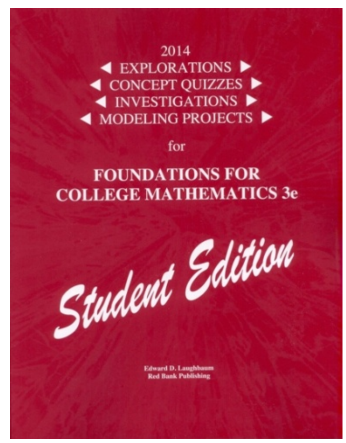
◀ EXPLORATIONS ▶
◀ CONCEPT QUIZZES ▶
◀ INVESTIGATIONS ▶
◀ MODELING PROJECTS ▶

for

FOUNDATIONS FOR COLLEGE MATHEMATICS 3e

Edward D. Laughbaum
Marysville, Ohio



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PREFACE

Understanding mathematics often requires investigations into the concepts underlying the mathematics. These investigations may be personal or as part of a small group. Understanding mathematics is totally different from memorizing mathematical algorithms -- a series of precisely prescribed steps for solving specific types of problems, -- or "knowing" a series of facts. Many of the mathematical algorithms that are useful in the applied fields and within mathematics are available on calculators and computers; thus, the need for memorizing the algorithms has diminished. However, the need for *understanding* mathematics has increased dramatically because of the need to utilize this technology and correctly interpret the results. The need to understand mathematics is also manifested in the changes in the world economy. U.S. students are now competing in a world market for jobs, and we can no longer expect a good job with benefits unless we understand, recall, and are able to use mathematics. *Explorations*, *Concept Quizzes, Investigations, and Modeling Projects for Foundations for College Mathematics 3e* contains a variety of assessment/teaching tools that can be used to help students understand mathematics. Inherent to the brain-based pedagogy and mathematical processes in *Foundations 3e* and this ancillary activity book, improved understanding is not the only benefit; that is, long-term memories with enhanced recall are fostered by the same brain-function-based techniques.

The activities included in this book can be challenging and may require the neural processing of group work. The graphing calculator is <u>required</u> for all activities. However, do not just think of a graphing calculator as a mathematical "answer finder." It is often the case that the study of the answers is where you will find the real mathematics. Look for what the answers tell you about the related mathematics. Look for patterns that will teach you how to do the related mathematics without a graphing calculator. Look for connections to previously learned mathematics and to real-world contextual situations. This will help understanding, memory, and recall of the related mathematics. Think about what the visualizations of the mathematics tell you. Visualizations promote understanding and provide variety in the otherwise symbolic processes found in algebra.

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January 4, 2017

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Laughbaum taught mathematics at the high school level for 5 years, at the two-year college level for 23 years, and at the university level for 16 years. Ed has given over 270 presentations in 10 countries and 34 states; he has published over 86 books or peer reviewed professional articles. His professional work has been referenced in over 30 countries.

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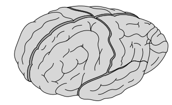
The Neuroscience in *Foundations 3e*

The following "articles" are embedded in *Foundations 3e*. Research indicates that students must understand basic functioning of the brain. So these are in the text to help students understand brain function.

Basic Brain Function – A Beginning



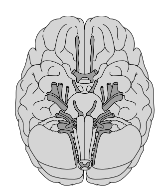
On a cellular and a molecular level, your brain is designed to learn and remember. That is, the chemistry and physics of basic brain operation creates a memory (anything you learn) and then you can recall what you have learned at a later time when needed. But in reality, you know you are not able to remember everything you have learned – if you did, there would be no reason to review before a midterm, final exam, state test, or college entrance exam. A part of the reason for "forgetting" is that the brain structure called the hippocampus (that processes your memory) determines what is important to you (and your brain) and what is not. Those things deemed not important are given this status based on your personal overall interest in, attention to, and value of, the "not important" memory. When you decide something is not important, the hippocampus does not store these memories in the long-term memory areas of your brain. That is, they may be forgotten. The memory may be gone within minutes, hours, days, or years.

Your brain is an extremely complicated organ, yet when you understand its basic rules of operation, you can make it perform better. You can help it remember – long-term. You can even help it understand and remember algebra! You control interest in, attention to, and value of the algebra in this book. At the same time, the author has written this book using visualizations, contextual situations, pattern building, connections (neural associations), and other brain-based techniques to help you with memory and understanding of the algebra in this book.

