

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Differentiation

Exercise A, Question 1

Question:

F is the point with co-ordinates $(3, 9)$ on the curve with equation $y = x^2$.

(a) Find the gradients of the chords joining the point F to the points with coordinates:

(i) $(4, 16)$

(ii) $(3.5, 12.25)$

(iii) $(3.1, 9.61)$

(iv) $(3.01, 9.0601)$

(v) $(3 + h, (3 + h)^2)$

(b) What do you deduce about the gradient of the tangent at the point $(3, 9)$?

Solution:

$$\text{a (i) Gradient} = \frac{16 - 9}{4 - 3} = \frac{7}{1} = 7$$

$$\text{(ii) Gradient} = \frac{12.25 - 9}{3.5 - 3} = \frac{3.25}{0.5} = 6.5$$

$$\text{(iii) Gradient} = \frac{9.61 - 9}{3.1 - 3} = \frac{0.61}{0.1} = 6.1$$

$$\text{(iv) Gradient} = \frac{9.0601 - 9}{3.01 - 3} = \frac{0.0601}{0.01} = 6.01$$

$$\text{(v) Gradient} = \frac{(3 + h)^2 - 9}{(3 + h) - 3} = \frac{9 + 6h + h^2 - 9}{h} = \frac{6h + h^2}{h} = \frac{h(6 + h)}{h} = 6 + h$$

(b) The gradient at the point $(3, 9)$ is the value of $6 + h$ as h becomes very small, i.e. the gradient is 6.

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Exercise A, Question 2

Question:

G is the point with coordinates $(4, 16)$ on the curve with equation $y = x^2$.

(a) Find the gradients of the chords joining the point G to the points with coordinates:

(i) $(5, 25)$

(ii) $(4.5, 20.25)$

(iii) $(4.1, 16.81)$

(iv) $(4.01, 16.0801)$

(v) $(4 + h, (4 + h)^2)$

(b) What do you deduce about the gradient of the tangent at the point $(4, 16)$?

Solution:

$$(a) (i) \text{ Gradient} = \frac{25 - 16}{5 - 4} = \frac{9}{1} = 9$$

$$(ii) \text{ Gradient} = \frac{20.25 - 16}{4.5 - 4} = \frac{4.25}{0.5} = 8.5$$

$$(iii) \text{ Gradient} = \frac{16.81 - 16}{4.1 - 4} = \frac{0.81}{0.1} = 8.1$$

$$(iv) \text{ Gradient} = \frac{16.0801 - 16}{4.01 - 4} = \frac{0.0801}{0.01} = 8.01$$

$$(v) \text{ Gradient} = \frac{(4 + h)^2 - 16}{4 + h - 4} = \frac{16 + 8h + h^2 - 16}{h} = \frac{8h + h^2}{h} = \frac{h(8 + h)}{h} = 8 + h$$

(b) When h is small the gradient of the chord is close to the gradient of the tangent, and $8 + h$ is close to the value 8. So the gradient of the tangent at $(4, 16)$ is 8.

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Exercise B, Question 1

Question:

Find the derived function, given that $f(x)$ equals:

$$x^7$$

Solution:

$$f(x) = x^7$$

$$f'(x) = 7x^6$$

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Exercise B, Question 2

Question:

Find the derived function, given that $f(x)$ equals:

$$x^8$$

Solution:

$$f(x) = x^8$$

$$f'(x) = 8x^7$$

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Exercise B, Question 3

Question:

Find the derived function, given that $f(x)$ equals:

$$x^4$$

Solution:

$$f(x) = x^4$$

$$f'(x) = 4x^3$$

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Exercise B, Question 4

Question:

Find the derived function, given that $f(x)$ equals:

$$x^{\frac{1}{3}}$$

Solution:

$$f(x) = x^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}}$$

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Exercise B, Question 5

Question:

Find the derived function, given that $f(x)$ equals:

$$x^{\frac{1}{4}}$$

Solution:

$$f(x) = x^{\frac{1}{4}}$$

$$f'(x) = \frac{1}{4}x^{\frac{1}{4} - 1} = \frac{1}{4}x^{-\frac{3}{4}}$$

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Exercise B, Question 6

Question:

Find the derived function, given that $f(x)$ equals:

$$\sqrt[3]{x}$$

Solution:

$$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}}$$

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Exercise B, Question 7

Question:

Find the derived function, given that $f(x)$ equals:

$$x^{-3}$$

Solution:

$$f(x) = x^{-3}$$

$$f'(x) = -3x^{-3-1} = -3x^{-4}$$

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Exercise B, Question 8

Question:

Find the derived function, given that $f(x)$ equals:

$$x^{-4}$$

Solution:

$$f(x) = x^{-4}$$

$$f'(x) = -4x^{-4-1} = -4x^{-5}$$

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Exercise B, Question 9

Question:

Find the derived function, given that $f(x)$ equals:

$$\frac{1}{x^2}$$

Solution:

$$f(x) = \frac{1}{x^2} = x^{-2}$$

$$f'(x) = -2x^{-2-1} = -2x^{-3}$$

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Exercise B, Question 10

Question:

Find the derived function, given that $f(x)$ equals:

$$\frac{1}{x^5}$$

Solution:

$$f(x) = \frac{1}{x^5} = x^{-5}$$

$$f'(x) = -5x^{-5-1} = -5x^{-6}$$

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Exercise B, Question 11

Question:

Find the derived function, given that $f(x)$ equals:

$$\frac{1}{\sqrt[3]{x}}$$

Solution:

$$f(x) = \frac{1}{\sqrt[3]{x}} = x^{-\frac{1}{3}}$$

$$f'(x) = -\frac{1}{3}x^{-\frac{1}{3}-1} = -\frac{1}{3}x^{-\frac{4}{3}}$$

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Exercise B, Question 12

Question:

Find the derived function, given that $f(x)$ equals:

$$\frac{1}{\sqrt{x}}$$

Solution:

$$f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$$

$$f'(x) = -\frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-\frac{3}{2}}$$

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Exercise B, Question 13

Question:

Find the derived function, given that $f(x)$ equals:

$$\frac{x^2}{x^4}$$

Solution:

$$f(x) = \frac{x^2}{x^4} = x^{2-4} = x^{-2}$$

$$f'(x) = -2x^{-2-1} = -2x^{-3}$$

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Exercise B, Question 14

Question:

Find the derived function, given that $f(x)$ equals:

$$\frac{x^3}{x^2}$$

Solution:

$$f(x) = \frac{x^3}{x^2} = x^{3-2} = x^1$$

$$f'(x) = 1x^{1-1} = 1x^0 = 1$$

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Exercise B, Question 15

Question:

Find the derived function, given that $f(x)$ equals:

$$\frac{x^6}{x^3}$$

Solution:

$$f(x) = \frac{x^6}{x^3} = x^{6-3} = x^3$$

$$f'(x) = 3x^2$$

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Exercise B, Question 16

Question:

