  $\lim_{x \rightarrow 1} f = 3$   
 $\lim_{x \rightarrow 1^+} 5-2x = 3$

②  $f(1) = 5-2(1) = 3 = \lim_{x \rightarrow 1} f$

so yes cont<sup>s</sup> at  $x=1$

Diff check

$$\lim_{x \rightarrow 1^-} \frac{4-x^2-3}{x-1} = \lim_{x \rightarrow 1^-} \frac{1-x^2}{x-1} = - \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{(x-1)}$$

$$= -2$$

$$\lim_{x \rightarrow 1^+} \frac{5-2x-3}{x-1} = \lim_{x \rightarrow 1^+} \frac{2-2x}{x-1}$$

$$= \lim_{x \rightarrow 1} \frac{-2(x-1)}{x-1} = -2 \quad \therefore \text{same}$$

$$f'(1) = -2$$