STATISTICS – MEASURES OF CENTRAL TENDENCY (AVERAGES) & DISPERSION

DEFINITION OF CENTRAL TENDENCY / AVERAGES:

Central tendency (tending to the central value), which helps for finding performance and comparison

X	f		
00-19	1	Minimum	
20-39	3	Gradually increasing	
40-59	7	Maximum	
60-79	2	Gradually decreasing	
80-99	1	Minimum	

X - (Any variable: Height, Weight, Marks, Profits, Wages, and so on)

f - Frequency, (Usually, repetitiveness, frequent happenings, number of times of occurrence)

List of Formula

Arithmetic Mean ($\bar{x}$)	Geometric Mean (GM)	Harmonic Mean (HM)
Weighted Average		
$\bar{X} = \frac{\sum wX}{\sum w}$	$G = (X_1^{w_1} \times X_2^{w_2} \times \ldots \times X_n^{w_n})^{\frac{1}{\sum w}}$ **Or** $G = Antilog\left(\frac{\sum w \log X}{\sum w}\right)$	$H = \frac{\sum w}{\sum \frac{w}{X}}$
Combined Mean		
$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$	$G = Antilog\left(\frac{n_1 \log G_1 + n_2 \log G_2}{n_1 + n_2}\right)$	$H = \frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$

Measures of Central Tendency (Averages)					
Mean			**Partition Values:** (Arrange the items in ascending order)		**Mode** (M_o)
Arithmetic (usual cases) (Direct Method)	**Geometric (Comparisons – ratios, Proportions and %)**	**Harmonic (Two units together E.g. speed = distance / time**	**Median** (M_e)	**Fractiles** (F_e)	
Individual					
$\bar{X} = \frac{X_1 + X_2 \dots X_n}{n}$ $\bar{X} = \frac{\sum_{i=1}^{N} X_i}{n}$ $\bar{x} = \frac{\sum X}{n}$	$GM = (X_1 . X_2 . \dots X_n)^{\frac{1}{n}}$ $\textbf{or } GM = Antilog \left(\frac{\sum \log X}{n} \right)$	$HM = \frac{n}{\sum \frac{1}{X}}$	**If 'n' is odd:** $M_e = \left(\frac{n+1}{2} \right)^{th} obs$ (i.e. the middle obs) **If 'n' is even:** $M_e = \frac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th} obs}{2}$	$F_e = \frac{e(n+1)}{F}$	$M_o = most\ usual$ $(location\ method)$
Discrete series					
$\bar{X} = \frac{\sum fX}{\sum f} = \frac{\sum fX}{N}$	$= (X_1^{f_1} . X_2^{f_2} . \dots X_n^{f_n})^{\frac{1}{N}}$ $\textbf{or } G = Antilog \left(\frac{\sum f \log X}{N} \right)$	$HM = \frac{n}{\sum \frac{f}{X}}$	$M_e = size\ of \left(Cf > \frac{N+1}{2} \right)$	$F_e = size\ of \left(Cf > \frac{e(N+1)}{F} \right)$	**Regular frequency** $M_o = location\ method$ **Irregular frequency** $M_o = grouping\ method$
Continuous / Grouped Frequency / (Interpolation Method)					
$\bar{X} = \frac{\sum fm}{\sum f} = \frac{\sum fm}{N}$	$G = (m_1^{f_1} . m_2^{f_2} . \dots m_n^{f_n})^{\frac{1}{N}}$ $\textbf{Or } G = Antilog \left(\frac{\sum f \log m}{N} \right)$	$HM = \frac{n}{\sum \frac{m}{X}}$	$M_e = l_1 + \left(\frac{\frac{N}{2} - N_l}{N_u - N_l} \right) \times C$ $\textbf{Or } l + \frac{\frac{N}{2} - m}{f} \times c$	$F_e = l_1 + \left(\frac{e\frac{N}{F} - N_l}{N_u - N_l} \right) \times C$ $\textbf{Or } l + \frac{e\frac{N}{F} - m}{f} \times c$	$M_o = l_1 + \left(\frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1} \right) \times C$

Note:

1. **Indirect / Shortcut / Assumed Mean (A) Method: Deviation Method** $(d = X - A)$: $\bar{X} = A + \frac{\sum d}{n}$ & **Step-Deviation Method** $\left(d = \frac{X-A}{C} \right)$: $\bar{x} = A + \frac{\sum d}{n} \times C$
2. **Empirical relationship (thumb rule):** If mode is ill-defined (*in case of moderately skewed distribution*): $\bar{X} - M_o = 3(\bar{X} - M_e)$ *or* $M_o = 3M_e - 2\bar{X}$
3. **Fractiles:** Quartiles (Q), Octiles (O), Deciles (D) and Percentiles (P)

	Measures of Dispersion	
	Absolute	**Relative**
(i)	**Range** (**R**) $= L - S$	**Coefficient of range**$(Co\ R) = \frac{L-S}{L+S} \times 100$
(ii)	**Quartile Deviation** (**QD**) $= \frac{Q_3 - Q_1}{2}$ (Otherwise Semi inter quartile range) **Inter quartile range** $= Q_3 - Q_1$	**Coefficient of Quartile Deviation (Co QD)** $Co\ QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$
(iii)	**Mean Deviation (MD) about A,** $(A = \overline{X},\ M_e,\ M_o)$	**Coefficient of Mean Deviation** $(Co\ MD_A)$
	Individual: $\mathrm{MD}_A = \frac{1}{n}\sum \lvert x - A \rvert$	$Co\ MD_A = \frac{MD_A}{A} \times 100$
	Discrete: $\mathrm{MD}_A = \frac{1}{N}\sum f \lvert x - A \rvert$	
	Continuous: $MD_A = \frac{1}{N}\sum f \lvert m - A \rvert$	
(iv)	**Standard Deviation (s)**	**Coefficient of Variation (CV)**
	Individual: $s = \sqrt{\frac{\sum (X - \overline{X})^2}{n}}$ or $\sqrt{\frac{\sum X^2}{n} - \overline{X}^2}$	$CV = \frac{s}{\overline{X}} \times 100$
	Discrete: $s = \sqrt{\frac{\sum f(X - \overline{X})^2}{N}}$ or $\sqrt{\frac{\sum fX^2}{N} - \overline{X}^2}$	
	Continuous: $s = \sqrt{\frac{\sum f(m - \overline{X})^2}{N}}$ or $\sqrt{\frac{\sum fm^2}{N} - \overline{X}^2}$	
	$Variance = s^2$	

Shortcut:

$$s = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}\ Where\ d = X - A\ (for\ individual\ and\ discrete)\ \&\ d = \frac{m - A}{C}\ for\ continuous$$

	Comparison	
	Absolute Measure	**Relative Measure**
1	Dependent of unit	Independent of unit
2	Not considered for comparison	considered for comparison
3	Not much difficult compared to Relative measure	Difficult to compute and comprehend.

INDIVIDUAL OBSERVATIONS

Question 1: From the Individual Observations: 3, 6, 48 & 24, find out the following

Measures of Averages	**Measures of Dispersion**	
Arithmetic Mean	**Absolute Measure**	**Relative Measure**
Geometric Mean	Range	Coefficient of Range
Harmonic Mean	Quartile Deviation	Coefficient of Quartile Deviation
Median	Mean Deviation	Coefficient of Mean Deviation
Fractiles (Q_1, Q_3, O_6, D_7 & P_{75})	Standard Deviation / Variation	Coefficient of Variation
Mode		

Answer:

Measures of Averages				
Mean	**Formula**	**Calculation**		**Answer**
AM	$\bar{X} = \frac{\sum X}{n}$	$\frac{3+6+24+48}{4}$	$\frac{81}{4}$	20.25
GM	$GM = (X_1 \times X_2 \times ... \times X)^{\frac{1}{n}}$	$(3 \times 6 \times 24 \times 48)^{\frac{1}{4}}$	$(3^4.4^4)^{\frac{1}{4}}$	12
HM	$HM = \frac{n}{\sum \frac{1}{X}}$	$\frac{4}{\frac{1}{3}+\frac{1}{6}+\frac{1}{24}+\frac{1}{48}}$	$\frac{4 \times 48}{16+8+2+1} = \frac{192}{27}$	7.11

