Authoritarian Power Sharing with Endogenous Mobilization

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July 18, 2023

Abstract

In existing theories, dictators share power to solve a commitment problem. The opposition can compel policy concessions only upon mobilizing a rebellion threat. If mobilization opportunities arise rarely, the commitment problem binds and the ruler shares power to prevent costly conflict. In these models, sharing power bolsters the opposition's spoils (commitment effect), but not the opposition's probability of winning a revolt (threat-enhancing effect) or the frequency of mobilization (treated as exogenous). Incorporating all these elements yields three implications for strategic power sharing. (1) Modeling endogenous mobilization eliminates the equilibrium relationship between the frequency of mobilization and conflict. (2) Sharing more power, despite enhancing commitment, does not necessarily reduce prospects for conflict. The threat-enhancing effect makes the overall relationship inverted U-shaped. (3) A credible threat for the opposition to revolt is a necessary but not sufficient condition to induce power sharing; the ruler may prefer instead to endure conflict.

1 Introduction

Authoritarian leaders frequently share power with the opposition. In contemporary regimes, it is common to give cabinet positions or seats in the legislature to members of rival parties or ethnic groups (Arriola 2009; Francois et al. 2015; Meng 2020). In many historical European regimes, monarchs allocated rights over land and allowed nobles to veto requests for extraordinary taxes, and later granted spending powers and expanded the franchise (Stasavage 2011; Cox and Dincecco 2021; Kenkel and Paine 2023). In some cases, institutional concessions were sufficiently deep to yield transitions from dictatorships to fully democratic regimes.

A commonly posited rationale is that sharing power solves a commitment problem. The opposition enjoys periodic windows of opportunity to mobilize a mass anti-regime movement. A credible threat of revolt compels the ruler to offer concessions. These can constitute a *temporary* redistribution of spoils and one-off policy concessions, or *permanent* institutional concessions such as democratization (Acemoglu and Robinson 2000, 2001, 2006*b*) or power sharing within an authoritarian regime (Castañeda Dower et al. 2018, 2020; Powell 2021). Institutional concessions convey a permanent flow of rents for the opposition. All else equal, the ruler prefers to distribute temporary transfers only; why also give up something tomorrow if the opposition no longer poses a threat? But herein lies the ruler's commitment problem. Because the opposition cannot constantly sustain anti-regime mobilization, accepting a temporary transfer today might result in forgoing redistribution tomorrow—if they are no longer in a position to compel another spoils transfer. When mobilization periods arise infrequently, the opposition prefers to fight during its fleeting moments in the sun and "lock in" its temporary advantage rather than to accept a transient concession and forgo consumption in the future (Powell 2004). Recognizing the need to alleviate the commitment problem, the ruler is compelled to offer permanent institutional concessions.[1]

[1]The idea that dictators (and nascent democrats) must share power to solve commitment problems is widespread in the broader substantive literature as well (North and Weingast 1989; Gandhi 2008; Magaloni 2008; Ansell and Samuels 2014; Helmke 2017; Albertus and Menaldo 2018).

This framework illuminates in a parsimonious way how sharing power solves the dictator's commitment problem. Nonetheless, these models overlook two key elements of the dilemma of authoritarian power sharing. First, sharing power not only bolsters the ruler's *commitment* to deliver future spoils to the opposition, but distributing valuable resources and bringing the opposition closer to the center of power also *enhances their threat* (Meng et al. 2023). Treating the distribution of coercive power as a function of the power-sharing choice contrasts with the standard assumption that, when mobilized, the opposition's probability of succeeding in a revolt is a fixed parameter (usually set to 1) that is unaffected by power sharing. Empirically, access to positions in the central government makes it more feasible to ally with the state military to stage a coup, as opposed to organizing a private military to fight a rebellion (Roessler 2016). Other forms of power sharing similarly shift empower a challenger vis-à-vis the ruler, such as granting regional autonomy (Cederman et al. 2015), naming a Minister of Defense (Meng and Paine 2022), or designating a successor (Meng 2021; Kokkonen et al. 2022). How do rulers share power in light of the countervailing commitment and threat-enhancing effects?

Second, the frequency with which the opposition mobilizes is a key parameter in existing models. Infrequent mobilization triggers the ruler's commitment problem and prompts an offer of permanent institutional concessions, whereas frequent mobilization enables the ruler to buy off the opposition with temporary transfers only. Existing models treat the frequency of mobilization as a fixed parameter, that is, unaffected by other strategic decisions. However, this is unsatisfying for understanding the consequences of authoritarian power sharing. Intuitively, we would anticipate that the degree of power sharing would affect the opposition's incentives to mobilize against the regime. The effect of sharing more power could, in principle, go in either direction. Higher guaranteed spoils would conceivably lessen motives to mobilize, the goal of which is to pressure the ruler for more spoils. By contrast, greater prospects for victory should enhance the opposition's desire to mobilize, given the greater pressure it can exert on the ruler. How does sharing power affect the opposition's mobilization choices? How, in turn, do these mobilization choices affect prospects for conflict and the ruler's incentives to share power?

This paper provides new insights into commitment problems and power sharing by (1) linking the amount of power sharing to both the division of spoils and the distribution of coercive power and (2) endogenizing the opposition's choice to mobilize. The ruler's power-sharing choice determines what fraction of societal wealth is permanently allocated to the opposition. A more generous allocation raises both the opposition's basement level of spoils in each period and the opposition's probability of succeeding in a revolt. The model endogenizes the frequency of mobilization by assuming the opposition chooses in each period whether to pay a cost to mobilize, which fluctuates stochastically over time. Whenever the opposition mobilizes, the ruler proposes a temporary transfer. Such *pure spoils* transfers differ from power-sharing deals because the temporary transfer does not affect the distribution of power and does not guarantee concessions in future periods.

Surprisingly, when mobilization is costly and endogenous, the frequency of mobilization *does not affect* equilibrium prospects for conflict or institutional reform. Less frequent mobilization raises the opposition's opportunity cost to accepting a temporary transfer, as in existing models, but also lowers the average cost of mobilizing, a novel effect in the present model. These two effects perfectly offset each other because, at a threshold cost that determines the opposition's mobilization decision in each period, it is indifferent about whether to mobilize. This finding also underscores that the direct commitment and threat-enhancing effects are more fundamental in the ruler's power-sharing calculus than is the indirect effect on the frequency of mobilization.

Contrary to the standard notion that sharing power resolves the ruler's commitment problem, sharing power does not necessarily succeed at buying off the opposition from revolting because of the threat-enhancing effect. The relationship between the degree of power sharing and whether conflict occurs along the equilibrium path is inverted U-shaped. Very low wealth for the opposition yields a very low probability of winning, and very high wealth creates a very high opportunity cost to fight (and to mobilize); but for intermediate levels, the ruler may be unable to buy off the opposition. This affects the ruler's decision regarding whether and how much power to share with the opposition. The power-sharing choice is subject to a lower bound which, substantively, corresponds with regional strongholds or already-acquired positions in the government that yield spoils and influence for the opposition independent of the ruler's strategic power-sharing choice (and that the ruler cannot take away). When this lower bound is very low, *opposition credibility* fails because the opposition will not fight even if the ruler refuses to share power. But even if the opposition is credible, the ruler does not necessarily share power. *Ruler willingness* fails if the ruler prefers to face a fight in the future rather than to buy off the opposition with sizable institutional concessions. The ruler shares power beyond the lower bound if and only if both conditions hold.

These findings advance our understanding of the strategic motives for and consequences of authoritarian power sharing by untangling how sharing power affects the ruler's commitment ability and the opposition's coercive power and frequency of mobilization. These innovations are crucial for widespread empirical research on power sharing (see Meng et al. 2023 for a recent review), amid discussions of the endogeneity of power-sharing arrangements (Pepinsky 2014; Meng 2020, 190–92). Regarding other formal models, a handful of conflict bargaining models incorporate endogenous shifts in the distribution of coercive power (Fearon 1996; Powell 2013; Gibilisco 2021). However, these authors model permanent shifts in power, in contrast to fluctuating threats in window-of-opportunity models. Others, such as Bueno de Mesquita (2010) and Casper and Tyson (2014), study endogenous mobilization in the form of coordination problems, although not in the similar context of dynamic bargaining and strategic institutional reform. No existing model, to my knowledge, treats opposition mobilization as endogenous in a similar manner to the present model.

The most closely related models are those that assume the (exogenous) frequency of mobilization is correlated with the probability of revolt success (Paine 2022*b*; Luo 2022; Little and Paine 2023). However, these models assume this relationship as opposed to characterizing it as an endogenous function of strategic mobilization decisions. Furthermore, these models assume a positive relationship between power sharing and the frequency of mobilization, whereas here I characterize an equilibrium relationship that is inverted U-shaped. Finally, these models incorporate only the threat-enhancing effect of power sharing and not the commitment effect, as the ruler either cannot alter the opposition's basement level of spoils or this parameter is unrelated to the distribution of coercive power.