Find the derived function, given that $f(x)$ equals:

$$x^3 \times x^6$$

Solution:

$$f(x) = x^3 \times x^6 = x^{3+6} = x^9$$

$$f'(x) = 9x^8$$

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Exercise B, Question 17

Question:

Find the derived function, given that $f(x)$ equals:

$$x^2 \times x^3$$

Solution:

$$f(x) = x^2 \times x^3 = x^{2+3} = x^5$$

$$f'(x) = 5x^4$$

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Exercise B, Question 18

Question:

Find the derived function, given that $f(x)$ equals:

$$x \times x^2$$

Solution:

$$f(x) = x \times x^2 = x^{1+2} = x^3$$

$$f'(x) = 3x^2$$

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Exercise C, Question 1

Question:

Find $\frac{dy}{dx}$ when y equals:

(a) $2x^2 - 6x + 3$

(b) $\frac{1}{2}x^2 + 12x$

(c) $4x^2 - 6$

(d) $8x^2 + 7x + 12$

(e) $5 + 4x - 5x^2$

Solution:

(a) $y = 2x^2 - 6x + 3$

$$\frac{dy}{dx} = 2(2x) - 6(1) + 0 = 4x - 6$$

(b) $y = \frac{1}{2}x^2 + 12x$

$$\frac{dy}{dx} = \frac{1}{2}(2x) + 12(1) = x + 12$$

(c) $y = 4x^2 - 6$

$$\frac{dy}{dx} = 4(2x) - 0 = 8x$$

(d) $y = 8x^2 + 7x + 12$

$$\frac{dy}{dx} = 8(2x) + 7 + 0 = 16x + 7$$

(e) $y = 5 + 4x - 5x^2$

$$\frac{dy}{dx} = 0 + 4(1) - 5(2x) = 4 - 10x$$

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Exercise C, Question 2

Question:

Find the gradient of the curve whose equation is

(a) $y = 3x^2$ at the point $(2, 12)$

(b) $y = x^2 + 4x$ at the point $(1, 5)$

(c) $y = 2x^2 - x - 1$ at the point $(2, 5)$

(d) $y = \frac{1}{2}x^2 + \frac{3}{2}x$ at the point $(1, 2)$

(e) $y = 3 - x^2$ at the point $(1, 2)$

(f) $y = 4 - 2x^2$ at the point $(-1, 2)$

Solution:

(a) $y = 3x^2$

$$\frac{dy}{dx} = 6x$$

At the point $(2, 12)$, $x = 2$.

Substitute $x = 2$ into the gradient expression $\frac{dy}{dx} = 6x$ to give

$$\text{gradient} = 6 \times 2 = 12.$$

(b) $y = x^2 + 4x$

$$\frac{dy}{dx} = 2x + 4$$

At the point $(1, 5)$, $x = 1$.

Substitute $x = 1$ into $\frac{dy}{dx} = 2x + 4$ to give

$$\text{gradient} = 2 \times 1 + 4 = 6$$

(c) $y = 2x^2 - x - 1$

$$\frac{dy}{dx} = 4x - 1$$

At the point $(2, 5)$, $x = 2$.

Substitute $x = 2$ into $\frac{dy}{dx} = 4x - 1$ to give

$$\text{gradient} = 4 \times 2 - 1 = 7$$

(d) $y = \frac{1}{2}x^2 + \frac{3}{2}x$

$$\frac{dy}{dx} = x + \frac{3}{2}$$

At the point $(1, 2)$, $x = 1$.

Substitute $x = 1$ into $\frac{dy}{dx} = x + \frac{3}{2}$ to give

$$\text{gradient} = 1 + \frac{3}{2} = 2\frac{1}{2}$$

(e) $y = 3 - x^2$

$$\frac{dy}{dx} = -2x$$

At $(1, 2)$, $x = 1$.

Substitute $x = 1$ into $\frac{dy}{dx} = -2x$ to give

$$\text{gradient} = -2 \times 1 = -2$$

(f) $y = 4 - 2x^2$

$$\frac{dy}{dx} = -4x$$

At $(-1, 2)$, $x = -1$.

Substitute $x = -1$ into $\frac{dy}{dx} = -4x$ to give

$$\text{gradient} = -4 \times -1 = +4$$

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Exercise C, Question 3

Question:

Find the y -coordinate and the value of the gradient at the point P with x -coordinate 1 on the curve with equation $y = 3 + 2x - x^2$.

Solution:

$$y = 3 + 2x - x^2$$

When $x = 1$, $y = 3 + 2 - 1$
 $\Rightarrow y = 4$ when $x = 1$

Differentiate to give

$$\frac{dy}{dx} = 0 + 2 - 2x$$

When $x = 1$, $\frac{dy}{dx} = 2 - 2$

$$\Rightarrow \frac{dy}{dx} = 0 \text{ when } x = 1$$

Therefore, the y -coordinate is 4 and the gradient is 0 when the x -coordinate is 1 on the given curve.

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Exercise C, Question 4

Question:

Find the coordinates of the point on the curve with equation $y = x^2 + 5x - 4$ where the gradient is 3.

Solution:

$$y = x^2 + 5x - 4$$

$$\frac{dy}{dx} = 2x + 5$$

$$\text{Put } \frac{dy}{dx} = 3$$

$$\text{Then } 2x + 5 = 3$$

$$\Rightarrow 2x = -2$$

$$\Rightarrow x = -1$$

Substitute $x = -1$ into $y = x^2 + 5x - 4$:

$$y = (-1)^2 + 5(-1) - 4 = 1 - 5 - 4 = -8$$

Therefore, $(-1, -8)$ is the point where the gradient is 3.

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Exercise C, Question 5

Question:

Find the gradients of the curve $y = x^2 - 5x + 10$ at the points A and B where the curve meets the line $y = 4$.

Solution:

The curve $y = x^2 - 5x + 10$ meets the line $y = 4$ when

$$x^2 - 5x + 10 = 4$$

$$x^2 - 5x + 6 = 0$$

$$(x - 3)(x - 2) = 0$$

$$x = 3 \text{ or } x = 2$$

The gradient function for the curve is given by

$$\frac{dy}{dx} = 2x - 5$$

$$\text{when } x = 3, \frac{dy}{dx} = 2 \times 3 - 5 = 1$$

$$\text{when } x = 2, \frac{dy}{dx} = 2 \times 2 - 5 = -1$$

So the gradients are -1 and 1 at $(2, 4)$ and $(3, 4)$ respectively.

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Exercise C, Question 6

Question:

Find the gradients of the curve $y = 2x^2$ at the points C and D where the curve meets the line $y = x + 3$.

Solution:

The curve $y = 2x^2$ meets the line $y = x + 3$ when

$$2x^2 = x + 3$$

$$2x^2 - x - 3 = 0$$

$$(2x - 3)(x + 1) = 0$$

$$x = 1.5 \text{ or } -1$$

The gradient of the curve is given by the equation $\frac{dy}{dx} = 4x$.

The gradient at the point where $x = -1$ is $4 \times -1 = -4$.