Note:

X	3	6	24	48
log X	0.4771	0.7782	1.3802	1.6812
$\sum \log X$	4.3167			
	Formula	**Calculation**	**Answer**	
GM	$Antilog\left(\frac{\sum \log X}{n}\right)$	$Antilog\left(\frac{4.3167}{4}\right)$	11.94	

Positional Average				
	Formula	**Calculations**		**Answer**
M_e	$size\ of \left(\frac{n+1}{2}\right)^{th} obs$	$Size\ of\ 2.5^{th}\ obs$ $2^{nd}\ obs\ +\ 0.5\ (3^{rd}\ obs - 2^{nd}\ obs)$	$6 + 0.5(24 - 6)$	**15**
Q_1	$size\ of \left(\frac{n+1}{2}\right)^{th} obs$	$Size\ of\ 1.25^{th}\ obs$ $1^{st}\ obs\ +\ 0.25\ (2^{nd}\ obs - 1^{st}\ obs)$	$3 + 0.25(6 - 3)$	**3.75**
Q_3	$size\ of \left(\frac{3(n+1)}{4}\right)^{th} obs$	$Size\ of\ 3.75^{th}\ obs$ $3^{rd}\ obs\ +\ 0.75\ (4^{th}\ obs - 3^{rd}\ obs)$	$24 + 0.75\ (48 - 24)$	**42**
P_{75}	$size\ of \left(\frac{75(n+1)}{100}\right)^{th} obs$			
O_6	$size\ of \left(\frac{6(n+1)}{8}\right)^{th} obs$			
	Note: $Q_3 = O_6 = P_{75}$			

D_7	$size\ of\ \left(\frac{7(n+1)}{10}\right)^{th}\ obs$	$Size\ of\ 3.5^{th}\ obs$ $3^{rd}\ obs\ +\ 0.5\ (4^{th}\ obs - 3^{rd}\ obs)$	$24 + 0.5(48 - 24)$	**36**

Mode Mode is ill-defined (Since all the observation has equal appearance) Hence, the empirical relation is used to arrive M_o			
	Formula	**Calculations**	**Answer**
M_o	$Mean - Mode = 3(Mean - Median)$	$20.25 - Mode = 3(20.25 - 15)$	**4.5**

Measures of Dispersion (Absolute and Relative)

		Formula	**Calculation**	**Answer**
1	**Range (R)**	$L - S$	$48 - 3$	45
	Co – efficient of Range	$\frac{L-S}{L+S}$	$= \frac{48-3}{48+3}$	0.8823
2	**Quartile Deviation (QD)**	$\frac{Q_3 - Q_1}{2}$	$\frac{42 - 3.75}{2}$	19.125
	Coefficient of Quartile Deviation	$\frac{Q_3 - Q_1}{Q_3 + Q_1}$	$\frac{42 - 3.75}{42 + 3.75}$	0.84
3	**Mean Deviation ($MD_{\bar{X}}$)**	$\frac{1}{n}\sum \lvert X - \bar{X} \rvert$	$\frac{63}{4}$	15.75
	Co – efficient of MD	$\frac{MD_{\bar{X}}}{Mean}$	$\frac{15.75}{20.25}$	0.778
4	**Standard Deviation (s)**	$\sqrt{\frac{\sum(X - \bar{X})^2}{n}}$	$\sqrt{\frac{1284.75}{5}}$	17.921
	Or			
		$\sqrt{\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2}$	$\sqrt{\frac{2925}{4} - \left(\frac{81}{4}\right)^2}$	17.921
	$Var\ (X)$	S^2	17.921^2	321.16
	$Coefficient\ of\ variation, var(x)$	$\frac{s}{\bar{X}} \times 100$	$\frac{17.921}{20.25} \times 100$	88.49%

Working note

X	$\lvert X - \bar{X} \rvert$	$X - \bar{X}$	$(X - \bar{X})^2$	X^2
3	17.25	**17.25**	**297.5625**	9
6	14.25	14.25	203.0625	36
24	3.75	-3.75	14.0625	576
48	27.25	-27.75	770.0625	2304
Total	**63**		1284.75	2925

Question 2: Find Median, Q_1, Q_3, O_6, D_7, P_{75} for the observations: 1, 3, 6, 24, 48.