Visualizations	Help improve understanding and reduce memory loss
Contextual situations	Add meaning which improves memory and understanding
Pattern building	Pattern generalizing creates memories and assists understanding
Associations	The primary reason you can recall is because of neural associations

A history of interactions with peers, family, movies, music, school, television, etc. become integrated in your brain and influence your thoughts and decisions. This textbook is structured to help you integrate algebra into your thought processes. It is designed to help you understand the algebra through capitalizing on common brain processes. As you might guess, remembering the algebra you learned is crucial when it comes test time. The overall structure of *Foundations 3e* has been designed to help you with this very issue. Memory recall comes from neural connections of mathematical ideas, and this text creates connections to all algebraic concepts and skills through function representation, function behaviors, and real-world contextual situations.

Basic Brain Function – Connections



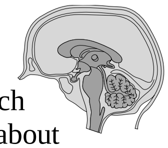
When you learn something new (whether in math class or anywhere), your brain connects what you have just learned to existing and related neural circuits already in your brain. When you learn something absolutely unrelated to anything stored in your brain, your brain still tries to connect it to existing circuits. Looking at this another way, when you first learn something new, it may take 15,000 neural synapses (places where two neurons connect) to contain the memory of what you have just learned. As a point of information, the 15,000 synapses is an educated guess as no one can say exactly how many synapses are involved and how many neurons are required.

Continuing … So, the next time you learn something related to what just required 15,000 synapses, will now require many fewer synapses because of the overlap in the two related memories (something learned). And when you learn something else new, but connected to the previous will now require even fewer synapses. And so on. Therefore, each time you learn something connected to something you already learned; it is EASIER to learn. It takes less energy. It is easier to recall.

Why is this important to you? Because in this textbook, ALL concepts and skills/procedures are CONNECTED through the use of function representation and function behaviors. That is, your brain will learn in less time and it will get easier as you progress through the text.

Your brain is the most complicated structure in the universe. Neuroscientists however, have a really good idea of how memory works and how it is created. This textbook capitalizes on how your brain works to make understanding and long-term memory more accessible to you.

Basic Brain Function – Can Algebra be Simple?

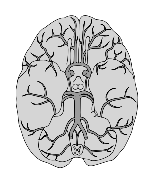


As your brain is developing, from about the second month after conception to your second birthday, your brain is extremely busy creating connections (called synapses) among neurons. During this time, each neuron forms about 15,000 connections to other neurons – all on their own – without learning. This is about 1.8 million synapses (connections) per second for over two years of time. Why is this important and what does this have to do with algebra being simpler than you might think? The answer is that when you learn something (not just algebra), the brain stores what you learned in a network of connected neurons. It is the series of connections (synapses) that hold the memory of something you just learned. When all the neurons in the neural circuit holding the memory fire (discharge electricity) from beginning to the end, you get the memory of what you are trying to remember. So, as a person under two years old, the circuits already exist. This is what makes learning extremely easy for a two year old, or three years old etc.

Now for the bad news: Which of these trillions of synapses (connections) remain, and which wither away, depends on whether they carry any traffic. That is, the traffic means learning and using what you learned. By one estimate, approximately 20 billion synapses are pruned every day between childhood and **early adolescence**. That is, after the age of two, the UNUSED connections start to wither – get cut off. With fewer connections available to process and hold a memory, the more difficult it becomes to learn. Adults have a more difficult time learning than do teenagers because most of the “free” circuits are gone.

Now for the good news: At around age 10-12, humans develop another burst of “free” synaptic growth. This growth is not as prolific as in the new brain, but because of the new neural circuit growth, learning again becomes simpler for teenagers. Adults are out of luck, because all the new unused neural connections (synapses) are cut by the time they become young adults. So they must CREATE new synaptic connections to hold and process anything new they learn. Woops, no free lunch for adult learners.
Teenagers, on the other hand, are offered a free lunch. But are the young hungry?

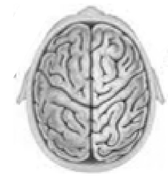
Basic Brain Function – The Prefrontal Lobes



Directly behind your forehead are the prefrontal lobes of your brain. The word lobes is used because there are two regions – one on left, and one on the right. These regions contain the decision-making circuits (connected neurons), as well as most all of the high-level functioning activity in your brain. The prefrontal lobes are the last circuits of your brain to develop. Actually, they are not finished developing until you are about 21 to 23 years of age. The implication is that people under this age sometimes do not make the best decisions. As you might be thinking, even adults do not always make the best decisions, or cannot always think clearly. You are right. But why, when they are over 23?