2 Setup

A ruler and opposition actor bargain over spoils throughout an infinite-horizon interaction. Periods are denoted by $t = 0, 1, 2, \ldots$ and the players share a common discount factor $\delta \in (0, 1)$. Total societal output equals 1 in every period. Before the bargaining interaction begins, the ruler makes a one-time power-sharing choice that determines the basement fraction of spoils that the opposition controls in every period, $\omega \in [\omega^{\text{min}}, 1]$, with $\omega^{\text{min}} > 0$. One way to interpret the one-time power-sharing choice is that critical junctures occur in which a ruler has sufficient agency to permanently alter the distribution of spoils and power vis-à-vis the opposition, perhaps because the ruler has recently ascended to power and has open cabinet positions or has newly confiscated land to redistribute. A single such critical juncture occurs in the present model, and we examine the consequences of this choice for the subsequent interaction.[2]

After ω is set, in every period of the infinite-horizon interaction, Nature determines the cost c_t that the opposition would pay upon mobilizing against the ruler. In a fraction $\alpha \in (0, 1)$ of periods, mobilization is feasible and Nature draws a cost from an iid distribution $F(c_t) \sim U[0, c^{\text{max}}]$, with $c^{\text{max}} \in (0, 1]$.[3] Conversely, in a fraction $1 - \alpha$ of periods, mobilization is infeasible (we can think

[2]Relaxing the assumption that the power-sharing choice occurs once creates technical problems because the level of power sharing affects the endogenous mobilization choice. By contrast, in models such as Acemoglu and Robinson (2006*b*), Castañeda Dower et al. (2018, 2020), and Powell (2021) in which the ruling faction makes a power-sharing choice in every period, the frequency of mobilization is fixed across time. Typically, window-of-opportunity models become intractable when the ruler can alter the distribution of power multiple times. For a recent exception (which holds fixed other moving pieces in the present model), see Luo (2022).

[3]The associated pdf is f. Most results hold for any continuous distribution with full support

of the cost as infinite in such periods). Existing models are equivalent to a setup in which $c^{\max} = 0$, and hence mobilization is costless in certain periods; and otherwise is infeasible, with α governing the frequency of each type of period.[4]

After observing the draw for the cost of mobilization, which is common knowledge, the opposition decides whether to mobilize. If not, then the ruler and opposition respectively consume $1 - \omega$ and ω, and the game moves to the next period.

If instead the opposition mobilizes, then bargaining occurs. The ruler proposes to transfer a fraction of its current-period wealth, $x_t \in [0, 1-\omega]$. Upon receiving a proposal, the opposition responds by accepting or initiating a conflict. Upon accepting, the ruler and opposition respectively consume $1 - \omega - x_t$ and $\omega + x_t - c_t$ and the game moves to the next period, with the average per-period continuation values denoted as V^R and V^O for the ruler and opposition, respectively.[5] The opposition wins a conflict with probability $p(\omega) \in (0, 1]$ and the ruler wins with complementary probability. We assume that $p(\omega)$ is continuous, strictly increasing, and strictly concave and satisfies standard boundary conditions: $p(0) = 0$, $p(1) = 1$, $\lim_{\omega \to 0} p'(\omega) = \infty$, and $\lim_{\omega \to 1} p'(\omega) < 1$.[6] The winner of a conflict controls all domestic wealth forever, although a fraction $\mu \in (0, 1)$ of wealth is permanently destroyed.[7] Figure 1 presents the tree for the stage game.

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over $[0, c^{\max}]$, and thus I express most results using the general distribution function. The uniformity assumption is used to ensure the ruler's utility function is strictly monotonic in ω (see Propositions 2 and 3).

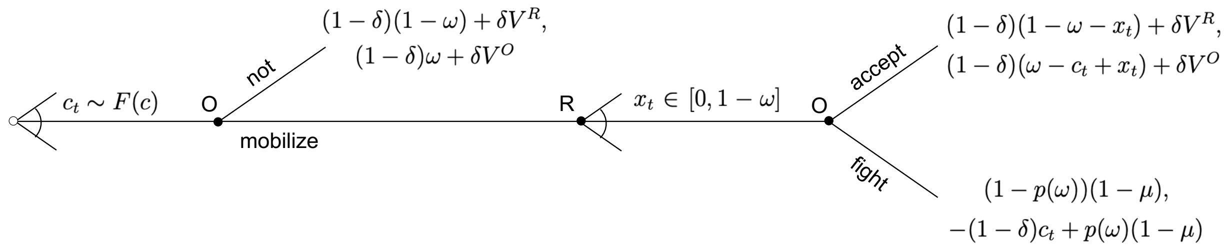
[4]Most findings for the present model go through for any $\alpha \in (0, 1]$. However, later, we impose an upper bound on α to ensure the ruler's utility function is strictly monotonic in ω (see Propositions 2 and 3).

[5]The findings are qualitatively identical if mobilizing requires the opposition to invest some or all its endowment ω; see footnote 9 for details.

[6]An example of a functional form that satisfies all these assumptions is $p(\omega) = \omega^{\gamma}$ for any $\gamma \in (0, 1)$.

[7]In Lemma 3, we impose an additional restriction on μ.

Figure 1: Tree of Stage Game



3 BARGAINING WITH FIXED POWER SHARING

Fixing the opposition's baseline level of wealth ω (which expresses the degree of power sharing) enables us to analyze how bargaining unfolds along the infinite horizon. Unlike existing models, the optimal transfer does not depend on the frequency with which the opposition mobilizes. Instead, whether the ruler can buy off the opposition hinges solely on the relationship between countervailing commitment and threat-enhancing effects. Raising the opposition's wealth exhibits diminishing marginal returns on bolstering its probability of winning, which implies that conflict occurs along the equilibrium path of play for intermediate levels of ω. The solution concept is Markov Perfect Equilibrium (MPE), and most proofs are in the appendix.

Characterizing the optimal transfer. In any period the opposition does not mobilize, its lifetime average per-period expected consumption is is $(1-\delta)\omega+\delta V^O$. If instead the challenger mobilizes, its incentive-compatibility condition to accept a transfer is

$$(1-\delta)(\omega-c_t+x_t)+\delta V^O \geq -(1-\delta)c_t+p(\omega)(1-\mu). \tag{1}$$

We examine the interior-optimal transfer that makes the opposition indifferent between accepting and fighting; later we explain the conditions under which this is in fact the equilibrium transfer offer. We write this transfer amount as x^*, which is identical in any period the opposition mobilizes, and restate the previous condition as an equality

$$(1-\delta)(\omega + x^*) + \delta V^O = p(\omega)(1-\mu). \tag{2}$$

Comparing consumption amounts demonstrates that, along a peaceful path, the opposition mobilizes if and only if the cost of doing so is lower than the temporary transfer it will receive, $c_t \leq x^*$.[8] For some purposes, it is helpful to express this threshold as $\hat{c} \equiv x^*$. Given the distribution of draws of c_t, the opposition will mobilize in a fraction $\alpha F(\hat{c})$ of periods and pay an average cost of mobilization $c^{\text{avg}} \equiv \frac{\int_0^{\hat{c}} c_t dF(c)}{F(\hat{c})}$ in such periods. Thus its continuation value is

$$V^O = \omega + \alpha F(\hat{c})(x^* - c^{\text{avg}}). \tag{3}$$

Substituting this term into Equation 2 enables us to implicitly characterize the interior-optimal transfer as a function of parameters:

$$\Theta(\omega) \equiv \underbrace{\omega}_{\text{Commitment}} - \underbrace{p(\omega)(1-\mu)}_{\text{Threat enhancing}} + \underbrace{\big(1-\delta(1-\alpha F(\hat{c}))\big)x^*}_{\text{Changes frequency of mobilization}} - \underbrace{\delta\alpha F(\hat{c})c^{\text{avg}}}_{\text{Changes average cost}} = 0. \tag{4}$$

The opposition's wealth ω affects x^* through four distinct channels, two direct and two indirect. The first direct effect is the *commitment effect*. Higher ω bolsters the opposition's basement level of consumption in every period. This raises the opposition's opportunity cost to fighting, and therefore lowers the transfer needed to secure acquiescence. The second direct effect is the *threat-enhancing effect*. Higher ω improves the opposition's prospects for winning a conflict, which prompts a (credible) demand for a larger transfer. Two indirect effects arise because ω also alters the threshold cost at which the opposition mobilizes, $\hat{c}$. This impacts both the fraction of periods in which the opposition mobilizes, $\alpha F(\hat{c})$, and hence the frequency with which the opposition

[8]Later we demonstrate that the opposition's optimal mobilization choice is identical along a conflictual path.

consumes the transfer; and the average cost the opposition pays when mobilizing, c^{avg}.[9]

Frequency of mobilization does not affect equilibrium outcomes. The indirect effects cancel out in equilibrium and do not affect the equilibrium transfer, as stated in Lemma 1.