Answer:

Positional Average				
	Formula	**Calculations**		**Answer**
M_e	$size\ of \left(\frac{n+1}{2}\right)^{th} obs$	$Size\ of\ 3^{rd}\ obs$	$6 + 0.5(24 - 6)$	**6**
Q_1	$size\ of \left(\frac{n+1}{4}\right)^{th} obs$	$Size\ of\ 1.5^{th}\ obs$ $1^{st}\ obs\ +\ 0.5\ (2^{nd}\ obs - 1^{st}\ obs)$	$1 + 0.5(3 - 1)$	**2**
Q_3	$size\ of \left(\frac{3(n+1)}{4}\right)^{th} obs$	$Size\ of\ 4.5^{th}\ obs$ $4^{th}\ obs\ +\ 0.5\ (5^{th}\ obs - 4^{th}\ obs)$	$24 + 0.5\ (48 - 24)$	**36**
P_{75}	$size\ of \left(\frac{75(n+1)}{100}\right)^{th} obs$			
O_6	$size\ of \left(\frac{6(n+1)}{8}\right)^{th} obs$			
	Note: $Q_3 = O_6 = P_{75}$			
D_7	$size\ of \left(\frac{7(n+1)}{10}\right)^{th} obs$	$Size\ of\ 4.2^{th}\ obs$ $4^{th}\ obs\ +\ 0.2\ (5^{th}\ obs - 4^{th}\ obs)$	$24 + 0.2(48 - 24)$	**28.8**

Question 3: Discrete Frequency Distribution

x	10	11	12	13	14	15	16	17	18	19
f	8	15	20	100	98	95	90	75	50	30

Answer:

	Measures of Averages			
		Formula	**Calculation**	**Answer**
1	Arithmetic Mean($\bar{x}$)	$\frac{\sum fX}{N}$	$\frac{8727}{581}$	15.02
2	Geometric Mean(GM)	Antilog $\left(\frac{\sum f\ \log X}{N}\right)$	Antilog $\left(\frac{682.4203}{581}\right)$	14.95
3	Harmonic Mean (HM)	$\frac{N}{\sum \frac{f}{X}}$	$\frac{581}{39.25}$	14.802

Working Note:

X	f	fX	$\log X$	$f \log X$	$\frac{f}{X}$
10	8	80	1.0000	8.0000	0.800
11	15	165	1.0414	15.6210	1.360
12	20	240	1.0792	21.5840	1.670
13	100	1300	1.1139	111.3900	7.690

14	98	1372	1.1461	112.3178	7.000
15	95	1425	1.1761	111.7295	6.330
16	90	1440	1.2041	108.3690	5.625
17	75	1275	1.2304	92.2800	4.411
18	50	900	1.2553	62.7650	2.780
19	30	570	1.2788	38.364	1.578
Total	**581**	**8727**		**682.4203**	**39.25**

Positional Average

	Formula	Calculations	Answer
M_e	$size\ of \left(\frac{N+1}{2}\right)^{th}\ obs$	$Size\ of\ 291^{st}\ obs$ $(i.e.cf > 291)$	**15**
Q_1	$size\ of \left(\frac{1(N+1)}{4}\right)^{th}\ obs$	$Size\ of\ 145.5^{th}\ obs$ $(i.e.cf > 145.5)$	**14**
Q_3	$size\ of \left(\frac{3(N+1)}{4}\right)^{th}\ obs$	$Size\ of\ 436.5^{th}\ obs$ $(i.e.cf > 436.5)$	**17**
P_{75}	$size\ of \left(\frac{75(N+1)}{100}\right)^{th}\ obs$		
O_6	$size\ of \left(\frac{6(N+1)}{8}\right)^{th}\ obs$		
	Note: $Q_3 = O_6 = P_{75}$		
D_7	$size\ of \left(\frac{7(N+1)}{10}\right)^{th}\ obs$	$Size\ of\ 407.4^{th}\ obs$ $(i.e.cf > 407.4)$	**16**

