Neural/Mathematical Connections in *Foundations for College Mathematics 3e*

Foundations for College Mathematics 3e

I have been especially excited about the potential for neuroscience to give us direction in remedial algebra education. Since traditional algebra textbooks are a huge business, I don't see changes in their equation-solving approach in the light of scientific research because of the cost of the change and because most authors are not aware of brain function applied to teaching/learning of algebra. So I am focusing on an audience where there is great potential for providing a very positive learning experience showcasing how neuroscience can improve student success. Below is a brief description of how and why I used neuroscientific-based research on core brain function in *Foundations 3e*.

After reading the TOC and reviewing the homework pages, you realize that *Foundations 3e* is not like other remedial algebra textbooks (or traditional algebra textbooks). But *Foundations 3e* is not different just to be different. Its structure is designed around core functionality of the brain. This does NOT include how the brain memorizes, because memorization memory is not dependable and requires maintenance in the form of continuous review. At the core of understanding and long-term memory is brain function that is automatic and ubiquitous – neural associations (like connections), visual processing, and pattern recognition. Long-term memory, recall, and understanding are processed through neural associations. To create a neural association between two ideas/concepts/procedures, you must simultaneously present the brain with both. Actually, simultaneous means within 1/5 second - the length of time required for the physical/chemical process to take place within the synapses. The process of creating neural associations is AUTOMATIC in the brain; so when left to create associations on its own, it can result in poor mathematical memory and understanding because the connected neuronal circuits are likely not both mathematical in nature nor are they likely meaningful. The unstructured and automatic association creation usually becomes severed because they are commonly not ever used again; therefore, students are left with no mechanism for recalling the mathematics taught except memorization. In *Foundations 3e*, I have capitalized on basic brain function to make the neural circuits representing two mathematical ideas become neurally associated (connected), as opposed to a math concept being connected to whatever may be going through the student's brain during class. The primary reason for controlling connections is that long-term memory RECALL is processed through associations (connections). The more connections to a math concept the more likely students will be able to recall. Another important aspect in *Foundations 3e* is connecting math concepts to simple real-world contexts because the brain must couch abstract concepts in concrete terms before it can understand the abstraction. By controlling the daily connections, the teacher can use them in teaching algebra. Further, for associations to survive, they must be used or re-visited like in *Foundations 3e*. Using core brain function is powerful pedagogy.

On an un-related note: Research is clear that students get better grades when they know how their brain works (learns) when compared to students given an intensive "how-to-study" course. So you will find brain information on 17 pages of *Foundations 3e*. The pages are devoted to describing a wide variety of basic brain function. In addition, the section exercise sets are designed to remind students about the brain's involvement in their education – see page viii of *Foundations 3e*.

Algebra Education

As I understand algebra education, the last 60 years of reform has been about making refinement after refinement, or simply "dumbing down" the algebra curriculum – at both the high school and college levels. Yet there have been NO national improvements in remediation rates for college-intending high school graduates and NO improvements in passing rates in college developmental education programs. The real problem is that educators originally developed a curriculum without knowing how the brain functions. Without knowledge of brain function, the design of the traditional equation-solving curriculum was/is much like designing a glove without knowing how a hand works. It turns out the equation-based remedial algebra curriculum is not a very good fit from a learning perspective (memory creation and understanding abstract ideas), but it did satisfy the needs of the engineers and scientist who had to solve equations. As you might expect, the percent of students going into the STEM fields was small and algebra curriculum success was restricted to those interested in science/engineering/mathematics. In the traditional equation-solving curriculum/pedagogy, one might think that equation is the common theme and every concept/procedure is connected in a neuroscientific way through neural associations. But for example, do we teach students to factor by connecting it to solving a quadratic equation? Do we connect the teaching of addition of radical expressions to solving radical equations? Is, for example, the $\sqrt{7}$ ever connected to the source function $y = \sqrt{x}$? Etc. Teaching algebra as a series of unconnected "topics" is very hard on long-term memory to the point where it is almost non-existent, except in the case where the student is interested in algebra – like the STEM students of the mid-1900's. But now, we expect all students to learn algebra, yet we did not change the curriculum to match the new audience.