There is a common saying in life, “use it or lose it.” Or saying it another way: mental activity increases the production of molecules that are thought to enhance mental activity. This applies directly to brain function – especially thinking and decision-making. It may not be obvious, but the items under the heading **Developing the Pre-Frontal Lobes** can sometimes be a little more challenging. Actually there are a variety of explorations, concept quizzes, investigations, or modeling projects (found in the ancillary workbook) that can be challenging. Even many of the other exercises in this textbook can be challenging. Why are they in the textbook if they are “hard?” Because the only way to have, and keep properly functioning prefrontal lobes is to do challenging stuff – like the exercises in this textbook and in the ancillary activity workbook. As it turns out, if you do not do challenging mathematics, or anything that is challenging, it is akin to having a lobotomy. When an adult has a lobotomy (for unrelated medical reasons), they can no longer think critically nor can they make complicated decisions. So we are back to why some (many) adults cannot think clearly and are poor at decision-making. We are also back to why you need to do challenging stuff – like algebra. Well, all this information is something to think about.

Basic Brain Function – Visualizations I



In this textbook you will find that, when possible, visualizations are used early in the teaching lesson of a skill or concept. Almost all other textbooks use symbols to start a lesson on, for example, factoring. They may eventually use a picture (graph), but mostly not. The process of using visualizations last, or not at all, has a negative impact on memory. Further, not using visualizations at all has a negative impact on your understanding of an algebraic concept or skill. You might ask why?

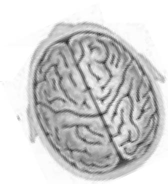
Neuroscience tells us that to create a new memory while minimizing loss of over time; you should try to do something eventful during the early moments of the encoding process as this will influence the fate of the new memory. What is eventful? Well, it is not using symbols (unless you are already interested in algebra). There are various things one can do to improve memory loss due to passage of time, and one of them is to use visualizations in the early moments of memory creation as the event the brain needs. Brains have an unbelievable ability to remember images. Your brain is an image processor. On a second-by-second basis, it uses images. So visualizations are extremely important to the brain, and just as important to the memory creation process. We can even go so far to say that if the average brain is not presented with visualizations, it will ignore the lesson – in total or partially.

Basic Brain Function – Generalizing

Your brain operation is mostly unknown to you since you only know about thoughts that reach your consciousness. However, a primary mode of operation of your brain is to generalize patterns it senses through vision, hearing, etc. In algebra, for example, what is the next term in the sequence x, x^2, x^3, …? You know doubt knew in less than 1/5 of a second that the next term is x^4. Why did you get it so quickly? Because this is what your brain does millions of times an hour – all without you ever being aware of the decision your brain made about x^4. Your brain is extremely good at generalizing patterns. With 120 billion neurons in your brain, 100 trillion glial cells, and trillions of connections (synapses) among neurons, neurotransmitters (chemicals) are oozing and sparks flying constantly, whether you are awake or asleep, during active thinking and during boredom, it doesn't matter. At any one instant, billions of synapses are active.

Given this plain and simple fact, this textbook incorporates pattern building processes throughout so that your brain generalizes the desired mathematical pattern. Why is this important to you? As soon as you "figure out" a pattern (mathematical or not), your brain creates a memory of it. This is learning, the creation of a memory of a mathematical concept (idea) or a mathematical skill. Learning through pattern generalizing is significantly richer than learning through memorization. Mathematics learned through memorization is extremely fragile. When you memorize mathematics you are much more likely to forget quickly without review. When you memorize, you must review, and then review again, and again. And then, you will still forget what you learned by not using the mathematics.

You have likely never been asked to learn through generalizing patterns before, but in this book you will find pattern-building activities in the end of section exercises, in introductory materials, and in the activity book for this text. The more you do pattern generalizing, the better you get, not just in mathematics, but in all your work.



Basic Brain Function – Visualizations II

You may have noted that this textbook makes extensive use of mathematical visualizations as a teaching/learning tool. All homework exercises, all teaching/learning activities – student or teacher-centered, and all modeling projects, explorations, and investigations rely on the use of visualizations found on the graphing calculator. Why are visualizations so prevalent? What power do visualizations hold in algebra?