The gradient at the point where $x = 1.5$ is $4 \times 1.5 = 6$.

So the gradient is -4 at $(-1, 2)$ and is 6 at $(1.5, 4.5)$.

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Exercise D, Question 1

Question:

Use standard results to differentiate:

(a) $x^4 + x^{-1}$

(b) $\frac{1}{2}x^{-2}$

(c) $2x^{-\frac{1}{2}}$

Solution:

(a) $f(x) = x^4 + x^{-1}$
 $f'(x) = 4x^3 + (-1)x^{-2}$

(b) $f(x) = \frac{1}{2}x^{-2}$
 $f'(x) = \frac{1}{2}(-2)x^{-3} = -x^{-3}$

(c) $f(x) = 2x^{-\frac{1}{2}}$
 $f'(x) = 2 \left(-\frac{1}{2} \right) x^{-1\frac{1}{2}} = -x^{-\frac{3}{2}}$

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Exercise D, Question 2

Question:

Find the gradient of the curve with equation $y = f(x)$ at the point A where:

(a) $f(x) = x^3 - 3x + 2$ and A is at $(-1, 4)$

(b) $f(x) = 3x^2 + 2x^{-1}$ and A is at $(2, 13)$

Solution:

(a) $f(x) = x^3 - 3x + 2$

$$f'(x) = 3x^2 - 3$$

At $(-1, 4)$, $x = -1$.

Substitute $x = -1$ to find $f'(-1) = 3(-1)^2 - 3 = 0$

Therefore, gradient = 0.

(b) $f(x) = 3x^2 + 2x^{-1}$

$$f'(x) = 6x + 2(-1)x^{-2} = 6x - 2x^{-2}$$

At $(2, 13)$, $x = 2$.

$$f'(2) = 6(2) - 2(2)^{-2} = 12 - \frac{2}{4} = 11 \frac{1}{2}$$

Therefore, gradient = $11 \frac{1}{2}$.

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Exercise D, Question 3

Question:

Find the point or points on the curve with equation $y = f(x)$, where the gradient is zero:

(a) $f(x) = x^2 - 5x$

(b) $f(x) = x^3 - 9x^2 + 24x - 20$

(c) $f(x) = x^{\frac{3}{2}} - 6x + 1$

(d) $f(x) = x^{-1} + 4x$

Solution:

(a) $f(x) = x^2 - 5x$

$f'(x) = 2x - 5$

When gradient is zero, $f'(x) = 0$.

$$\Rightarrow 2x - 5 = 0$$

$$\Rightarrow x = 2.5$$

As $y = f(x)$, $y = f(2.5)$ when $x = 2.5$.

$$\Rightarrow y = (2.5)^2 - 5(2.5) = -6.25$$

Therefore, $(2.5, -6.25)$ is the point on the curve where the gradient is zero.

(b) $f(x) = x^3 - 9x^2 + 24x - 20$

$f'(x) = 3x^2 - 18x + 24$

When gradient is zero, $f'(x) = 0$.

$$\Rightarrow 3x^2 - 18x + 24 = 0$$

$$\Rightarrow 3(x^2 - 6x + 8) = 0$$

$$\Rightarrow 3(x - 4)(x - 2) = 0$$

$$\Rightarrow x = 4 \text{ or } x = 2$$

As $y = f(x)$, $y = f(4)$ when $x = 4$.

$$\Rightarrow y = 4^3 - 9 \times 4^2 + 24 \times 4 - 20 = -4$$

Also $y = f(2)$ when $x = 2$.

$$\Rightarrow y = 2^3 - 9 \times 2^2 + 24 \times 2 - 20 = 0$$

Therefore, at $(4, -4)$ and at $(2, 0)$ the gradient is zero.

(c) $f(x) = x^{\frac{3}{2}} - 6x + 1$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}} - 6$

When gradient is zero, $f'(x) = 0$.

$$\Rightarrow \frac{3}{2}x^{\frac{1}{2}} - 6 = 0$$

$$\Rightarrow x^{\frac{1}{2}} = 4$$

$$\Rightarrow x = 16$$

As $y = f(x)$, $y = f(16)$ when $x = 16$.

$$\Rightarrow y = 16^{\frac{3}{2}} - 6 \times 16 + 1 = -31$$

Therefore, at $(16, -31)$ the gradient is zero.

$$(d) f(x) = x^{-1} + 4x$$

$$f'(x) = -1x^{-2} + 4$$

For zero gradient, $f'(x) = 0$.

$$\Rightarrow -x^{-2} + 4 = 0$$

$$\Rightarrow \frac{1}{x^2} = 4$$

$$\Rightarrow x = \pm \frac{1}{2}$$

$$\text{When } x = \frac{1}{2}, y = f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^{-1} + 4\left(\frac{1}{2}\right) = 2 + 2 = 4$$

$$\text{When } x = -\frac{1}{2}, y = f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^{-1} + 4\left(-\frac{1}{2}\right) = -2 - 2 = -4$$

Therefore, $\left(\frac{1}{2}, 4\right)$ and $\left(-\frac{1}{2}, -4\right)$ are points on the curve where the gradient is zero.

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Exercise E, Question 1

Question:

Use standard results to differentiate:

(a) $2\sqrt{x}$

(b) $\frac{3}{x^2}$

(c) $\frac{1}{3x^3}$

(d) $\frac{1}{3}x^3(x-2)$

(e) $\frac{2}{x^3} + \sqrt{x}$

(f) $3\sqrt{x} + \frac{1}{2x}$

(g) $\frac{2x+3}{x}$

(h) $\frac{3x^2-6}{x}$

(i) $\frac{2x^3+3x}{\sqrt{x}}$

(j) $x(x^2-x+2)$

(k) $3x^2(x^2+2x)$

(l) $(3x-2)\left(4x + \frac{1}{x}\right)$

Solution:

(a) $y = 2\sqrt{x} = 2x^{\frac{1}{2}}$

$$\frac{dy}{dx} = 2 \left(\frac{1}{2} \right) x^{-\frac{1}{2}} = x^{-\frac{1}{2}}$$

$$(b) y = \frac{3}{x^2} = 3x^{-2}$$

$$\frac{dy}{dx} = 3(-2)x^{-3} = -6x^{-3}$$

$$(c) y = \frac{1}{3x^3} = \frac{1}{3}x^{-3}$$

$$\frac{dy}{dx} = \frac{1}{3}(-3)x^{-4} = -x^{-4}$$

$$(d) y = \frac{1}{3}x^3(x-2) = \frac{1}{3}x^4 - \frac{2}{3}x^3$$

$$\frac{dy}{dx} = \frac{4}{3}x^3 - \frac{2}{3} \times 3x^2 = \frac{4}{3}x^3 - 2x^2$$

$$(e) y = \frac{2}{x^3} + \sqrt{x} = 2x^{-3} + x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = -6x^{-4} + \frac{1}{2}x^{-\frac{1}{2}}$$