In all this time, no one has restructured the remedial algebra curriculum to capitalize on neuroscience so that we can develop real understanding and long-term memory of the algebra taught – until now. Unfortunately, this new remedial algebra curriculum designed with function as the common connection (capitalizing on neuroscience) is a new experience for teachers and can be unsettling. As a result, many choose to stay the course and argue that we are making progress with the mainstream algebra curriculum. But this thinking hasn't improved the success rate for remedial students in the last 50 years, and the remediation rate is still growing. Another problem with this thinking is that math teachers were not likely remedial students, and there are differences in brain processing between someone who likes and is interested in math and those who struggle with it and do not like it. (It has to do with the internal neuron/neurotransmitter-based value system.) Teachers often think that if today's students would study more-like they did, students would succeed. But studying more as a remedial student can actually cause more difficulties because they often study algebra based on their misconceptions, misunderstandings, and false memories from their initial experiences in algebra. *Foundations 3e* provides a major break with the misconceptions developed by students using the "topics" based algebra curriculum.

Mathematical standards for algebra have been around since 1989 for high school and 1995 for the first two years of college. They are based on the traditional equation-solving algebra curriculum. What we know is that teaching to the standards has not decreased the number of high school students needing remediation nor has it improved graduation rates and passing rates in college. But the issue for adopting a curriculum using a function approach is that we think it will not satisfy the conditions of the current standards. While the sequencing may be different, *Foundations 3e* is still all about algebra. I guess the real issue is why we would want to replicate a curriculum that has helped to produce the level of failure as has the traditional equation-based curriculum. Remedial students need something different.

Analysis by Chapter

Neuroscience tells us that appropriate neurons release dopamine upon being presented with a concept/idea. The release of dopamine signals the learning circuits that the external event is good (dopamine in this situation acts as a reward to the brain) and it should be alert and pay attention now and to similar events (concepts) in the future. The brain CANNOT learn without the action caused by dopamine. Therefore, Chapters 2 and 3 focus on concepts. Once the brain understands that algebra is about ideas/concepts, we can teach processes and procedures at a later time through connections. If processes and procedures are taught before the related concepts, they are commonly assigned a low value in the average brain (little or no release of dopamine), resulting in poor memory with little understanding. It is sad that many textbooks do just this – teach procedures before concepts, or only teach procedures to be memorized.

Chapter Two uses contextual data sets (functions in numeric form) that can be modeled by linear, quadratic, exponential, absolute value, or square root functions. The data sets are used to teach function representation and function behaviors. But it is more than that. The data sets are contextual situations that provide an emotional tag AND they become the first link in a chain of neural associations that will help understanding and long-term memory. Emotional tags are also a tool that helps students understand concepts more quickly. Chapter Three, uses the same functions, but since symbolic form is developed in Section 2.2, Chapter Three focuses on behaviors of these functions starting with symbolic form instead of numeric form, and parameter-behavior connections are developed. A variety of functions are included because they are accessible through simple contextual situations that are used to teach them. Conceptually, function representation and behaviors flow naturally from the contexts. The pattern-building activity used to create the symbolic form of a function is the same no matter the function type. Unfortunately, my early experience with traditionally educated remedial students is that they think every relationship (functional) is linear. Coincidentally, the nearly simultaneous work with a variety of functions solves this problem early on.

Chapter Four begins with the traditional symbolic manipulations on the functions developed in Chapters 2 & 3. There are major differences found in *Foundations 3e*. The symbols to be operated upon have attached meaning derived from the contextual situations in Chapters 2 & 3. The symbols have connections through function representation and behaviors. The concept of symbols is developed in Chapter 2 (through the pattern-building activity using the list editor of the TI-84), so procedural operations can follow with a connection. Arithmetic operations with symbols are developed as operations on functions and they are presented using visualizations and/or pattern-building activities to a generalization. (See the Concept Quizzes and Investigations.) Homework exercises reinforce the ability to generalize. Generalizing is the primary mode of thinking, so operations are taught using this core brain function. You will notice that the process of factoring starts in Chapters 2 & 3 in the concepts quizzes centered on the behavior of function zeros. The concept quizzes continue in Chapter Four using pattern building to develop a connection between the parameters of $(cx + d)$ and the zero $–d/c$. Students use the parameter-behavior connections to relate the zero(s) of a factorable polynomial function found with trace or the zero finder to the binomial factors of the form $(cx + d)$. This connection is then used to teach the pencil-and-paper method of factoring. The process capitalizes on distributed learning, visual processing, pattern building, and connections through function behaviors-all brain friendly actions. Note: visual processing in NOT related to the notion that some students are “visual learners.”