The reason visualizations (dynamic graphical visualizations found on the graphing calculator) are used throughout this text is that they are CRUCIAL to understanding and memory (with recall) of the algebra being taught. Research in the neurosciences concludes that visualizations allow us to understand the mathematics through our eyes and our mind's eye. Vision processes mathematical thinking, which helps us understand the world, although you may not have experienced this yet.

As you study pictures (graphs) along with the words, you get improved memory. And just as important, your brain easily rejects or ignores items that do not contain the visual information it is seeking. A psychology professor from Canada asked his psychology students to view 100 pictures for 5 seconds each. He brought them back in a week, and showed them the pictures again, mixed with 100 new pictures. The students correctly recognized more than 90% of the pictures, having seen them only once, for just five seconds. The same 90% results were returned for 1,000 pictures, and then for 10,000. This demonstrates the power of visualizations on memory. Because of memory considerations, visualizations are used early in the teaching/learning process of each concept – not at the end to confirm the concept.
The lack of use of visualizations makes learning of algebra more difficult, and causes a lack of "big picture" thinking. Because this text focuses on visualizations, typical brains are less likely to reject the algebra included.

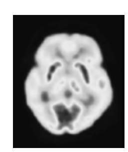
Visualizations are all about understanding and memory with recall!

Basic Brain Function – Unconscious Processing

As you know, you are aware of your own thoughts. But you may not know that you are only aware of thoughts through your consciousness. If you are conscious, like now as you read this text, you have a continuous series of thoughts you are aware of. They just keep coming. As you watch a movie or TV, your thoughts are influenced by what you see and you will likely have thoughts about the movie or TV show. Likewise, the same is true when you listen to music. But what if there is no obvious external stimulus? Whoa, you still have thoughts coming to your consciousness. It turns out your brain has thousands upon thousands of thoughts every second of the day. But how can this be when your consciousness only allows you to think one thought at a time? It is your unconscious processing of thoughts that exists thousands at a time, and you are not aware of any of them. The vast majority of these unconscious thoughts are NOT just about controlling basic body functions like breathing, keeping the heart pumping, moving muscles, etc. They are the same kinds of thoughts that reach consciousness. But there is still more, your unconscious brain is processing information, it is making decisions, and it can process algebra with the end outcome of you being more likely to understand and remember it.

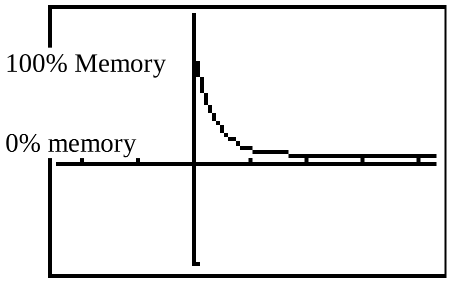
What a thought, learning algebra on an unconscious level! The problem is how can your teacher or your textbook help you think about and process algebra unconsciously? The answer is not simple because there are thousands and thousands of thoughts all happening at the same time. Further, your brain has a built in value system. That is, it decides what is important to you, and it thinks more about the things that are important to you. On the other hand, external stimuli can also cause your brain to think about them (a scary movie, theft of your cell phone, an embarrassing incident, etc.). This is an important neural function for learning algebra. Some of the ancillary activities that are a part of this text were designed to be assigned to you BEFORE the concept is taught by your teacher. This helps your brain to be ready for the teacher because you will have already been thinking (processing) the ideas and are more ready to learn – but only if you decide the ideas are of value.

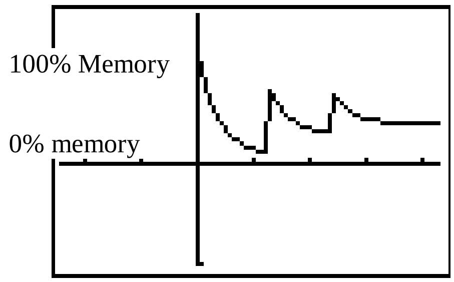
Basic Brain Function – Distributed Learning



If you have ever taken an algebra course before this one, you are aware that this text is different in structure. One of several differences is that the sequencing used to teach (and you learn) many algebraic concepts is distributed over various sections and several chapters. For example, in Section 2.3 you first learned about function behaviors. But you learned more about function behaviors in every section of Chapter Three, and in Chapters 4-10 plus Chapter 14, you use function behaviors to learn new concepts/skills and in the process learned even more about behaviors of functions. Certainly in the process of doing this you learned more and more about functions and algebra. Why is this important?