Lemma 1 (Effects of opposition wealth)**.** *The opposition's wealth ω affects the interior-optimal transfer x^* only through its direct effects.*

The proof follows from disaggregating the total derivative into its constituent effects, which yields

$$\frac{dx^*(\omega, p(\omega), \hat{c}(\omega), F(\hat{c}(\omega))}{d\omega} = \underbrace{\frac{\partial x^*}{\partial \omega} + \frac{\partial x^*}{\partial p} p'(\omega)}_{\text{Direct effects}} + \underbrace{\left(\frac{\partial x^*}{\partial \hat{c}} + \frac{\partial x^*}{\partial F} f(\hat{c}) \right) \frac{d\hat{c}}{d\omega}}_{\text{Indirect effects}}. \tag{5}$$

The indirect effects cancel out because

$$\frac{\partial x^*}{\partial \hat{c}} + \frac{\partial x^*}{\partial F} f(\hat{c}) = \frac{\delta \alpha f(\hat{c})}{1 - \delta(1 - \alpha F(\hat{c}))} (\underbrace{x^* - \hat{c}}_{=0}) = 0, \tag{6}$$

which proves the lemma. As in existing models, mobilizing less frequently prompts the opposition to demand more in each mobilization period, given the lower fraction of future periods in which they will gain the additional transfer. This is captured by the term x^* in parentheses in Equation

[9]The last term is multiplied by δ. The cost of mobilization has already been sunk at the bargaining stage and, therefore, is not subtracted out in the present period (this is straightforward to see in Equation 1). This explains why the following alternative assumption would not qualitatively change the findings. If mobilizing required the opposition to invest some or all of its endowment ω, it would have already sunk this cost by the bargaining stage. The additional cost to mobilizing would, nonetheless, alter the opposition's calculus by affecting consumption in *future* periods, but would simply reduce in magnitude the effect of ω on the opportunity cost of conflict (which arises because the opposition permanently relinquishes ω upon losing a conflict). Thus, I prefer the simpler setup without this redundant moving piece.

6. However, the present model contains an additional, countervailing effect because mobilization is endogenous. A opposition who mobilizes less frequently has a lower cost threshold $\hat{c}$ and, therefore, pays a lower average cost in periods it chooses to mobilize. This lowers its transfer demand. The two effects perfectly offset each other because, at the threshold cost $\hat{c}$, the opposition is indifferent between (a) mobilizing and gaining x^* and (b) not mobilizing. This contrasts with an (implicit) assumption in existing models with exogenous mobilization: the opposition never pays a cost to mobilizing, and therefore strictly prefers to mobilize whenever possible.

Consequently, the present model explains a shortcoming with the standard finding that an opposition who mobilizes infrequently has greater bargaining leverage. Such an opposition actor also pays lower average costs to mobilizing, which decreases its propensity to fight. The two effects cancel out when mobilization is costly and these costs are drawn from a continuous distribution.

Instead, ω affects the equilibrium transfer only through the commitment and threat-enhancing effects. The overall relationship between ω and each of x^* and $F(\hat{c})$ is negative quadratic, the intuition for which we discuss later when analyzing the conditions under which the equilibrium path of play is peaceful.

Lemma 2 (Effect of opposition wealth on transfer and frequency of mobilization)**.** *The relationship between ω and each of x^* and $F(\hat{c})$ is negative quadratic. Each function reaches a unique maximum at $\omega = \hat{\omega} \in (0, 1)$ and equals 0 at $\omega = \{0, \omega_0\}$, for a unique $\omega_0 \in (\hat{\omega}, 1)$.*

Ruler's preference to buy off the opposition. A standard result in conflict bargaining models is that the player making the bargaining offers always prefers to buy off the other player, if possible (e.g., Fearon 1995). Because the offerer can hold the offeree down to indifference, the offerer consumes the entire surplus saved by preventing fighting, which is assumed to be costly. This is not guaranteed in the present model, though, because conflict is not necessarily (net) costly. Fighting permanently destroys a fraction μ of societal output, but total surplus is not 1 along a peaceful path, either, because the opposition periodically pays a cost to mobilize. The ex ante expected cost in each period is $\alpha F(\hat{c})c^{\text{avg}}$, which is discounted by one period from the perspective of any bargaining interaction because the opposition would have already sunk the present-period cost c_t (see footnote 9). Costly mobilization lowers the opposition's opportunity cost to fight, which detracts from the ruler's consumption by enabling the opposition to credibly demand a larger transfer in return for forgoing a conflict. Lemma 3 characterizes the critical cost-of-conflict threshold, which we assume to hold throughout the analysis.

> **Lemma 3** (Threshold cost of fighting)**.** *A unique threshold $\hat{\mu} \in (0, 1)$ exists such that if $\mu > \hat{\mu}$, then the ruler strictly prefers to buy off the opposition with the optimal transfer than to trigger conflict.*

However, other standard results hold for all parameter values. The ruler never proposes a transfer that the opposition strictly prefers to accept; the ruler could do better by deviating to a slightly lower offer that would still be accepted. Furthermore, the ruler prefers to buy off the opposition if possible even if ω is very large. Although the ruler is tempted in this circumstance to trigger fighting and hence to probabilistically gain a larger fraction of societal wealth upon winning a conflict, this effect never dominates because $p(\omega) > \omega$.[10]

Intermediate wealth levels yield a peaceful equilibrium. An equilibrium in which conflict occurs with probability 0 in every period, referred to as a peaceful equilibrium, is possible if and only if the optimal transfer does not exceed the ruler's wealth, $x^* \leq 1 - \omega$. If this inequality fails, then the ruler cannot buy off the opposition in a period it has mobilized, yielding conflict along the equilibrium path. Conversely, if the inequality holds, then there is a unique peaceful equilibrium.

To understand how the opposition's wealth ω affects the equilibrium budget surplus $1 - \omega - x^*$,

[10]The inequality follows from the conditions placed on $p(\omega)$, in particular strict concavity.

we restrict attention to the direct effects on x^* because of Lemma 1:

$$\frac{d}{d\omega}(1-\omega-x^*) = -\left[1+\left(\frac{\partial x^*}{\partial \omega}+\frac{\partial x^*}{\partial p}p'(\omega)\right)\right] =$$

$$-1+\frac{1}{1-\delta(1-\alpha F(\hat{c}))}\Big(\underbrace{1}_{\text{Commitment}}-\underbrace{p'(\omega)(1-\mu)}_{\text{Threat-enhancing}}\Big). \quad (7)$$

The commitment and threat-enhancing effects, and not the frequency of mobilization, determine prospects for conflict in equilibrium. The overall effect of ω can go in either direction because the commitment and threat-enhancing effects cut in opposite directions; the former pushes toward peace whereas the latter pushes toward conflict.[11]

Proposition 1 characterizes the equilibrium strategies and path of play for an exogenously set value of ω, showing that conflict occurs along the equilibrium path only for intermediate values of ω. The opposition wins with very low probability at low ω, as $p(0) = 0$, and therefore conflict cannot occur when ω is too low. However, the magnitude of the threat-enhancing effect is largest at small values of ω because the opposition's probability of winning a conflict, $p(\omega)$, exhibits diminishing marginal returns in ω. By contrast, the commitment effect is linear in ω and, at high levels of ω, the opposition's basement level of spoils is so high that fighting is suboptimal regardless of its probability of winning. Combining these two effects implies that only at intermediate levels is it possible to have $x^* > 1-\omega$. This also explains why the relationship between ω and each of x^* and $F(\hat{c})$ is negative quadratic, presented earlier in Lemma 2.

[11]An additional effects arises because higher permanent spoils for the opposition reduces the available budget for the ruler to make a temporary transfer in a mobilization period. It is straightforward to show that the commitment effect dominates this additional effect, $1 < \frac{1}{1-\delta(1-\alpha F(\hat{c}))}$. A unit increase in the opposition's wealth increases the opportunity cost of fighting in the present and in every future mobilization period, whereas a unit loss in the ruler's per-period budget affects the equilibrium budget surplus only in the present period.

Proposition 1 (Equilibrium for fixed opposition wealth)**.**

In any period t, the opposition mobilizes if $c_t \leq \hat{c}$ and does not mobilize otherwise. Upon mobilizing, the opposition accepts any $x_t \geq x^*$ and fights otherwise.