Equations provide another opportunity to apply core brain function in the teaching/learning process. Solving equations is found in the chapters: 6, 7, 8, 9, 10, 12, 13, and 14. Equations are directly related to functions so we have an immediate connection. Chapter Six focuses on the concept of a solution and roots; and then proceeds to find roots of equations in a visual manner using function representation. The four visual/function based methods become available for use in the remainder of the text, and the pencil-and-paper methods are derived from the function-based methods. Students seem to prefer the zeros method for solving equations. This takes us directly back to Chapters 2 & 3 where they learned how to find zeros of a function.

In the traditional algebra curriculum, you usually find a chapter on function which includes function notation. This is just another example of a curriculum based on unrelated topics because, for example, function notation is never used again in the text. But in *Foundations 3e*, every chapter is about some aspect of function – including the use of function notation. Function is a common theme connecting every concept and procedure. No concept or procedure stands alone. As a result, we make great gains on long-term memory, recall, and mathematical understanding.

Chapters 5, 7, 8, 9, 10, and 14 have a common structure, and each represents a final analysis of the related function which is a continuation of discussions found in Chapters 2 & 3. Each chapter starts with a section on the analysis of a particular function. This section builds on (and connects to) what students learned from Chapters 2 & 3 and most include a contextual situation centered on the chapter function. The section continues with geometric transformations and further refinements in the general characteristics of the function – including parameter-behavior connections. This section is followed by symbol manipulation sections, then a section on solving equations, and ending with applications of the chapter function. This structure allows symbol manipulation, equation solving and applications to be connected through function. Applications no longer seem to be a major point of difficulty like in the traditional curriculum because all of the contextual situations have prepared students for what looks like just more contextual situations. The chapter structure capitalizes on distributed learning, visualizations, pattern building, and connections through function behaviors-all brain friendly actions.

Graphing Calculator

The graphing calculator provides the dynamic visualizations that neuroscience tells us are used by the brain to help it understand the mathematics and to pay attention because of the motion on the screen. Chapters 2 & 3 provide the opportunity for teaching the technology. As each section unfolds, the necessary functionality of the graphing calculator is taught. When you use the 83/84 calculator data programs, or the Nspire data documents, you create interactive data sets for every contextual data set in the textbook and ancillary activity book. These contextual situations are used because they provide a connection to the algebra, they add meaning, and they help students understand the algebra. One method of understanding comes from connecting the unknown (algebra) to something known (the context). Another method comes from visualizations of the algebra processed on the graphing calculator. An extremely important point about the philosophy and practically of *Foundations 3e* is that the graphing calculator is used as a teaching tool! It and function are used to teach the pencil and paper methods needed to succeed on high stakes tests, and in future courses in mathematics or in the connected fields.

In the process of teaching Chapters 2 & 3, students also learn about the parameter behavior connection which is used to model data found in the modeling projects, investigations, quizzes, and selected homework exercises – all graphing calculator active. This kind of distributed learning has strong implications for improving memory.