Many things you learn, you forget. It could be that you forget in a few minutes or it might be a few years, and some things you never forget. Let's suppose you learned how to write $3\frac{1}{5}\%$ as a fraction in middle school. Your teacher told you how to do it and then he/she assigned practice homework for you to do 20 or 30 more conversions of percents to fractions. Unless you constantly review, your memory of how to convert percents to fractions looks like the graph on the left. BUT, if you learn something where you distribute the learning over time, your memory of what you learned looks like the graph on the right.





Note that you are left with a higher percent memory under distributed learning that you have under "one shot" learning. The time-axis is left blank because time depends on what you learned and your interest in what you learned. As noted above, it could be minutes, days, or years.

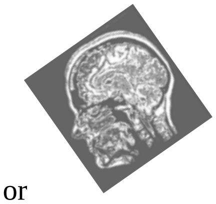
Basic Brain Function – Connections Revisited

The central theme of this textbook is the mathematical idea of "function." The reason is that every algebraic concept or procedure in the textbook can be connected through function representation (numeric, graphic, or symbolic) or function behaviors (increasing/decreasing, maximum/minimum, zero/positive/negative, rate of change, and domain/range). Why is this important? Because of the way the brain stores and recalls something learned – memories.

When your brain is presented with something new, it AUTOMATICALLY tries to connect the new stuff to what you already know. The problem in an algebra course like this one is that there may be nothing in your brain related to the algebra you are learning. But this does not stop your brain from connecting to something – anything. And here is the problem for teaching and learning. What we really want to happen is for your brain to connect to previously learned algebra and to simple real-world contexts (like an I. V. drip or a ball tossed straight up) that have mathematical properties that are related to the topic at hand. Otherwise your brain may connect the current algebraic concept/procedure to a wide variety of totally unrelated ideas already in your brain. If teachers knew what your brain likely connects an algebraic concept to, that information could be used to teach other mathematical ideas. Anything you learn is stored in your brain as clusters of related (connected) ideas. Therefore, this textbook connects every new idea or procedure with previously learned ideas or to contextual real-world situation to help your brain store and retrieve memories. Why is this important when you know people who could learn without the teacher and textbook connecting everything?

It turns out that ALL recall of something learned is processed in the brain through a series of connections. If every neuron in the networks storing a concept or procedure fires (discharges electricity to the next neuron in the circuit) it is likely that the thought will reach consciousness – you will recall it. Further, to be able to recall better, you need lots of connections to the concept/procedure you need to recall. There are many connections to every mathematical concept presented in this text. As for people who can recall without the connections being made in the textbook or by the teacher, he/she has very likely learned how to make good connections without thinking about it. But not many people are like this.

Basic Brain Function – Learning



Many educators who devote their professional lives to figuring out "learning" often describe learning as being complicated. They break it down into parts, hoping the parts are simpler to understand than the whole. But nothing seems to work.

To neuroscientists, learning is not necessarily complicated on a macro level, but it may be on a cellular or molecular level. Learning is the process of creating a memory and the brain does it automatically and it is an on-going process. However, you are most certainly aware that you do not remember every memory you have created. Forgetting is another normal process. So the issue becomes how to create a long-term memory, for example, of how to solve any equation. But long-term memory isn't worth much unless you understand what it means to solve an equation. That is, in mathematics education you need to learn (create memories) and understand what you learned, keep the memory long term, and be able to recall it. All of this is done on a cellular/molecular level.

The brain has a built-in electro/chemical value system. It assigns a value to the things we think about, $2x - 5$, faces, $f(x)$, where I parked my car yesterday, $3x + 7 = 12$, my friend's name, etc. The value system helps determine if we will store a particular memory. The brain structure called the hippocampus is, generally speaking, the part of our brain that processes the storing and reconstruction of our memories. Based on the value of a particular memory (something learned), it will process the storage of the memory or not. Memories assigned "of no value," like what you had for dinner 3 days ago, will likely not be processed into long-term memory and will fade within several days. Recall of the stored memory is a function of time, what it is connected to, and value of the memory. Symbol manipulations (like what you may think of as algebra) typically may not be assigned a high positive value. But algebra can have a high value if you are interested in it.