- *Peaceful path of play: If $x^* \leq 1 - \omega$, then the ruler proposes a transfer of*

$$x_t = \begin{cases} x^* & \text{if } \omega < \omega_0 \\ 0 & \text{if } \omega \geq \omega_0, \end{cases} \tag{8}$$

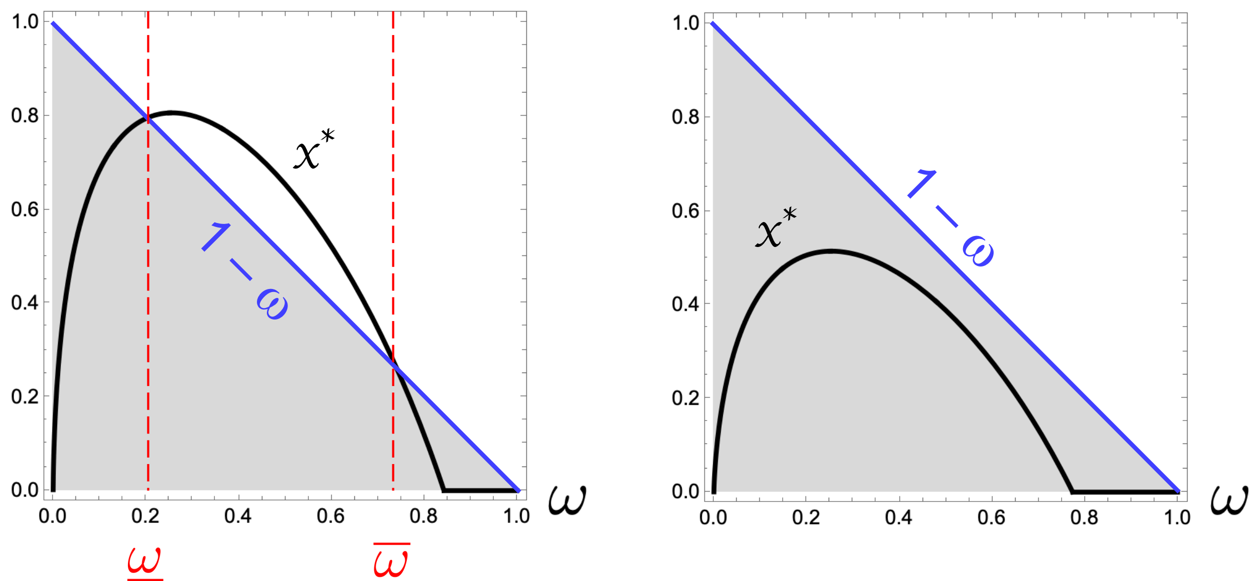
  for ω_0 characterized in Lemma 2. In equilibrium, the opposition accepts.

- *Conflictual path of play: If $x^* > 1 - \omega$, then the ruler is indifferent among all feasible transfers and the opposition fights in response to any offer.*

The equilibrium path of play is peaceful either for all values of ω, or for two non-connected and compact intervals $\omega \in [0, \underline{\omega}] \cup [\overline{\omega}, 1]$, with $0 < \underline{\omega} < \overline{\omega} < \omega_0$. Otherwise, the path of play is conflictual.

Figure 2 illustrates the two qualitatively distinct possibilities. The black line is the equilibrium transfer x^* and the gray shaded region is the set of feasible offers, which shrinks in ω because permanently giving away more to the opposition reduces the remaining spoils that the ruler can offer as a temporary transfer. All other parameters are fixed at values listed in the note accompanying the figure. In both panels, the optimal transfer initially rises in ω before dropping, but the panels differ in the steepness with which x^* rises. In the left panel, the threat-enhancing effect is sufficiently large in magnitude (causing the spike in x^*) that the transfer exceeds the budget constraint for intermediate levels of ω. By contrast, the magnitude of the threat-enhancing effect is lower in the right panel, and hence the transfer is always feasible. Later, when analyzing comparative statics, we discuss how to substantively interpret these differences.

Figure 2: Equilibrium Offer and Conflict Region



Notes: Functional forms: $p(\omega) = \omega^\gamma$ and $F \sim U[0, c^{\max}]$. Parameter values: $\delta = 0.9$, $\mu = 0.05$, $c^{\max} = 1$, $\alpha = 0.55$. Left panel: $\gamma = 0.7$. Right panel: $\gamma = 0.8$.

4 Optimal Power Sharing

The ruler shares power beyond the lower bound, that is, sets $\omega > \omega^{\min}$, if and only if two conditions are met: opposition credibility and ruler willingness.[12]

4.1 Opposition Credibility

The opposition is credible if the ruler must share power above the lower bound to prevent conflict, that is, if the equilibrium path of play is conflictual for $\omega = \omega^{\min}$. As we know from Proposition 1, this is true for any $\omega^{\min} \in [\underline{\omega}, \overline{\omega}]$. To show that opposition credibility is necessary for power sharing, it suffices to demonstrate that the ruler's average per-period consumption strictly decreases in ω along a peaceful path. Thus, if ω is not needed to switch the bargaining path from conflictual to peaceful, the ruler will not increase ω in the opposition's favor. Formally, we need the following

[12]See also Kenkel and Paine (2023) and Meng et al. (2023).

derivative to be negative

$$\frac{d}{d\omega}\Big(1-\omega-\alpha F(\hat{c})x^*\Big) = \underbrace{-1}_{\text{ⓐ}}\underbrace{-\alpha F(\hat{c})\frac{dx^*}{d\omega}}_{\text{ⓑ}}\underbrace{-\alpha f(\hat{c})x^*\frac{d\hat{c}}{d\omega}}_{\text{ⓒ}}. \tag{9}$$

This is similar to the derivative in Equation 7, which evaluates the effect of ω on the equilibrium budget surplus (and hence the ruler's consumption) in a period in which the opposition has mobilized. The present derivative, by contrast, multiplies the optimal transfer by the ex ante per-period probability of opposition mobilization, $\alpha F(\hat{c})$. Terms a and b can be rewritten as

$$-1-\alpha F(\hat{c})\frac{dx^*}{d\omega} = -1 + \frac{\alpha F(\hat{c})}{1-\delta(1-\alpha F(\hat{c}))}\Big(\underbrace{1}_{\text{Commitment}} - \underbrace{p'(\omega)(1-\mu)}_{\text{Threat-enhancing}}\Big) < 0. \tag{10}$$

A marginal increase in ω gives away one unit of consumption from the ruler to the opposition in every period. This is partially offset by the commitment effect, which lowers the transfer needed to buy off the opposition in a mobilization period. However, because mobilization occurs in only a fraction $\alpha F(\hat{c})$ of all periods, the net effect is negative.[13] Finally, the threat-enhancing effect enables the opposition to credibly demand a higher transfer, which also lowers the ruler's consumption.

The complication with signing Equation 9 arises because of term c, which captures the effect of ω on the equilibrium frequency of mobilization periods. This final term reinforces the aforementioned negative effects for $\omega < \hat{\omega}$ (see Lemma 2). In this range, marginal increases in ω prompt the opposition to mobilize more frequently, which lowers the ruler's consumption. However, for

[13]To square this observation with the discussion in footnote 11, note that

$$\frac{\alpha F(\hat{c})}{1-\delta(1-\alpha F(\hat{c}))} < 1 < \frac{1}{1-\delta(1-\alpha F(\hat{c}))}.$$

$\omega \in (\hat{\omega}, \omega_0)$, the effect flips and thus the last term in Equation 9 cuts in the opposite direction. In Proposition 2, we demonstrate that Equation 9 is net negative for all values of ω if the cost-of-mobilization threshold $\hat{c}$ is not too responsive to changes in ω. This is true if the fraction of periods in which mobilization is possible, α, is sufficiently low. Consequently, when this is true, we recover the standard intuition that a credible threat by the opposition is a necessary condition for power sharing.

Proposition 2 (Opposition credibility)**.** *Assume $\alpha < \tilde{\alpha}$, for $\tilde{\alpha} > 0$ defined in the proof. Suppose $\omega^{min} \in [0, \underline{\omega}] \cup [\overline{\omega}, 1]$, which implies that opposition credibility fails because the equilibrium path of play is peaceful for $\omega = \omega^{min}$. Then the ruler sets $\omega = \omega^{min}$.*

4.2 Ruler Willingness

A credible threat by the opposition is a necessary, but not sufficient, condition for power sharing. This surprising because, if opposition credibility holds, we might anticipate that the ruler would always set ω high enough to buy off the opposition. Doing so is always feasible because the upper bound of the conflict range satisfies $\overline{\omega} < 1$. As discussed earlier, the standard logic in conflict bargaining models is that the player making the offers always prefers to induce peace, if possible, because that player consumes the surplus saved by preventing fighting.

This logic does not apply to the power-sharing choice, however. Raising ω bolsters the opposition's probability of winning, which enables it to demand higher transfers even along a peaceful equilibrium path. Because of the threat-enhancing effect, the ruler compares two scenarios. At $\omega = \omega^{\min}$, a conflict will occur eventually but the ruler wins with relatively high probability. Alternatively, the ruler can raise ω to $\overline{\omega}$, which yields peace but requires the ruler to compensate the opposition for its higher probability of winning with a larger transfer in every mobilization period.[14]

[14]Proposition 2 implies that the ruler will never set ω above the level needed to induce a peaceful path of play.