Visualizations

A perceived issue with *Foundations 3e* is that it does not contain glitzy visualizations as do traditional textbooks. There is a reason for this; besides the obvious benefit of a $45 book compared to a $90 book. Imagine what your students are thinking about when they look at a color image of, let's say a baseball diamond in a field with trees lining the horizon, players in the field and on bases, people in the stands, and the left-fielder is throwing a ball to home plate as a runner rounds third base. The actual mathematics is printed near the left-fielder as a couple lines of print, and the unknown "*x*" is near the bottom of the visualization. Scientists analyzed how long students looked at an item and what items they looked at. Back to the question, "What were the students looking at?" Well, the analysis does show some students were looking at the math – some for a very short period and a few for a little longer. However, by far, students were looking at everything not related to the mathematics. No one even looked at the *x*. What this suggests is that the images in a textbook should only contain the needed mathematics like in *Foundations 3e*. Why distract your students? Why not focus on mathematics that is also available dynamically on the graphing calculator? As a point of information, the researchers used graphics from many textbook publishers with the same results. This research may also be related to other studies telling us that humans in developed countries have maxed out their working memory capacity sacrificing learning and memory consolidation for the daily glitz on their cell phones, main-stream textbooks, etc.

Another important aspect of visualizations is that *Foundations 3e* places them at the beginning of a lesson when possible, or in conjunction with a contextual situation at the beginning of a lesson. There is a very good neuroscientific reason for this. Any attempt to reduce memory loss over time should take control of what happens in the early moments of memory formation, when encoding processes influence the fate of the memory. That is, during memory formation, some form of external action on our part assists with the formation of the memory by influencing the cellular and molecular activity. Since the brain is so attentive to visualizations, we start a lesson with visualization and end with symbolic work. The reverse does not provide the influential and early external action needed.

Pattern Building

Pattern generalizing is the pervasive mode of thought taking a giant lead over reasoning in the average brain. This basic fact gives rise to the considerable use of pattern-building activities in the concept quizzes, explorations, homework exercises, ***e***-lesson activities (ancillary), and content development. This basic brain function is also why reasoning should be placed after students have generalized a definition, law, property, procedure, etc. Neuroscience also tells us that when a student generalizes a definition, law, property, procedure, etc., the brain creates a long-term memory of the generalization. This is something that rote practice does not do.

Ancillaries

The ancillaries for *Foundations 3e* are not just the typical assessment processors, but are integral to learning. For example, most of the concept quizzes are used as tools to prime the brain for a mathematical concept, and as a teaching tool of the concept so that when direct teaching is used, the brain is more likely to be ready and is more likely to be open to direct teaching. Priming also makes the mathematical concept available to the unconscious learning neural circuitry common to all brains.

A visualization in a textbook is static but neuroscience tells us that without the visualization, many students will ignore the mathematical symbols. However, if the visualization is dynamic, then we also get the attention of the brain. So, a graphing calculator makes this happen and students are expected to graph functions and data for this very reason. The same is true for data sets (used as a contextual situation). They are available as interactive tools when students simply run the data programs available on the ancillary CD.

Foundations 3e and the ancillaries provide so many opportunities to engage the brain that we call this an enriched teaching/learning environment. Why is this important? Because the more ways we have of teaching or processing a concept/procedure, the more neural circuits we have that are connected (associated) to the concept/procedure. Why is this important? Because, long-term memory recall is processed through neural associations making the likelihood of recall much greater. The brain has the wonderful ability to take just a piece of a pattern (idea/concept/procedure) and is able to recall the entire version when it is well connected through neural associations.

Observations

Educational research appears to be mostly anecdotal, and does not hold to a high scientific standard. The National Mathematics Advisory Panel seemed to draw the same conclusion in their report. The NCTM *Mathematics Teachers* has not changed my views. Their research articles are mostly really nothing but drawing educational conclusions based on anecdotal educational research.

Having had early training in the science field as an undergraduate physics major; I have some appreciation for the scientific method and the requirements of scientific research. Neuroscience research results and interpretation by any scientists must be reproducible. Once confirmed and conclusions meticulously analyzed by various labs, we can be as certain as we can be that the research is correct. In neuroscience there is a common consensus about basic brain function. That is, there is no question that pattern-generalizing is a primary mode of thinking in the average brain. Scientists know that the brain has unconscious learning circuitry that can be primed. Scientific research has made storage and retrieval of long-term memory via neural associations fact-not conjecture. *Foundations 3e* is based on this research.

Reference Note

I have not obtained permissions to quote neuroscientists, but you can find many of my fully referenced and peer reviewed published papers on my home page or through a Google (Google Scholar, or Bing) search. So I simply argue in this paper using what neuroscientists have written without reference.