The common thinking, within mathematics education, is that if you practice you will learn. This is likely the case for physical learning like driving a car, a dance move, hitting a ball, etc. As for the learning (creating a memory) of an abstract idea, or a concrete mathematical procedure, practice will only take you so far – as used in mathematics education. That is, practicing a procedure (like factoring, equation solving, multiplying polynomials, etc.) for a day and then moving on to practicing another procedure tomorrow and a different one the day after is not enough to create a recallable long-term memory with understanding. If you review (practice) the procedures right before a test, and again before the final exam, you should be able to recall temporarily. But this leaves you with intermittent or false memory reconstruction in the long run.

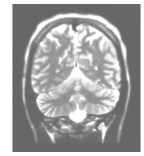
Basic Brain Function – Enriched Environment

Before neuroscientists discovered how the brain works, those in education thought that students were only capable of learning through one method of teaching, like lecturing. It was commonly used so as to not overwhelm the brain, or confuse the brain. However, this thinking was totally wrong.

An enriched teaching environment means teaching multiple methods for doing algebra with a variety of teaching tools, and using a variety of teaching methods. Because this textbook requires the graphing calculator, we have many more opportunities for enriching the environment. Examples include electronic teaching of concepts and skills, reviewing, homework, assessing through TI StudyCard e-activities, Texas Instruments LearningCheck™ e-activities, Cabri Jr. e-activities, and teaching understandings through dynamic visualizations. You also have access to explorations, concept quizzes, investigations, and modeling projects. But why do we need such a variety?

The **enriched environment** promotes correct memory of math learned. The enriched environment provides the brain with more ways of recalling the algebra, and it produces more synapses, and more dendrite growth than does lecture used all the time. The depth or richness of the options you have in this course is just what the brain ordered.

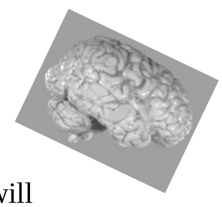
Basic Brain Function – Graphing Technology



Obviously, the graphing calculator does some algebra. In addition to graphing and table-making, it can find maximums, minimums, zeros, etc. However, the graphing calculator also provides the brain with novelty. Why is this important? Because a basic brain function is to look for something new. Ten thousand years ago, this was important to humans, because if you didn't look for novelty, you may find yourself dead. That is, motion is novelty in your field of vision, and the brain is constantly looking for motion. When motion is sensed, the brain responds. Motion gets the brains attention. To capitalize on this basic brain function, the author assumes you have a graphing calculator with you during class time, and when you do homework, because it helps you focus on the algebra when doing homework. That is, the brain is looking for motion/novelty and the graphing calculator offers it. So you find yourself using it even when you don't intend to use it.

Given the brain's attraction to electronic technology, this text was written with the idea that its objectives would be to get your attention to algebra through technology, to help you focus on the algebra, to help teach algebra, etc. Because of the diverse functionality of the graphing calculator, teachers can help you learn algebra with it, by using the functionality in a novel way, like using the mathematical concept of function as an underlying theme instead of the un-novel concept of equation. It may be difficult for you, as a student, to recognize that this textbook uses function behaviors and function representation to help you learn algebra, but it does. Most of the teaching and learning in this text are dependent on the graphing calculator. But the book has not focused on doing the algebra with the graphing calculator. We just get to the pencil and paper doing of algebra through a more interesting route.

Basic Brain Function – Attention



Do you give your teacher your attention when he/she is teaching a lesson (let's say lecturing)? It may depend on whether or not you are interested in the topic. Being attentive in class or while studying is a complicated issue from a neuroscience perspective. But on the other hand neuroscientists do know some things about attention. For example, they know you cannot learn anything if you are not paying attention to the task at hand (learning algebra). Further, neuroscientists have discovered that the brain will automatically shut down non-related conscious neural circuits if you focus intently on studying or learning. This allows you to concentrate on what you are studying or learning. It turns out that when you are first learning a skill/procedure you must actually think about it, or learning will soon be lost. Well, there is more, but the question is how can a textbook or teacher help you focus and pay attention?

The best neuroscience can offer (based on the author's current knowledge) is that technology usually gets your attention and interesting content will help keep it active. Novelty is actually at the root of attention. That is, technology is interpreted as being novel and since the brain must learn new "stuff" to survive, it is attracted to technology. As for interesting content, the author has tried to provide real-world contexts that help more students to find algebra more interesting.