Which of these two choices the ruler prefers depends on the precise value of $\omega^{\min}$ within the range $[\underline{\omega}, \overline{\omega}]$. If $\omega^{\min}$ is relatively close to $\underline{\omega}$, then the ruler must shift substantially more power in the opposition's favor to garner peace. In this circumstance, the ruler prefers to endure conflict, and ruler willingness fails. If instead $\omega^{\min}$ is relatively close to $\overline{\omega}$, then raising ω to this level only minimally bolsters the opposition's probability of winning. Consequently, this outweighs in magnitude the costliness of fighting. The ruler shares power in this circumstance, and thus we say that ruler willingness holds. When both opposition credibility and ruler willingness hold, the ruler sets $\omega = \overline{\omega} > \omega^{\min}$.

Proposition 3 (Ruler willingness)**.** *Assume $\alpha < \min\left\{\tilde{\alpha}, \tilde{\alpha}', \frac{1}{2\delta}\right\}$, for $\tilde{\alpha}' > 0$ defined in the present proof and $\tilde{\alpha} > 0$ defined in the proof of Proposition 2.*

Suppose $\omega^{min} \in [\underline{\omega}, \overline{\omega}]$, which implies that opposition credibility holds because the equilibrium path of play is conflictual for $\omega = \omega^{min}$.

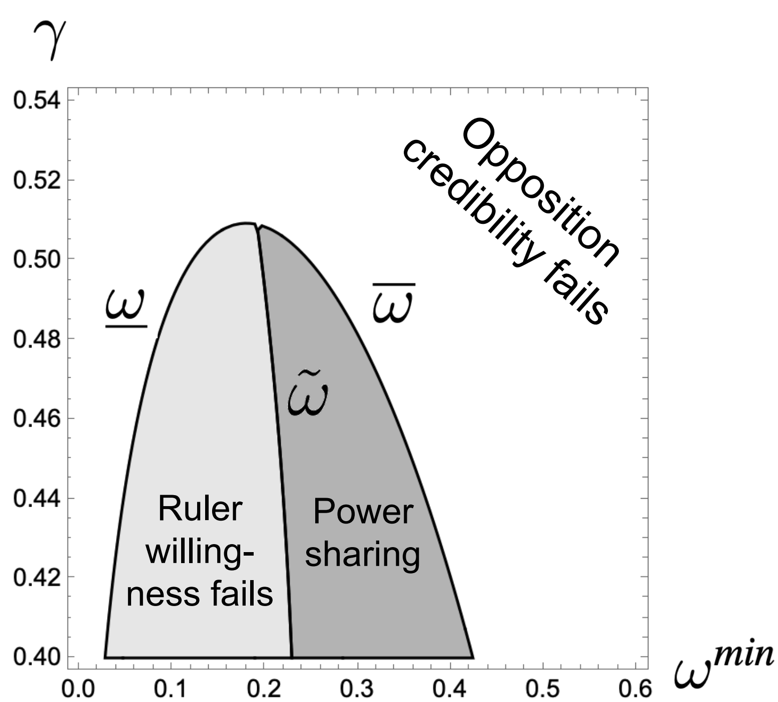
- *If $\omega^{min} < \tilde{\omega}$, for a threshold $\tilde{\omega} < \overline{\omega}$ defined in the proof, then ruler willingness fails and the ruler sets $\omega = \omega^{min}$.*
- *If $\omega^{min} > \tilde{\omega}$, then ruler willingness holds and the ruler sets $\omega = \overline{\omega}$.*

4.3 Comparative Statics

A key determinant of equilibrium outcomes is the relationship between the permanent division of spoils, ω, and the opposition's probability of succeeding in a revolt, $p(\omega)$. If $p'(\omega)$ is insufficiently steep at low values of ω, then fighting does not occur along the equilibrium path regardless of the value of ω (see Figure 2). By contrast, if $p'(\omega)$ is steeper, then opposition credibility holds over a non-empty range $\omega^{\min} \in (\underline{\omega}, \overline{\omega})$.

Figure 3 provides more insight into this relationship by presenting a region plot to illustrate comparative statics (formal analysis in progress). The x-axis is $\omega^{\min}$ and the y-axis captures the steepness of $p(\omega)$. Using the functional form $p(\omega) = \omega^{\gamma}$, lower values of γ correspond with a steeper function. All other parameter values are set to values listed in the accompanying note. The lines depict the three threshold values of ω from the preceding propositions, each of which is a function of γ. The envelope created by the $\underline{\omega}(\gamma)$ and $\overline{\omega}(\gamma)$ functions encompasses the parameter space in which opposition credibility holds. This parameter space is partitioned by $\tilde{\omega}(\gamma)$, which separates the region in which ruler willingness holds (and hence the ruler sets $\omega = \overline{\omega}$) from the region in which this condition fails and thus the ruler sets $\omega = \omega^{\min}$ despite conflict occurring at this value of ω.

Figure 3: Comparative Statics



Notes: Functional forms: $p(\omega) = \omega^{\gamma}$ and $F \sim U[0, c^{\max}]$. Parameter values: $\delta = 0.9$, $\mu = 0.3$, $c^{\max} = 1$, $\alpha = 0.55$.

As the figure shows, increasing the opposition's threat by lowering γ exhibits two countervailing effects. Compellence is indeed a necessary condition for power sharing. At $\omega^{\min} = 0.30$, a low enough value of γ is needed to prompt power sharing, because otherwise opposition credibility fails. However, as γ decreases, the range of parameters in which ruler willingness fails widens. Lower γ raises the threshold $\overline{\omega}$, which makes it more costly for the ruler to share power. At $\omega^{\min} = 0.22$, lowering γ from 0.50 to 0.40 dissuades power sharing by causing ruler willingness to fail.

The following three substantive conditions should correspond with high γ, and hence raising ω

should yield relatively small shifts in $p(\omega)$. Consistent with the non-monotonic effects just described, high γ can either undermine power sharing by causing opposition credibility to fail or promote power sharing by causing ruler willingness to hold; depending on the value of ω^{min}. First, a stronger and more reliable state military. When the military is competent and acts in the interest of the regime, then sharing power is less dangerous. Opposition actors have fewer opportunities to gain a foothold within the state apparatus and to challenge the ruler. Thus, sharing power is unlikely to effectively safeguard the ruler unless the ruler has already solved the guardianship dilemma to some degree (Roessler 2016), perhaps because the regime has origins in violent rebellion (Meng and Paine 2022).

Second, greater state control over the economy. Tighter state control reduces opportunities for the opposition to use its wealth to challenge the regime. The core of the state-society bargain in rentier states such as Saudi Arabia is that citizens receive employment by the state and other forms of largesse in return for not engaging in anti-regime subversion, which is defined widely and punished harshly (Gause 1994). Similarly, command economies and former communist regimes such as Belarus often use public employment as a tool to prevent citizens from challenging the regime (Way 2020).

Third, more credible institutions. In some settings, constitutional promises such as periodic elections and electoral rules that do not overwhelmingly favor the incumbent are credible, and therefore constitute increases in ω without greatly shifting $p(\omega)$. In other settings, formal institutions are weak and only force constitutes a credible means to uphold promises. Sharing power in the context of weak institutions is difficult because the ruler must, in effect, give away guns to the opposition to make any promise credible (Meng et al. 2023). Thus, permanent increases in ω necessitate large increases in $p(\omega)$. The ongoing crisis in Sudan, which stems from a failed transition to democracy in 2019, highlights the perils of sharing power amid weak formal institutions (Powell 2021).

5 CONCLUSION

Existing theories have developed the compelling mechanism that institutional power-sharing concessions are a response to the generic commitment problem faced by authoritarian leaders. But most existing models do not consider how sharing power can backfire by empowering the opposition or by affecting the frequency of anti-regime mobilization. Incorporating these components into a model shows that the countervailing commitment and threat-enhancing effects determine the degree of power sharing and the opposition's response, whereas the frequency of mobilization does not affect equilibrium outcomes. Because of the threat-enhancing effect, (a) sharing more power does not necessarily reduce prospects for conflict, and (b) the ruler may refuse to share power even if doing so is necessary and sufficient to prevent conflict.

In future research, the model could be extended in various directions to address other pertinent questions related to authoritarian power sharing. In the present version, the ruler can only increase ω relative to a baseline level of wealth for the opposition, $\omega^{\min}$. One extension would be to assume that the ruler has an additional option to coerce the opposition and thereby lower ω below this baseline. However, by making the regime more reliant on the military, this would trigger the canonical guardianship dilemma and leave the ruler vulnerable to coups or defection (Paine 2022*a*). Thus, when contemplating the use of repression, the ruler would trade off between the gains from weakening the opposition and the risks from empowering the coercive apparatus.