$$(f) y = \sqrt[3]{x} + \frac{1}{2x} = x^{\frac{1}{3}} + \frac{1}{2}x^{-1}$$

$$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{2}x^{-2}$$

$$(g) y = \frac{2x+3}{x} = \frac{2x}{x} + \frac{3}{x} = 2 + 3x^{-1}$$

$$\frac{dy}{dx} = 0 - 3x^{-2}$$

$$(h) y = \frac{3x^2-6}{x} = \frac{3x^2}{x} - \frac{6}{x} = 3x - 6x^{-1}$$

$$\frac{dy}{dx} = 3 + 6x^{-2}$$

$$(i) y = \frac{2x^3+3x}{\sqrt{x}} = \frac{2x^3}{x^{\frac{1}{2}}} + \frac{3x}{x^{\frac{1}{2}}} = 2x^2 \frac{1}{2} + 3x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 5x^{\frac{1}{2}} + 1.5x^{-\frac{1}{2}}$$

$$(j) y = x(x^2 - x + 2) = x^3 - x^2 + 2x$$

$$\frac{dy}{dx} = 3x^2 - 2x + 2$$

$$(k) y = 3x^2(x^2 + 2x) = 3x^4 + 6x^3$$

$$\frac{dy}{dx} = 12x^3 + 18x^2$$

$$(1) y = (3x - 2)\left(4x + \frac{1}{x}\right) = 12x^2 - 8x + 3 - \frac{2}{x} = 12x^2 - 8x + 3 - 2x^{-1}$$

$$\frac{dy}{dx} = 24x - 8 + 2x^{-2}$$

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Exercise E, Question 2

Question:

Find the gradient of the curve with equation $y = f(x)$ at the point A where:

(a) $f(x) = x(x + 1)$ and A is at $(0, 0)$

(b) $f(x) = \frac{2x-6}{x^2}$ and A is at $(3, 0)$

(c) $f(x) = \frac{1}{\sqrt{x}}$ and A is at $\left(\frac{1}{4}, 2\right)$

(d) $f(x) = 3x - \frac{4}{x^2}$ and A is at $(2, 5)$

Solution:

(a) $f(x) = x(x + 1) = x^2 + x$

$f'(x) = 2x + 1$

At $(0, 0)$, $x = 0$.

Therefore, gradient $= f'(0) = 1$

(b) $f(x) = \frac{2x-6}{x^2} = \frac{2x}{x^2} - \frac{6}{x^2} = \frac{2}{x} - 6x^{-2} = 2x^{-1} - 6x^{-2}$

$f'(x) = -2x^{-2} + 12x^{-3}$

At $(3, 0)$, $x = 3$.

Therefore, gradient $= f'(3) = -\frac{2}{3^2} + \frac{12}{3^3} = -\frac{2}{9} + \frac{12}{27} = \frac{2}{9}$

(c) $f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$f'(x) = -\frac{1}{2}x^{-\frac{3}{2}}$

At $\left(\frac{1}{4}, 2\right)$, $x = \frac{1}{4}$.

Therefore, gradient $= f'\left(\frac{1}{4}\right) = -\frac{1}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}} = -\frac{1}{2} \times 2^3 = -4$

(d) $f(x) = 3x - \frac{4}{x^2} = 3x - 4x^{-2}$

$f'(x) = 3 + 8x^{-3}$

At $(2, 5)$, $x = 2$.

Therefore, gradient $= f'(2) = 3 + 8(2)^{-3} = 3 + \frac{8}{8} = 4$.

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Differentiation

Exercise F, Question 1

Question:

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when y equals:

$$12x^2 + 3x + 8$$

Solution:

$$y = 12x^2 + 3x + 8$$

$$\frac{dy}{dx} = 24x + 3$$

$$\frac{d^2y}{dx^2} = 24$$

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Differentiation

Exercise F, Question 2

Question:

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when y equals:

$$15x + 6 + \frac{3}{x}$$

Solution:

$$y = 15x + 6 + \frac{3}{x} = 15x + 6 + 3x^{-1}$$

$$\frac{dy}{dx} = 15 - 3x^{-2}$$

$$\frac{d^2y}{dx^2} = 0 + 6x^{-3}$$

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Edexcel Modular Mathematics for AS and A-Level

Differentiation

Exercise F, Question 3

Question:

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when y equals:

$$9\sqrt{x} - \frac{3}{x^2}$$

Solution:

$$y = 9\sqrt{x} - \frac{3}{x^2} = 9x^{\frac{1}{2}} - 3x^{-2}$$

$$\frac{dy}{dx} = 4\frac{1}{2}x^{-\frac{1}{2}} + 6x^{-3}$$

$$\frac{d^2y}{dx^2} = -2\frac{1}{4}x^{-\frac{3}{2}} - 18x^{-4}$$

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Differentiation

Exercise F, Question 4

Question:

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when y equals:

$$(5x + 4)(3x - 2)$$

Solution:

$$y = (5x + 4)(3x - 2) = 15x^2 + 2x - 8$$

$$\frac{dy}{dx} = 30x + 2$$

$$\frac{d^2y}{dx^2} = 30$$

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Differentiation

Exercise F, Question 5

Question:

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when y equals:

$$\frac{3x+8}{x^2}$$

Solution:

$$y = \frac{3x+8}{x^2} = \frac{3x}{x^2} + \frac{8}{x^2} = \frac{3}{x} + 8x^{-2} = 3x^{-1} + 8x^{-2}$$

$$\frac{dy}{dx} = -3x^{-2} - 16x^{-3}$$

$$\frac{d^2y}{dx^2} = 6x^{-3} + 48x^{-4}$$

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Differentiation

Exercise G, Question 1

Question:

Find $\frac{d\theta}{dt}$ where $\theta = t^2 - 3t$

Solution:

$$\theta = t^2 - 3t$$

$$\frac{d\theta}{dt} = 2t - 3$$

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Differentiation

Exercise G, Question 2

Question:

Find $\frac{dA}{dr}$ where $A = 2 \pi r$

Solution:

$$A = 2 \pi r$$

$$\frac{dA}{dr} = 2 \pi$$

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Exercise G, Question 3

Question:

Find $\frac{dr}{dt}$ where $r = \frac{12}{t}$

Solution:

$$r = \frac{12}{t} = 12t^{-1}$$

$$\frac{dr}{dt} = -12t^{-2}$$

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Differentiation

Exercise G, Question 4

Question:

Find $\frac{dv}{dt}$ where $v = 9.8t + 6$

Solution:

$$v = 9.8t + 6$$

$$\frac{dv}{dt} = 9.8$$

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Differentiation

Exercise G, Question 5

Question:

Find $\frac{dR}{dr}$ where $R = r + \frac{5}{r}$

Solution:

$$R = r + \frac{5}{r} = r + 5r^{-1}$$

$$\frac{dR}{dr} = 1 - 5r^{-2}$$

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Differentiation

Exercise G, Question 6

Question:

Find $\frac{dx}{dt}$ where $x = 3 - 12t + 4t^2$

Solution:

$$x = 3 - 12t + 4t^2$$

$$\frac{dx}{dt} = 0 - 12 + 8t$$

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Differentiation

Exercise G, Question 7

Question:

Find $\frac{dA}{dx}$ where $A = x(10 - x)$

Solution:

$$A = x(10 - x) = 10x - x^2$$

$$\frac{dA}{dx} = 10 - 2x$$

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Differentiation

Exercise H, Question 1

Question:

Find the equation of the tangent to the curve:

(a) $y = x^2 - 7x + 10$ at the point $(2, 0)$

(b) $y = x + \frac{1}{x}$ at the point $\left(2, 2\frac{1}{2}\right)$

(c) $y = 4\sqrt{x}$ at the point $(9, 12)$

(d) $y = \frac{2x-1}{x}$ at the point $(1, 1)$

(e) $y = 2x^3 + 6x + 10$ at the point $(-1, 2)$

(f) $y = x^2 + \frac{-7}{x^2}$ at the point $(1, -6)$

Solution:

(a) $y = x^2 - 7x + 10$

$$\frac{dy}{dx} = 2x - 7$$

At $(2, 0)$, $x = 2$, gradient $= 2 \times 2 - 7 = -3$.

Therefore, equation of tangent is

$$y - 0 = -3(x - 2)$$

$$y = -3x + 6$$

$$y + 3x - 6 = 0$$

(b) $y = x + \frac{1}{x} = x + x^{-1}$

$$\frac{dy}{dx} = 1 - x^{-2}$$

At $\left(2, 2\frac{1}{2}\right)$, $x = 2$, gradient $= 1 - 2^{-2} = \frac{3}{4}$.

Therefore, equation of tangent is

$$y - 2\frac{1}{2} = \frac{3}{4}(x - 2)$$

$$y = \frac{3}{4}x - 1\frac{1}{2} + 2\frac{1}{2}$$

$$y = \frac{3}{4}x + 1$$

$$4y - 3x - 4 = 0$$

(c) $y = 4\sqrt{x} = 4x^{\frac{1}{2}}$

$$\frac{dy}{dx} = 2x^{-\frac{1}{2}}$$

At $(9, 12)$, $x = 9$, gradient $= 2 \times 9^{-\frac{1}{2}} = \frac{2}{3}$.

Therefore, equation of tangent is

$$y - 12 = \frac{2}{3}(x - 9)$$

$$y = \frac{2}{3}x - 6 + 12$$

$$y = \frac{2}{3}x + 6$$

$$3y - 2x - 18 = 0$$

(d) $y = \frac{2x-1}{x} = \frac{2x}{x} - \frac{1}{x} = 2 - x^{-1}$

$$\frac{dy}{dx} = 0 + x^{-2}$$

At $(1, 1)$, $x = 1$, gradient $= 1^{-2} = 1$.

Therefore, equation of tangent is

$$y - 1 = 1 \times (x - 1)$$

$$y = x$$

(e) $y = 2x^3 + 6x + 10$

$$\frac{dy}{dx} = 6x^2 + 6$$

At $(-1, 2)$, $x = -1$, gradient $= 6(-1)^2 + 6 = 12$.

Therefore, equation of tangent is

$$y - 2 = 12[x - (-1)]$$

$$y - 2 = 12x + 12$$

$$y = 12x + 14$$

(f) $y = x^2 - \frac{7}{x^2} = x^2 - 7x^{-2}$

$$\frac{dy}{dx} = 2x + 14x^{-3}$$

At $(1, -6)$, $x = 1$, gradient $= 2 + 14 = 16$.

Therefore, equation of tangent is

$$y - (-6) = 16(x - 1)$$

$$y + 6 = 16x - 16$$

$$y = 16x - 22$$

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Differentiation

Exercise H, Question 2

Question:

Find the equation of the normal to the curves:

(a) $y = x^2 - 5x$ at the point $(6, 6)$

(b) $y = x^2 - \frac{8}{\sqrt{x}}$ at the point $(4, 12)$

Solution:

(a) $y = x^2 - 5x$

$$\frac{dy}{dx} = 2x - 5$$

At $(6, 6)$, $x = 6$, gradient of curve is $2 \times 6 - 5 = 7$.

Therefore, gradient of normal is $-\frac{1}{7}$.

The equation of the normal is

$$y - 6 = -\frac{1}{7}(x - 6)$$

$$7y - 42 = -x + 6$$

$$7y + x - 48 = 0$$

(b) $y = x^2 - \frac{8}{\sqrt{x}} = x^2 - 8x^{-\frac{1}{2}}$

$$\frac{dy}{dx} = 2x + 4x^{-\frac{3}{2}}$$

At $(4, 12)$, $x = 4$, gradient of curve is $2 \times 4 + 4(4)^{-\frac{3}{2}} = 8 + \frac{4}{8} = \frac{17}{2}$

Therefore, gradient of normal is $-\frac{2}{17}$.

The equation of the normal is

$$y - 12 = -\frac{2}{17}(x - 4)$$

$$17y - 204 = -2x + 8$$

$$17y + 2x - 212 = 0$$

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Differentiation

Exercise H, Question 3

Question:

Find the coordinates of the point where the tangent to the curve $y = x^2 + 1$ at the point $(2, 5)$ meets the normal to the same curve at the point $(1, 2)$.

Solution:

$$y = x^2 + 1$$

$$\frac{dy}{dx} = 2x$$

At $(2, 5)$, $x = 2$, $\frac{dy}{dx} = 4$.

The tangent at $(2, 5)$ has gradient 4.

Its equation is

$$y - 5 = 4(x - 2)$$

$$y = 4x - 3 \text{ ①}$$

The curve has gradient 2 at the point $(1, 2)$.

The normal is perpendicular to the curve. Its gradient is $-\frac{1}{2}$.

The equation of the normal is

$$y - 2 = -\frac{1}{2}(x - 1)$$

$$y = -\frac{1}{2}x + 2\frac{1}{2} \text{ ②}$$

Solve Equations ① and ② to find where the tangent and the normal meet.

Equation ① – Equation ②:

$$0 = 4\frac{1}{2}x - 5\frac{1}{2}$$

$$x = \frac{11}{9}$$

Substitute into Equation ① to give $y = \frac{44}{9} - 3 = \frac{17}{9}$.

Therefore, the tangent at $(2, 5)$ meets the normal at $(1, 2)$ at $\left(\frac{11}{9}, \frac{17}{9}\right)$.

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Differentiation

Exercise H, Question 4

Question:

Find the equations of the normals to the curve $y = x + x^3$ at the points $(0, 0)$ and $(1, 2)$, and find the coordinates of the point where these normals meet.

Solution:

$$y = x + x^3$$

$$\frac{dy}{dx} = 1 + 3x^2$$

At $(0, 0)$ the curve has gradient $1 + 3 \times 0^2 = 1$.

The gradient of the normal at $(0, 0)$ is $-\frac{1}{1} = -1$.

The equation of the normal at $(0, 0)$ is

$$y - 0 = -1(x - 0)$$

$$y = -x \text{ ①}$$

At $(1, 2)$ the curve has gradient $1 + 3 \times 1^2 = 4$.