Basic Brain Function – Practice Practice Practice

When a person practices a skill like factoring or solving equations, etc., the neuronal circuits required to factor/solve may become more myelinated. Myelination is a process where brain cells called oligodendrocytes wrap a white substance (myelin) around the axons of the neurons involved in the process of factoring/solving. This process makes the neurons more likely to fire to the end of the neural circuit – meaning you don't get lost half way through. So the more you use a circuit (practice), the more myelin is placed around the axons making them more and more likely to fire to completion. This process eventually means you can do algebra automatically. That is, the related circuits become automated – taking less neural energy to process – while still getting the work of polynomials factored or equations solved.

UNFORTUNATELY as soon as the practicing stops, the neural circuits start to lose the myelin. With reduced myelin, recall becomes hit-or-miss, and eventually the memory is severed. Further, if you don't really practice enough in the first place, or do not practice problems that are hard enough, the myelination is limited and does not have the desired effect of being able to cause you to remember how to factor or solve equations on the final exam. That is, passage of time without practice will reduce your ability to recall and actually factor or solve correctly.

As you may have noted in this textbook, we use many other brain tools/techniques to produce memory with recall. Practice is fine in the short term, but we need more for the long haul.

Basic Brain Function – The Role of Dopamine

The cells in your brain are primarily all neurons and glial cells. Unlike most other cells in your body, neurons do not divide to produce more neurons. There are around 100 billion neurons in your brain, and nearly a trillion glial cells. One type of glial cells eliminates unwanted bacteria and viruses in the brain, but they have many other functions. The neuronal connections give us memory, process our thoughts, and control our muscles and speech, interpret what we see and hear, and many other functions. Among all these cells you will find about 30 different neurotransmitters. These are molecules that, for example, control whether a neuron discharges electricity or not. When a neuronal circuit of, maybe 1500 connections (synapses) fires to the end of the circuit, you may have a thought come to your consciousness. That is, you are aware of the thought. Most thoughts are of the unconscious nature with you never being aware of their existence. Of course there are thousands of books written about brain function, so in the little space left on this page, you will learn very little about brain function. However, in this space, we will learn just a little about the function of the neurotransmitter dopamine, and how it is significant in your life of learning.

Dopamine has many uses in the brain, as do most of the neurotransmitters, but what is significant here is that the release of dopamine causes you to have a sensation of "feeling" good. Neurons release dopamine when you know the answer to a question, or when you actually answer the question in class, or tell a classmate or parent – likewise for doing your homework correctly. Or for example, if you recognize a mathematical pattern and make a prediction based on your generalization of the pattern, neurons will release dopamine and you will feel good. You may not feel great, but there will be a sensation somewhere within your brain that feels good. On the down side, if you give an answer or make a generalization that is wrong, dopamine is not released. So you don't feel good. The brain prefers feeling good and will repeat actions that release more dopamine; as a result, the related neural memory circuits become stronger and are more likely to fire in the future. This brain function is basic to learning. Without the dopamine reward, you could not learn anything. It is a great feeling to be right about what you do in algebra class. Wow, knowing algebra has a significant impact on the brain.

Basic Brain Function – Meaning and Understanding

Hopefully you have noticed that many new concepts/skills have been presented using a related real-world situation used to help you understand the algebra. Real-world situations are concrete ideas as opposed to algebra which is abstract in nature. The problem with abstractions (algebra/math) is that the average brain needs help in trying to understand the abstraction. The brain tries to figure out abstract ideas by interpreting them in concrete (real) terms. But for many students, what is "real" about, for example, an abstract idea like function, or $2x + 5x$, or the root of an equation? So the problem is that we may have difficulty understanding algebra because of its abstractness. This is the reason this textbook teaches new ideas within real-world contexts. That is, it provides you with the desperately needed concrete connection – making abstract algebra more understandable. There is more to the story. These emotional stakes enable you to understand certain concepts more quickly, AND reason about algebra at a higher cognitive level than where there isn't the personal stake.

It gets even better. These real-world contexts (like the I.V. drip Exploration in the Chapter Two ancillary activity book) give you additional neural associations (connections) that are required to be able to create correct long-term memory with recall. Perhaps the single most important ingredient in improving your memory of the algebra in this textbook is to encode your learning by adding meaning—what you get from real-world contextual situations.