The present model assumes that the only outside option for the opposition is to fight a war to capture the central government. However, many revolts seek to establish a separate state or to gain regional autonomy. Furthermore, producers can withhold investments in valuable production when exploited by the state. These considerations could be analyzed in the present framework. In Paine (2019), the ruler taxes output from the opposition, who can hide their economic production (in every period) and sometimes are able to fight to separate (determined by an exogenous frequency-of-mobilization parameter akin to that in Acemoglu and Robinson 2006*b* and related models). These aspects could be integrated into the present framework with endogenous mobilization. Power sharing in this context could constitute the ruler choosing an upper bound on the tax rate in every period (a basement level of spoils for the opposition), which would also raise the opposition's prospects for successfully fighting to separate.

The present discussion of the power-sharing tradeoff has analogs to the dictator's modernization dilemma (e.g., Acemoglu and Robinson 2006*a*; see also the discussion of growth under extractive institutions in Acemoglu and Robinson 2013). Allowing for more efficient production technologies and implementing market reforms can boost total economic production, but these gains are typically concentrated outside the state itself. Thus, economic reform can exhibit similar effects as a power-sharing deal by redistributing resources that empower the opposition. One possible difference, though, is that some gains from greater economic production do indeed accrue to the state. Thus, continuing to conceive of economic reform as akin to sharing power, the opposition credibility condition may not hold. Even if the opposition does not threaten the regime, the ruler may nonetheless implement economic reforms to reap economic gains (which may in turn aid with other goals such as deterring international threats). The present framework is sufficiently broad to enable studying these disparate variants of the dictator's power-sharing dilemma.

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Online Appendix

Proof of Lemma 1. See text. ■

Proof of Lemma 2. The proof proceeds in three steps. 1. Characterizing the maximizer $\hat{\omega}$. 2. Characterizing the 0 point at $\omega = 0$. 3. Characterizing the 0 point at $\omega = \omega_0$.

1. Characterizing the maximizer $\hat{\omega}$. The first derivative is

$$\frac{dx^*}{d\omega} = \frac{-1 + p'(\omega)(1-\mu)}{1 - \delta(1 - \alpha F(\hat{c}))}.$$

Applying the intermediate value theorem establishes the existence of an extreme point $\hat{\omega} \in (0, 1)$ that satisfies the first-order condition $p'(\hat{\omega})(1 - \mu) = 1$.

- At the lower bound, $p'(0)(1 - \mu) > 1$ is guaranteed by $\lim_{\omega \to 0} p'(\omega) = \infty$ and $\mu < 1$.
- At the upper bound, $p'(1)(1 - \mu) < 1$ is guaranteed by $\lim_{\omega \to 1} p'(\omega) < 1$ and $\mu > 0$.
- The continuity of the overall term is ensured by the continuity of $p(\cdot)$ in ω.

The unique maximizer claim follows from the strictly negative second derivative:

$$\frac{d^2x^*}{d\omega^2} = -\frac{1}{1 - \delta(1 - \alpha F(\hat{c}))}\left(\underbrace{-p''(\omega)}_{>0}(1-\mu) + \delta\alpha f(\hat{c})\left(\frac{dx^*}{d\omega}\right)^2\right) < 0.$$

The proof for $F(\hat{c})$ is similar. The first derivative is:

$$\frac{dF(\hat{c})}{d\omega} = f(\hat{c})\frac{d\hat{c}}{d\omega}.$$

The equivalence of $\hat{c}$ and x^* ensures that x^* and $F(\hat{c})$ have the same maximizer, given the same first-order condition. The second derivative is:

$$\frac{d^2F(\hat{c})}{d\omega^2} = f(\hat{c})\frac{d^2\hat{c}}{d\omega^2} + f'(\hat{c})\left(\frac{d\hat{c}}{d\omega}\right)^2 < 0.$$

Again using the equivalence of $\hat{c}$ and x^*, we just established that $\frac{d^2\hat{c}}{d\omega^2} < 0$. With the uniform distribution, $f'(\hat{c}) = 0$, and thus the second term cancel out (NB: any $f' \leq 0$ would ensure that this term is weakly negative). Therefore, the overall term is strictly negative.

These results also establish that x^* and $F(\hat{c})$ are each negative quadratic in ω. Consequently, $\frac{dx^*}{d\omega} = \frac{d\hat{c}}{d\omega} > 0$ for $\omega < \hat{\omega}$ and $\frac{dx^*}{d\omega} < 0$ for $\omega < \hat{\omega}$.

2. Characterizing the 0 point at $\omega = 0$. To demonstrate that $x^*(0) = F(\hat{c}(0)) = 0$, suppose not and assume each is strictly positive. We easily obtain a contradiction because the optimal transfer satisfies

$$\underbrace{(1-\delta)x^*}_{>0} + \delta\alpha \underbrace{F(\hat{c})}_{>0}(\underbrace{x^* - c^{\text{avg}}}_{>0}) = \underbrace{\overbrace{-\omega}^{0} + \overbrace{p(\omega)}^{0}(1-\mu)}_{0}.$$

A similar proof establishes that x^* cannot be strictly negative.

3. Characterizing the 0 point at $\omega = \omega_0$. Applying the intermediate value theorem establishes the existence of $\omega_0 \in (\hat{\omega}, 1)$ such that $\Theta(\omega_0) = 0$, for

$$\Theta(\omega) = -\omega_0 + p(\omega_0)(1-\mu) + \delta\alpha \int_0^{\hat{c}(\omega_0)} c_t dF(c).$$

- At the lower bound of $\omega = \hat{\omega}$, the strict positivity of x^* follows from the 0 point at $\omega = 0$ and the fact that x^* is strictly increasing at $\omega = \epsilon$, for an arbitrarily small $\epsilon > 0$ (see the previous steps of the present proof).
- At the upper bound of $\omega = 1$, suppose $x^*(1) = \hat{c}(1) \geq 0$. We easily obtain a contradiction because the optimal transfer satisfies

$$\underbrace{(1-\delta)x^*}_{>0} + \delta\alpha \underbrace{F(\hat{c})}_{>0}(\underbrace{x^* - c^{\text{avg}}}_{>0}) = \underbrace{\overbrace{-\omega}^{-1} + \overbrace{p(\omega)}^{1}(1-\mu)}_{-\mu}.$$

- The continuity of the function is ensured by the continuity of the distribution function.

This point is unique because $\frac{dx^*}{d\omega} < 0$ for $\omega > \hat{\omega}$, as established in part 1 of the present proof. ■

Proof of Lemma 3. If feasible, the ruler prefers to buy off the opposition with the interior-optimal transfer if

$$1 - \omega - \big(1 - \delta(1 - \alpha F(\hat{c}))\big)x^* > (1 - p(\omega))(1-\mu).$$

Substituting in for x^* and simplifying yields $\mu - \delta\alpha F(\hat{c})c^{\text{avg}} > 0$, which we can rewrite as

$$\Omega(\mu) \equiv \mu - \delta\alpha \int_0^{\hat{c}(\mu)} c_t dF(c) > 0.$$

Applying the intermediate value theorem establishes the existence of $\hat{\mu} \in (0, 1)$ that satisfies

$\Omega(\hat{\mu}) = 0$.

- The lower bound requires $\Omega(0) = -\delta\alpha \int_0^{\hat{c}(0)} c_t dF(c) < 0$. It suffices to show $\int_0^{\hat{c}(0)} c_t dF(c) > 0$, which in turn requires showing that x^* (and hence $\hat{c}$) is strictly positive at $\mu = 0$. Suppose instead that $x^* = \hat{c} = 0$ at $\mu = 0$. We easily obtain a contradiction because the optimal transfer satisfies

$$\underbrace{(1-\delta)x^* + \delta\alpha F(\hat{c})(x^* - c^{\text{avg}})}_{=0} = \underbrace{-\omega + p(\omega)}_{>0}.$$

- At the upper bound, we have $\Omega(1) = 1 - \delta\alpha \int_0^{\hat{c}(1)} c_t dF(c) > 0$, which follows from the fact that F is a distribution function and $c^{\text{max}} \leq 1$.
- $\Omega(\mu)$ satisfies the conditions for the implicit function theorem and is therefore continuous in μ.