The gradient of the normal at $(1, 2)$ is $-\frac{1}{4}$.

The equation of the normal at $(1, 2)$ is

$$y - 2 = -\frac{1}{4}(x - 1)$$

$$4y - 8 = -x + 1$$

$$4y + x - 9 = 0 \text{ ②}$$

Solve Equations ① and ② to find where the normals meet.

Substitute $y = -x$ into Equation ②:

$$-4x + x = 9 \Rightarrow x = -3 \text{ and } y = +3.$$

Therefore, the normals meet at $(-3, 3)$.

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Differentiation

Exercise H, Question 5

Question:

For $f(x) = 12 - 4x + 2x^2$, find an equation of the tangent and normal at the point where $x = -1$ on the curve with equation $y = f(x)$. [E]

Solution:

$$y = 12 - 4x + 2x^2$$

$$\frac{dy}{dx} = 0 - 4 + 4x$$

$$\text{when } x = -1, \frac{dy}{dx} = -4 - 4 = -8.$$

The gradient of the curve is -8 when $x = -1$.

As $y = f(x)$, when $x = -1$

$$y = f(-1) = 12 + 4 + 2 = 18$$

The tangent at $(-1, 18)$ has gradient -8 . So its equation is

$$y - 18 = -8(x + 1)$$

$$y - 18 = -8x - 8$$

$$y = 10 - 8x$$

The normal at $(-1, 18)$ has gradient $\frac{-1}{-8} = \frac{1}{8}$. So its equation is

$$y - 18 = \frac{1}{8} (x + 1)$$

$$8y - 144 = x + 1$$

$$8y - x - 145 = 0$$

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Differentiation

Exercise I, Question 1

Question:

A curve is given by the equation $y = 3x^2 + 3 + \frac{1}{x^2}$, where $x > 0$.

At the points A , B and C on the curve, $x = 1$, 2 and 3 respectively.
Find the gradients at A , B and C . **[E]**

Solution:

$$y = 3x^2 + 3 + \frac{1}{x^2} = 3x^2 + 3 + x^{-2}$$

$$\frac{dy}{dx} = 6x - 2x^{-3} = 6x - \frac{2}{x^3}$$

$$\text{When } x = 1, \frac{dy}{dx} = 6 \times 1 - \frac{2}{1^3} = 4$$

$$\text{When } x = 2, \frac{dy}{dx} = 6 \times 2 - \frac{2}{2^3} = 12 - \frac{2}{8} = 11 \frac{3}{4}$$

$$\text{When } x = 3, \frac{dy}{dx} = 6 \times 3 - \frac{2}{3^3} = 18 - \frac{2}{27} = 17 \frac{25}{27}$$

The gradients at points A , B and C are 4 , $11 \frac{3}{4}$ and $17 \frac{25}{27}$, respectively.

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Differentiation

Exercise I, Question 2

Question:

Taking $f(x) = \frac{1}{4}x^4 - 4x^2 + 25$, find the values of x for which $f'(x) = 0$. [E]

Solution:

$$f(x) = \frac{1}{4}x^4 - 4x^2 + 25$$

$$f'(x) = x^3 - 8x$$

When $f'(x) = 0$,

$$x^3 - 8x = 0$$

$$x(x^2 - 8) = 0$$

$$x = 0 \text{ or } x^2 = 8$$

$$x = 0 \text{ or } \pm \sqrt{8}$$

$$x = 0 \text{ or } \pm 2\sqrt{2}$$

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Differentiation

Exercise I, Question 3

Question:

A curve is drawn with equation $y = 3 + 5x + x^2 - x^3$. Find the coordinates of the two points on the curve where the gradient of the curve is zero. **[E]**

Solution:

$$y = 3 + 5x + x^2 - x^3$$

$$\frac{dy}{dx} = 5 + 2x - 3x^2$$

Put $\frac{dy}{dx} = 0$. Then

$$5 + 2x - 3x^2 = 0$$

$$(5 - 3x)(1 + x) = 0$$

$$x = -1 \text{ or } x = \frac{5}{3}$$

Substitute to obtain

$$y = 3 - 5 + 1 - (-1)^3 \text{ when } x = -1, \text{ i.e.}$$

$$y = 0 \text{ when } x = -1$$

and

$$y = 3 + 5 \left(\frac{5}{3} \right) + \left(\frac{5}{3} \right)^2 - \left(\frac{5}{3} \right)^3 \text{ when } x = \frac{5}{3}, \text{ i.e.}$$

$$y = 3 + \frac{25}{3} + \frac{25}{9} - \frac{125}{27} = 9 \frac{13}{27} \text{ when } x = \frac{5}{3}$$

So the points have coordinates $(-1, 0)$ and $\left(1 \frac{2}{3}, 9 \frac{13}{27} \right)$.

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Differentiation

Exercise I, Question 4

Question:

Calculate the x -coordinates of the points on the curve with equation $y = 7x^2 - x^3$ at which the gradient is equal to 16. **[E]**

Solution:

$$y = 7x^2 - x^3$$

$$\frac{dy}{dx} = 14x - 3x^2$$

$$\text{Put } \frac{dy}{dx} = 16, \text{ i.e.}$$

$$14x - 3x^2 = 16$$

$$3x^2 - 14x + 16 = 0$$

$$(3x - 8)(x - 2) = 0$$

$$x = \frac{8}{3} \text{ or } x = 2$$

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Differentiation

Exercise I, Question 5

Question:

Find the x -coordinates of the two points on the curve with equation $y = x^3 - 11x + 1$ where the gradient is 1. Find the corresponding y -coordinates. [E]

Solution:

$$y = x^3 - 11x + 1$$

$$\frac{dy}{dx} = 3x^2 - 11$$

As gradient is 1, put $\frac{dy}{dx} = 1$, then

$$3x^2 - 11 = 1$$

$$3x^2 = 12$$

$$x^2 = 4$$

$$x = \pm 2$$

Substitute these values into $y = x^3 - 11x + 1$:

$$y = 2^3 - 11 \times 2 + 1 = -13 \text{ when } x = 2 \text{ and}$$

$$y = (-2)^3 - 11(-2) + 1 = 15 \text{ when } x = -2$$

The gradient is 1 at the points $(2, -13)$ and $(-2, 15)$.

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Differentiation

Exercise I, Question 6

Question:

The function f is defined by $f(x) = x + \frac{9}{x}$, $x \in \mathbb{R}$, $x \neq 0$.