The unique threshold claim for $\hat{\mu}$ follows from:

$$\frac{d\Omega(\mu)}{d\mu} = 1 - \delta\alpha \frac{d\hat{c}}{d\mu}\hat{c}f(\hat{c}) = 1 + \frac{\delta\alpha p(\omega)\hat{c}f(\hat{c})}{1 - \delta(1 - \alpha F(\hat{c}))} > 0.$$

In addition to the positive direct effect of raising μ that arises from lowering the ruler's consumption along a conflictual path, higher μ also raises the ruler's consumption along a peaceful path through an indirect channel: undercutting the challenger's bargaining leverage by diminishing the value of winning a conflict. ■

Proof of Proposition 1. Given the analysis in the text, the following claims remain to be proved. First, we rule out mixed equilibria. Second, we demonstrate that the opposition has an identical mobilization threshold along peaceful and conflictual paths. Third, we characterize the thresholds $\underline{\omega}$ and $\overline{\omega}$ that determine the conflict region.

Step 1. No mixed equilibria. In any equilibrium path of play, conflict either occurs with probability 0 in every period or occurs with probability 1 in every period that the opposition mobilizes.

- Mixing never occurs in a mobilization period if there exists an offer that the opposition accepts with positive probability. The ruler cannot be indifferent if the transfer is strictly less than $1 - \omega$, as the ruler could profitably deviate to raising the offer to one that would be accepted.
- Generically, for all parameter values, we have either $x^* < 1 - \omega$ or $x^* > 1 - \omega$, which implies that the opposition is never indifferent about its acceptance/conflict decision.
- Any equilibrium requires that the opposition accepts with probability 1 when indifferent

between accepting and fighting. If the opposition mixed over this decision, then the ruler would deviate to an infinitesimally smaller offer to secure acceptance with probability 1, which creates an open set problem.

Step 2. Equivalent mobilization thresholds along peaceful and conflictual paths. Along a conflictual path, the opposition's per-period consumption is governed by the following recursive equation:

$$V^O_{\text{conflict}} = \underbrace{(1-\alpha F(\hat{c}))\big((1-\delta)\omega + \delta V^O_{\text{conflict}}\big)}_{\text{No mobilization}} + \underbrace{\alpha F(\hat{c})\big(-(1-\delta)c^{\text{avg}} + p(\omega)(1-\mu)\big)}_{\text{Opposition mobilizes}}.$$

This easily solves to:

$$V^O_{\text{conflict}} = \frac{(1-\alpha F(\hat{c}))(1-\delta)\omega + \alpha F(\hat{c})\big(-(1-\delta)c^{\text{avg}} + p(\omega)(1-\mu)\big)}{1-\delta(1-\alpha F(\hat{c}))}.$$

Along a peaceful path, the opposition's per-period consumption is

$$V^O_{\text{peace}} = \omega + \alpha F(\hat{c})(x^* - c^{\text{avg}}).$$

Substituting in for x^* and rearranging shows this is identical to V^O_{conflict}.

Step 3. Characterizing the conflict region. Proving that $1-\omega-x^*$ is a negative quadratic function implies (generically) that it equals 0 either zero or two times. For the case of two intersections, we denote the smaller point as $\underline{\omega}$ and the larger as $\overline{\omega}$, with

$$1-\underline{\omega}-x^*(\underline{\omega}) = 0 \quad \text{and} \quad 1-\overline{\omega}-x^*(\overline{\omega}) = 0.$$

The proof for the claim follows directly from step 1 of the proof for Lemma 2. The bounds on each ω threshold follow from the proof in Lemma 2 that the optimal transfer equals 0 for $\omega \in \{0, \omega_0\}$. ■

Proof of Proposition 2. To show that the ruler minimizes ω when the path of play is fixed as peaceful, it suffices to demonstrate that its average per-period consumption strictly decreases in ω. Thus, we restate Equation 9 with a negative sign:

$$\frac{d}{d\omega}\Big(1-\omega-\alpha F(\hat{c})x^*\Big) = \underbrace{-1}_{\text{(a)}} \underbrace{-\alpha F(\hat{c})\frac{dx^*}{d\omega}}_{\text{(b)}} \underbrace{-\alpha f(\hat{c})x^*\frac{d\hat{c}}{d\omega}}_{\text{(c)}} < 0. \qquad \text{(A.1)}$$

As shown in the text, the sum of terms a and b is strictly negative for all values of ω; and term c reinforces the negative effect for $\omega < \hat{\omega}$ and cancels out for $\omega \geq \omega_0$ (and $\omega = \hat{\omega}$), but cuts in the opposite direction for $\omega \in (\hat{\omega}, \omega_0)$. For this intermediate region, the claim holds for any $\alpha < \hat{\alpha}$, for a unique $\hat{\alpha} > 0$ implicitly characterized as

$$-1 - \hat{\alpha}\Big(F\big(\hat{c}(\hat{\alpha})\big) + f\big(\hat{c}(\hat{\alpha})\big)x^*(\hat{\alpha})\Big)\frac{dx^*}{d\omega}\bigg|_{\alpha=\hat{\alpha}} = 0. \tag{A.2}$$

To establish $\hat{\alpha} > 0$, it is straightforward to see that the left-hand side of Equation A.2 is strictly negative at $\alpha = 0$. To prove uniqueness, it suffices to establish

$$\frac{d^2}{d\omega d\alpha}\big(1 - \omega - \alpha F(\hat{c})x^*\big) =$$

$$-\Bigg(\underbrace{F(\hat{c}) + f(\hat{c})x^*}_{①} + \underbrace{\alpha\frac{dx^*}{d\alpha}\big(2f(\hat{c}) + f'(\hat{c})x^*\big)}_{②}\Bigg)\frac{dx^*}{d\omega} - \alpha\big(F(\hat{c}) + f(\hat{c})x^*\big)\underbrace{\frac{d^2x^*}{d\omega d\alpha}}_{③} > 0. \tag{A.3}$$

Terms 1 and 2 in Equation A.3. $\omega > \hat{\omega}$ implies $\frac{dx^*}{d\omega} < 0$, and therefore we need to show that the sum of terms 1 and 2 is positive. Term 1 is obviously positive, and term 2 is negative because

$$\frac{dx^*}{d\alpha} = \frac{d\hat{c}}{d\alpha} = -\frac{\delta F(\hat{c})(\hat{c} - c^{\text{avg}})}{1 - \delta(1 - \alpha F(\hat{c}))} < 0. \tag{A.4}$$

As with $\frac{dx^*}{d\omega}$, discussed in the text, the indirect effects on the frequency of mobilization cancel out in this derivative.

With the uniformity assumption, the sum of terms 1 and 2 simplifies to $\frac{2}{c^{\max}}\frac{d}{d\alpha}(\alpha x^*) = \frac{2}{c^{\max}}(x^* + \alpha\frac{dx^*}{d\alpha})$. The uniformity assumption guarantees that the direct effect from raising α strictly dominates the indirect effect (by lowering x^*) by ensuring that the lower-order moments of the distribution function are not too large in magnitude at any point. Substituting Equation A.4 into $x^* + \alpha\frac{dx^*}{d\alpha}$ and simplifying yields

$$\frac{(1-\delta)x^* + \delta\alpha F(\hat{c})c^{\text{avg}}}{1 - \delta(1 - \alpha F(\hat{c}))} > 0.$$

Term 3 in Equation A.3. It suffices to demonstrate $\frac{d^2x^*}{d\omega d\alpha} < 0$. The derivative is

$$\frac{d^2x^*}{d\omega d\alpha} = -\frac{1 - \delta(1 - \alpha F(\hat{c}))}{\delta\big(F(\hat{c}) + \alpha f(\hat{c})\frac{d\hat{c}}{d\alpha}\big)}\left(\frac{dx^*}{d\omega}\right)^2.$$

Therefore, the claim requires $\frac{d}{d\alpha}\big(\alpha F(\hat{c})\big) = F(\hat{c}) + \alpha f(\hat{c})\frac{d\hat{c}}{d\alpha} > 0$. Substituting Equation A.4

into $F(\hat{c}) + \alpha f(\hat{c})\frac{d\hat{c}}{d\alpha} > 0$ and rearranging yields

$$1 - \delta + \delta\alpha\big(F(\hat{c}) - f(\hat{c})x^*\big) + \delta\alpha f(\hat{c})c^{\text{avg}} > 0.$$

Imposing the uniformity assumption for F yields the result because $F(\hat{c}) - f(\hat{c})x^* = \frac{1}{c^{\max}}(\underbrace{\hat{c} - x^*}_{=0})$. By minimizing the maximum value of f, uniform distribution ensures that the negative term is not too large in magnitude for any parameter values. ■

Proof of Proposition 3. The proof proceeds in three steps. First, we demonstrate that the ruler's utility is strictly higher along a peaceful than a conflictual path (holding constant all parameter values). Second, as in the proof of Proposition 2, we demonstrate that the ruler's average per-period consumption strictly decreases in ω along a conflictual path. Third, we characterize the ruler's optimal choice of ω.

1. Peaceful path preferred to conflictual path. Along a peaceful path, the ruler's average per-period consumption is:

$$V^R_{\text{peace}} = 1 - \omega - \alpha F(\hat{c})\frac{-\omega + p(\omega)(1-\mu) + \delta\alpha F(\hat{c})c^{\text{avg}}}{1 - \delta(1 - \alpha F(\hat{c}))}.$$

Along a conflictual path, the ruler's per-period consumption is governed by the following recursive equation:

$$V^R_{\text{conflict}} = \underbrace{(1 - \alpha F(\hat{c}))\big((1-\delta)(1-\omega) + \delta V^R_{\text{conflict}}\big)}_{\text{No mobilization}} + \underbrace{\alpha F(\hat{c})(1 - p(\omega))(1-\mu)}_{\text{Opposition mobilizes}}.$$

This easily solves to:

$$V^R_{\text{conflict}} = \frac{(1 - \alpha F(\hat{c}))(1-\delta)(1-\omega) + \alpha F(\hat{c})(1 - p(\omega))(1-\mu)}{1 - \delta(1 - \alpha F(\hat{c}))}.$$

Taking the difference between these terms yields:

$$V^R_{\text{peace}} - V^R_{\text{conflict}} = \frac{\alpha F(\hat{c})}{1 - \delta(1 - \alpha F(\hat{c}))}\big(\mu - \delta\alpha F(\hat{c})c^{\text{avg}}\big). \tag{A.5}$$

This is strictly positive for all $\mu > \hat{\mu}$ (see Lemma 3), which we assume to hold. This term expresses the net loss surplus from conflict, $\mu - \delta\alpha F(\hat{c})c^{\text{avg}}$, weighted by the value of consumption in the expected period in which the conflict will occur, $\frac{\alpha F(\hat{c})}{1-\delta(1-\alpha F(\hat{c}))}$.

2. Decreasing utility in ω along conflictual path. To show that the ruler minimizes ω when the path of play is fixed as conflictual, we can follow the same structure of the proof for Proposition 2.

$$\frac{d}{d\omega}V^R_{\text{conflict}} = \underbrace{-\frac{1}{1-\delta(1-\alpha F(\hat{c}))}\Bigg(\overbrace{(1-\delta)(1-\alpha F(\hat{c}))}^{\text{Lost wealth}}+\overbrace{(1-\mu)\alpha F(\hat{c})p'(\omega)}^{\text{Lower Pr(win)}}\Bigg)}_{\text{Direct effects}}$$
$$\underbrace{-\frac{(1-\delta)\alpha f(\hat{c})}{\big(1-\delta(1-\alpha F(\hat{c}))\big)^2}\big(\overbrace{1-\omega-(1-\mu)(1-p(\omega))}^{>0 \text{ b/c } p(\omega)>\omega}\big)\frac{d\hat{c}}{d\omega}}_{\text{Indirect effect through affecting } \hat{c}} \tag{A.6}$$

The direct effects operate through lowering the ruler's per-period wealth and its chance of winning a conflict. Algebraic rearranging demonstrates

$$-\frac{1}{1-\delta(1-\alpha F(\hat{c}))}\Bigg((1-\delta)(1-\alpha F(\hat{c}))+(1-\mu)\alpha F(\hat{c})p'(\omega)\Bigg) = -1-\alpha F(\hat{c})\frac{d\hat{c}}{d\omega},$$

which is identical to the direct effects encompassed in terms a and b in Equation A.1. Thus, this component of Equation A.6 is strictly negative for all ω.

The indirect effects operate through affecting the mobilization threshold $\hat{c}$. This yields a similar intuition as term c in Equation A.1, although the exact expression differs because of the discrepancy in payoffs along peaceful and conflictual paths shown in Equation A.5. Thus, as in the proof of Proposition 2, we need an additional assumption that α is sufficiently low to ensure that Equation A.6 is strictly negative for $\omega \in (\hat{\omega}, \omega_0)$. We implicitly characterize a unique threshold $\hat{\alpha}' > 0$ as

$$-1-\hat{\alpha}'F\big(\hat{c}(\hat{\alpha}')\big)\frac{d\hat{c}}{d\omega}\bigg|_{\alpha=\hat{\alpha}'}-\frac{(1-\delta)\hat{\alpha}'f\big(\hat{c}(\hat{\alpha}')\big)}{\Big(1-\delta\big(1-\hat{\alpha}'F\big(\hat{c}(\hat{\alpha}')\big)\big)\Big)^2}\big(1-\omega-(1-\mu)(1-p(\omega))\big)\frac{d\hat{c}}{d\omega}\bigg|_{\alpha=\hat{\alpha}'} = 0. \tag{A.7}$$

To establish $\hat{\alpha}' > 0$, it is straightforward to see that the left-hand side of Equation A.7 is strictly negative at $\alpha = 0$. To prove uniqueness, it suffices to establish $\frac{d^2}{d\omega d\alpha}V^R_{\text{conflict}} > 0$. The proof for Proposition 2 established $\frac{d}{d\alpha}\big(-1-\alpha F(\hat{c})\frac{d\hat{c}}{d\omega}\big) > 0$ for $\omega > \hat{\omega}$. Therefore, it remains to establish that the indirect effect in the second line of Equation A.6 strictly increases in α. The derivative is

$$\frac{d}{d\alpha}\Bigg(-\frac{(1-\delta)\alpha f(\hat{c})}{\big(1-\delta(1-\alpha F(\hat{c}))\big)^2}\big(1-\omega-(1-\mu)(1-p(\omega))\big)\Bigg) =$$

$$-\frac{1-\omega-(1-\mu)(1-p(\omega))}{\big(1-\delta(1-\alpha F(\hat{c}))\big)^2}\Bigg(\underbrace{(1-\delta)\alpha f(\hat{c})\frac{d^2\hat{c}}{d\omega d\alpha}}_{①}$$

$$\underbrace{+\frac{d\hat{c}}{d\omega}(1-\delta)\Bigg(f(\hat{c})+\alpha f'(\hat{c})\frac{d\hat{c}}{d\alpha}-\frac{2\delta\alpha f(\hat{c})}{1-\delta(1-\alpha F(\hat{c})}\Big(F(\hat{c})+\alpha f(\hat{c})\frac{d\hat{c}}{d\alpha}\Big)\Bigg)}_{②}\Bigg)$$

It suffices to demonstrate that terms 1 and 2 are each strictly negative. For term 1, the proof of Proposition 2 demonstrated $\frac{d^2\hat{c}}{d\omega d\alpha} < 0$. For term 2, we know $\frac{d\hat{c}}{d\omega} < 0$ for $\omega \in (\hat{\omega}, \omega_0)$, and therefore we need to show that that the term in brackets is strictly positive. Imposing the uniform distribution assumption enables us to eliminate the term $\alpha f'(\hat{c})\frac{d\hat{c}}{d\alpha}$ and to simplify $F(\hat{c}) + \alpha f(\hat{c})\frac{d\hat{c}}{d\alpha}$ (see the proof for Proposition 2); in both cases, the uniform distribution ensures that lower-order moments are not too large in magnitude at any point. Thus the term in parentheses can be restated as

$$\frac{f(\hat{c})}{1-\delta(1-\alpha F(\hat{c}))}\Big((1-\delta)(1-2\delta\alpha)+\delta\alpha F(\hat{c})(1-\delta\alpha)\Big).$$

A sufficient condition for this condition to be strictly positive is $\alpha < \frac{1}{2\delta}$, which we impose as part of the upper bound on α stated in the proposition.

3. Characterizing the ruler's optimal choice of $\boldsymbol{\omega}$. We can implicitly define $\tilde{\omega}$ as the value of ω at which the ruler's lifetime expected consumption along a conflictual path at $\omega = \tilde{\omega}$ equals its lifetime expected consumption along a peaceful path at the minimum value of ω that secures such a path, $\omega = \overline{\omega}$. Formally, this is $V^R_{\text{peace}}(\overline{\omega}) = V^R_{\text{conflict}}(\tilde{\omega})$. Because $V^R_{\text{peace}}(\omega) > V^R_{\text{conflict}}(\omega)$ for a fixed ω (see step 1 of the present proof), we know $V^R_{\text{peace}}(\overline{\omega}) = V^R_{\text{conflict}}(\tilde{\omega}) < V^R_{\text{peace}}(\tilde{\omega})$. Thus, because V^R_{peace} strictly decreases in ω (see the proof of Proposition 2), $\tilde{\omega}$ is unique and satisfies $\tilde{\omega} < \overline{\omega}$, and the ruler's optimal choice is $\omega = \overline{\omega}$ for all $\omega^{\text{min}} \in (\tilde{\omega}, \overline{\omega})$. Finally, because V^R_{conflict} strictly decreases in ω (see step 2 of the present proof), the ruler's optimal choice is $\omega = \omega^{\text{min}}$ for all $\omega^{\text{min}} \in (\underline{\omega}, \tilde{\omega})$. ■