- (a) Find $f'(x)$.
- (b) Solve $f'(x) = 0$. **[E]**

Solution:

(a) $f(x) = x + \frac{9}{x} = x + 9x^{-1}$

$$f'(x) = 1 - 9x^{-2} = 1 - \frac{9}{x^2}$$

- (b) When $f'(x) = 0$,

$$1 - \frac{9}{x^2} = 0$$

$$\frac{9}{x^2} = 1$$

$$x^2 = 9$$

$$x = \pm 3$$

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Differentiation

Exercise I, Question 7

Question:

Given that

$$y = x^{\frac{3}{2}} + \frac{48}{x}, x > 0,$$

find the value of x and the value of y when $\frac{dy}{dx} = 0$. [E]

Solution:

$$y = x^{\frac{3}{2}} + \frac{48}{x} = x^{\frac{3}{2}} + 48x^{-1}$$

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - 48x^{-2}$$

Put $\frac{dy}{dx} = 0$, then

$$\frac{3}{2}x^{\frac{1}{2}} - \frac{48}{x^2} = 0$$

$$\frac{3}{2}x^{\frac{1}{2}} = \frac{48}{x^2}$$

Multiply both sides by x^2 :

$$\frac{3}{2}x^2 \cdot \frac{1}{2} = 48$$

$$x^2 \cdot \frac{1}{2} = 32$$

$$x = (32)^{\frac{2}{5}}$$

$$x = 4$$

Substitute to give $y = 4^{\frac{3}{2}} + \frac{48}{4} = 8 + 12 = 20$

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Differentiation

Exercise I, Question 8

Question:

Given that

$$y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}, x > 0,$$

find $\frac{dy}{dx}$. [E]

Solution:

$$y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} + \frac{4}{2}x^{-\frac{3}{2}} = \frac{3}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}$$

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Differentiation

Exercise I, Question 9

Question:

A curve has equation $y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}}$.

(a) Show that $\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}}(4 - x)$

(b) Find the coordinates of the point on the curve where the gradient is zero. **[E]**

Solution:

(a) $y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}}$

$$\frac{dy}{dx} = 12 \left(\frac{1}{2} \right) x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}x^{-\frac{1}{2}}(4 - x)$$

(b) The gradient is zero when $\frac{dy}{dx} = 0$:

$$\frac{3}{2}x^{-\frac{1}{2}}(4 - x) = 0$$

$$x = 4$$

Substitute into $y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}}$ to obtain

$$y = 12 \times 2 - 2^3 = 16$$

The gradient is zero at the point with coordinates (4 , 16) .

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Differentiation

Exercise I, Question 10

Question:

(a) Expand $\left(x^{\frac{3}{2}} - 1\right)\left(x^{-\frac{1}{2}} + 1\right)$.

(b) A curve has equation $y = \left(x^{\frac{3}{2}} - 1\right)\left(x^{-\frac{1}{2}} + 1\right)$, $x > 0$. Find $\frac{dy}{dx}$.

(c) Use your answer to **b** to calculate the gradient of the curve at the point where $x = 4$. **[E]**

Solution:

(a) $\left(x^{\frac{3}{2}} - 1\right)\left(x^{-\frac{1}{2}} + 1\right) = x + x^{\frac{3}{2}} - x^{-\frac{1}{2}} - 1$

(b) $y = x + x^{\frac{3}{2}} - x^{-\frac{1}{2}} - 1$

$$\frac{dy}{dx} = 1 + \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$$

(c) When $x = 4$, $\frac{dy}{dx} = 1 + \frac{3}{2} \times 2 + \frac{1}{2} \times \frac{1}{4^{\frac{3}{2}}} = 1 + 3 + \frac{1}{16} = 4 \frac{1}{16}$

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Differentiation

Exercise I, Question 11

Question:

Differentiate with respect to x :

$$2x^3 + \sqrt{x} + \frac{x^2 + 2x}{x^2} \quad \text{[E]}$$

Solution:

$$\text{Let } y = 2x^3 + \sqrt{x} + \frac{x^2 + 2x}{x^2}$$

$$\Rightarrow y = 2x^3 + x^{\frac{1}{2}} + \frac{x^2}{x^2} + \frac{2x}{x^2}$$

$$\Rightarrow y = 2x^3 + x^{\frac{1}{2}} + 1 + 2x^{-1}$$

$$\frac{dy}{dx} = 6x^2 + \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-2} = 6x^2 + \frac{1}{2\sqrt{x}} - \frac{2}{x^2}$$

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Differentiation

Exercise I, Question 12

Question:

The volume, $V \text{ cm}^3$, of a tin of radius $r \text{ cm}$ is given by the formula $V = \pi (40r - r^2 - r^3)$. Find the positive value of r for which $\frac{dV}{dr} = 0$, and find the value of V which corresponds to this value of r . [E]

Solution:

$$V = \pi (40r - r^2 - r^3)$$

$$\frac{dV}{dr} = 40\pi - 2\pi r - 3\pi r^2$$

Put $\frac{dV}{dr} = 0$, then

$$\begin{aligned}\pi (40 - 2r - 3r^2) &= 0 \\ (4 + r)(10 - 3r) &= 0\end{aligned}$$

$$r = \frac{10}{3} \text{ or } -4$$

As r is positive, $r = \frac{10}{3}$.

Substitute into the given expression for V :

$$V = \pi \left(40 \times \frac{10}{3} - \frac{100}{9} - \frac{1000}{27} \right) = \frac{2300}{27} \pi$$

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Differentiation

Exercise I, Question 13

Question:

The total surface area of a cylinder $A \text{ cm}^2$ with a fixed volume of 1000 cubic cm is given by the formula $A = 2 \pi x^2 + \frac{2000}{x}$, where $x \text{ cm}$ is the radius. Show that when the rate of change of the area with respect to the radius is zero, $x^3 = \frac{500}{\pi}$. [E]

Solution:

$$A = 2 \pi x^2 + \frac{2000}{x} = 2 \pi x^2 + 2000x^{-1}$$

$$\frac{dA}{dx} = 4 \pi x - 2000x^{-2} = 4 \pi x - \frac{2000}{x^2}$$

$$\text{When } \frac{dA}{dx} = 0,$$

$$4 \pi x = \frac{2000}{x^2}$$

$$x^3 = \frac{2000}{4 \pi} = \frac{500}{\pi}$$

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Differentiation

Exercise I, Question 14

Question:

The curve with equation $y = ax^2 + bx + c$ passes through the point $(1, 2)$. The gradient of the curve is zero at the point $(2, 1)$. Find the values of a , b and c . **[E]**

Solution:

The point $(1, 2)$ lies on the curve with equation $y = ax^2 + bx + c$.
Therefore, substitute $x = 1$, $y = 2$ into the equation to give

$$2 = a + b + c \text{ ①}$$

The point $(2, 1)$ also lies on the curve.
Therefore, substitute $x = 2$, $y = 1$ to give

$$1 = 4a + 2b + c \text{ ②}$$

Eliminate c by subtracting Equation ② – Equation ①:

$$-1 = 3a + b \text{ ③}$$

The gradient of the curve is zero at $(2, 1)$ so substitute $x = 2$ into the expression for $\frac{dy}{dx} = 0$.

$$\text{As } y = ax^2 + bx + c$$

$$\frac{dy}{dx} = 2ax + b$$

At $(2, 1)$

$$0 = 4a + b \text{ ④}$$

Solve Equations ③ and ④ by subtracting ④ – ③:

$$1 = a$$

Substitute $a = 1$ into Equation ③ to give $b = -4$.

Then substitute a and b into Equation ① to give $c = 5$.

Therefore, $a = 1$, $b = -4$, $c = 5$.

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Differentiation

Exercise I, Question 15

Question:

A curve C has equation $y = x^3 - 5x^2 + 5x + 2$.

(a) Find $\frac{dy}{dx}$ in terms of x .

(b) The points P and Q lie on C . The gradient of C at both P and Q is 2. The x -coordinate of P is 3.

(i) Find the x -coordinate of Q .

(ii) Find an equation for the tangent to C at P , giving your answer in the form $y = mx + c$, where m and c are constants.

(iii) If this tangent intersects the coordinate axes at the points R and S , find the length of RS , giving your answer as a surd. **[E]**

Solution:

$$y = x^3 - 5x^2 + 5x + 2$$

$$(a) \frac{dy}{dx} = 3x^2 - 10x + 5$$

$$(b) \text{ Given that the gradient is 2, } \frac{dy}{dx} = 2$$

$$3x^2 - 10x + 5 = 2$$

$$3x^2 - 10x + 3 = 0$$

$$(3x - 1)(x - 3) = 0$$

$$x = \frac{1}{3} \text{ or } 3$$

$$(i) \text{ At } P, x = 3. \text{ Therefore, at } Q, x = \frac{1}{3}.$$

$$(ii) \text{ At the point } P, x = 3, y = 3^3 - 5 \times 3^2 + 5 \times 3 + 2 = 27 - 45 + 15 + 2 = -1$$

The gradient of the curve is 2.

The equation of the tangent at P is

$$y - (-1) = 2(x - 3)$$

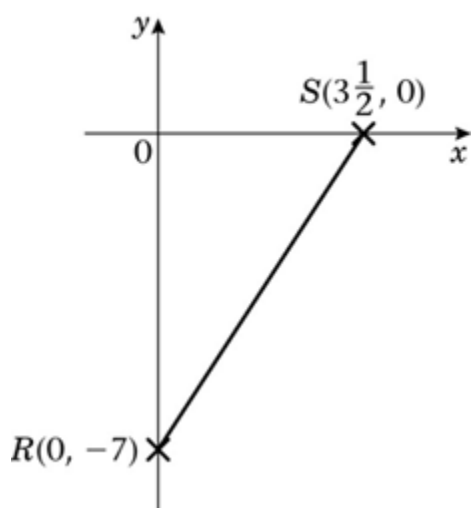
$$y + 1 = 2x - 6$$

$$y = 2x - 7$$

(iii) This tangent meets the axes when $x = 0$ and when $y = 0$.

$$\text{When } x = 0, y = -7. \text{ When } y = 0, x = 3\frac{1}{2}.$$

The tangent meets the axes at $(0, -7)$ and $\left(3\frac{1}{2}, 0\right)$.



The distance $RS = \sqrt{\left(3\frac{1}{2} - 0\right)^2 + [0 - (-7)]^2} = \sqrt{\frac{49}{4} + 49} = \frac{7}{2}\sqrt{1+4} = \frac{7}{2}\sqrt{5}$.

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Differentiation

Exercise I, Question 16

Question:

Find an equation of the tangent and the normal at the point where $x = 2$ on the curve with equation $y = \frac{8}{x} - x + 3x^2$, $x > 0$. [E]

Solution:

$$y = \frac{8}{x} - x + 3x^2 = 8x^{-1} - x + 3x^2$$

$$\frac{dy}{dx} = -8x^{-2} - 1 + 6x = -\frac{8}{x^2} - 1 + 6x$$

$$\text{when } x = 2, \frac{dy}{dx} = -\frac{8}{4} - 1 + 12 = 9$$

$$\text{At } x = 2, y = \frac{8}{2} - 2 + 3 \times 2^2 = 14$$

So the equation of the tangent through the point $(2, 14)$ with gradient 9 is

$$y - 14 = 9(x - 2)$$

$$y = 9x - 18 + 14$$

$$y = 9x - 4$$

The gradient of the normal is $-\frac{1}{9}$, as the normal is at right angles to the tangent.

So the equation of the normal is

$$y - 14 = -\frac{1}{9}(x - 2)$$

$$9y + x = 128$$

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Exercise I, Question 17

Question:

The normals to the curve $2y = 3x^3 - 7x^2 + 4x$, at the points $O(0, 0)$ and $A(1, 0)$, meet at the point N .

(a) Find the coordinates of N .

(b) Calculate the area of triangle OAN . **[E]**

Solution:

(a) $2y = 3x^3 - 7x^2 + 4x$

$$y = \frac{3}{2}x^3 - \frac{7}{2}x^2 + 2x$$

$$\frac{dy}{dx} = \frac{9}{2}x^2 - 7x + 2$$

At $(0, 0)$, $x = 0$, gradient of curve is $0 - 0 + 2 = 2$.

The gradient of the normal at $(0, 0)$ is $-\frac{1}{2}$.

The equation of the normal at $(0, 0)$ is $y = -\frac{1}{2}x$.

At $(1, 0)$, $x = 1$, gradient of curve is $\frac{9}{2} - 7 + 2 = -\frac{1}{2}$.

The gradient of the normal at $(1, 0)$ is 2.

The equation of the normal at $(1, 0)$ is $y = 2(x - 1)$.

The normals meet when $y = 2x - 2$ and $y = -\frac{1}{2}x$:

$$2x - 2 = -\frac{1}{2}x$$

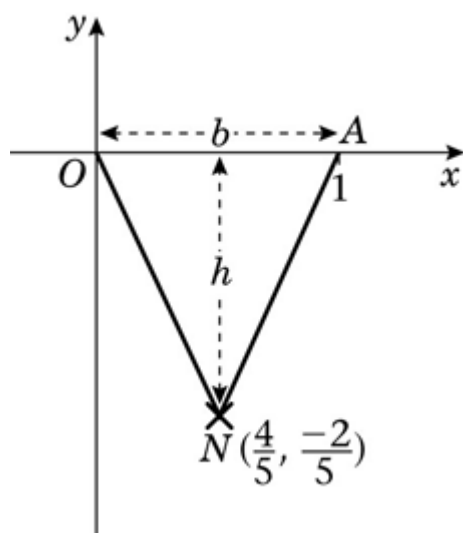
$$2\frac{1}{2}x = 2$$

$$x = 2 \div 2\frac{1}{2} = \frac{4}{5}$$

Substitute into $y = 2x - 2$ to obtain $y = -\frac{2}{5}$ and check in $y = -\frac{1}{2}x$.

N has coordinates $\left(\frac{4}{5}, -\frac{2}{5}\right)$.

(b)



The area of $\triangle OAN = \frac{1}{2} \text{base} \times \text{height}$

$$\text{base } (b) = 1$$

$$\text{height}(h) = \frac{2}{5}$$

$$\text{Area} = \frac{1}{2} \times 1 \times \frac{2}{5} = \frac{1}{5}$$

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