Working Notes

X	f	cf
10	8	8
11	15	23
12	20	43
13	100	143
14	98	241
15	95	336
16	90	426
17	75	501
18	50	551
19	30	581

Mode: Since there is a sudden increase in frequency from 20 to 100, we obtain mode by Grouping Table

Grouping Table							The highest frequency total in each of the six columns of the grouping table is identified and analyzed (Tally marks)						Total Tally Mark
	(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)	
X	f												
10	8	23		43									0
11	15		35		135								0
12	20	120				218							0
13	**100**		**198**	**293**			|		|	|			3
14	98	**193**			**283**			|	|	|	|		**4**
15	95		185			**260**		|		|	|	|	**4**
16	90	165		215							|	|	2
17	75		125		155							|	1
18	50	80											0
19	30												0

Explanation to column	
(1)	Original Frequency
(2)	grouping in "two's
(3)	Leaving the first and grouping the rest in "two's"
(4)	grouping in "three's"
(5)	Leaving the first and grouping in "three's"
(6)	Leaving the first & second and grouping in "three's"

Mode	
Mode is ill-defined or bi-modal (Since "14" and "15" occur equal number of times) Hence, the empirical relation is used to arrive M_o	
M_o	$Mean - Mode = 3(Mean - Median)$
	$15.02 - Mode = 3(15.02 - 15)$
	14.96

Points to Ponder:

Under Location Method, Mode = 13 (as the highest frequency is 100)

Under Grouping Method, Mode is ill- defined.

But, Under Empirical Relationship, Mode = 14.96, which brings the issues an accuracy

Measures of Dispersion

		Formula	Calculation	Answer
1	**Range (R)**	$L - S$	$19 - 10$	10
	Co – efficient of Range	$\frac{L-S}{L+S}$	$\frac{19-10}{19+10}$	0.31
2	**Quartile Deviation (QD)**	$\frac{Q_3 - Q_1}{2}$	$\frac{17-14}{2}$	1.5
	Coefficient of Quartile Deviation	$\frac{Q_3 - Q_1}{Q_3 + Q_1}$	$\frac{17-14}{17+14}$	0.0967
3	**Mean Deviation ($MD_{\bar{X}}$)**	$\frac{1}{N}\sum\lvert X - \bar{X}\rvert$	$\frac{969.82}{58.1}$	1.669
	Co – efficient of MD	$\frac{MD_{\bar{X}}}{Mean}$	$\frac{1.669}{15.02}$	0.111133
4	**Standard Deviation (s)**	$\sqrt{\frac{\sum f(X-\bar{X})^2}{N}}$	$\sqrt{\frac{2204.7628}{581}}$	3.80
	$Var\ (X)$	s^2	3.80^2	14.44
	$Coefficient\ of\ variation, var(X)$	$\frac{s}{\bar{X}} \times 100$	$\frac{3.80}{15.02} \times 100$	25.29%

Working Note:

		for MD		For SD	
X	f	$\lvert X-\bar{X}\rvert$	$f\lvert X-\bar{X}\rvert$	$(X-\bar{X})$	$f(X-\bar{X})^2$
10	8	5 .02	40.16	-5 .02	201.6032
11	15	4.02	60.30	-4.02	242.4060
12	20	3.02	68.40	-3.02	182.4080
13	100	2.02	202.00	-2.02	81.6080
14	98	1.02	99.96	-1.02	101.9592
15	95	0.02	1.90	-0.02	0.0380
16	90	0.98	88.20	0.98	86.4360
17	75	1.98	143.50	1.98	294.0300
18	50	2.98	1.49	2.98	444.0200
19	36	3.98	119.40	3.98	570.2544
Σ	**581**		**969.82**		**2204.7628**

Question 4: Continuous Frequency Distribution:

Marks	01-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Number of Students	3	7	13	17	12	10	8	8	6	6

Also verify the empirical relation

Answer:

Measures of Averages			
	Formula	**Calculation**	**Answer**
A.M. (Direct Method)	$\frac{\sum fm}{N}$	$\frac{4375}{90}$	48.61
A.M. (Short cut -Method)	$A+\frac{\sum fd}{N}\times c$ (A=45.5)	$45.5+\frac{28}{90}\times 10$	48.61
	$A+\frac{\sum fd}{N}\times c$ (A = 55.5)	$55.5+\frac{-620}{90}\times 10$	48.61
Geometric Mean, GM	$Antilog\left(\frac{\sum f\ \log m}{N}\right)$	$Antilog\,\frac{150.5439}{90}$	47.07
Harmonic Mean, HM	$\frac{N}{\sum\frac{f}{m}}$	$\frac{90}{2.7905}$	32.25

Working Note:

Marks (Class boundaries)	m	f	fm	$d = \frac{m-45.5}{10}$	fd	$\log m$	$f \log m$	$\frac{f}{m}$
0.5 – 10.5	5.5	3	16.5	-4	-12	0.7404	2.2212	0.5454
10.5 – 20.5	15.5	7	108.5	-3	-21	1.903	13.3210	0.4516
20.5 – 30.5	25.5	13	331.5	-2	-26	1.4065	18.2845	0.5098
30.5 – 40. 5	35.5	17	603.5	-1	-17	1.5502	26.3534	0.4789
40.5 – 50.5	45.5	12	546.0	0	0	1.6580	19.8960	0.2637
50.5 – 60.5	55.5	10	555.0	1	10	1.7443	17.4430	0.1801
60.5 – 70.5	65.5	8	524.0	2	16	1.8162	14.5296	0.1221
70.5 – 80.5	75.5	8	604.0	3	24	1.8779	15.0232	0.1060
80.5 – 90.5	85.5	6	513.0	4	24	1.9320	11.5920	0.0701
90.5 – 100.5	95.5	6	573.0	5	30	1.9800	11.8800	0.0628
Total		**90**	**4375.0**		**28**		**150.5439**	**2.7905**

Positional Average and Mode

	Formula	Calculation	Answer
M_e	$l + \frac{\frac{N}{2} - m}{f} \times c$	$40.5 + \frac{45-40}{12} \times 10$	44.67
Q_1	$l + \frac{\frac{1N}{4} - m}{f} \times c$	$20.5 + \frac{22.5-10}{13} \times 10$	30.12
Q_3	$l + \frac{\frac{3N}{4} - m}{f} \times c$	$60.5 + \frac{67.5-62}{8} \times 10$	67.38
O_6	$l + \frac{\frac{6N}{8} - m}{f} \times c$		
P_{75}	$l + \frac{\frac{75N}{100} - m}{f} \times c$		
	$O_3 = O_6 = P_{75} = 67.38$		
D_7	$l + \frac{\frac{7N}{10} - m}{f} \times c$	$60.5 + \frac{63-62}{8} \times 10$	61.75
M_o	$l_1 + \left(\frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1}\right) \times C$	$30.5 + \left(\frac{17-13}{2 \times 17 - 13 - 12}\right) \times 10$	34.94
	M_o *class is* (30.5 – 40.5), since 17 is the highest frequency		

Working Note

X	f	cf
0.5–10.5	3	3
10.5–20.5	7	10
20.5–30.5	13	23
30.5–40. 5	17	40
40.5–50.5	12	52
50.5–60.5	10	62
60.5–70.5	8	70
70.5–80.5	8	78
80.5–90.5	6	84
90.5–100.5	6	90

Graphical Method: Ogive Curves for Positional Average:

Marks	Number of Students	Less than ogive curve		More than ogive curve	
		UCL	< cf	LCL	>cf

0.5 – 10.5	3	10.5	3	0.5	**90** $(=\sum f)$
10.5 – 20.5	7	20.5	10	10.5	87
20.5 – 30.5	13	30.5	23	20.5	80
30.5 – 40. 5	17	40. 5	40	30.5	67
40.5 – 50.5	12	50.5	52	40.5	50
50.5 – 60.5	10	60.5	62	50.5	38
60.5 – 70.5	8	70.5	70	60.5	28
70.5 – 80.5	8	80.5	78	70.5	20
80.5 – 90.5	6	90.5	84	80.5	12
90.5 – 100.5	6	100.5	**90** $(=\sum f)$	90.5	6

Verification of Empirical relation:

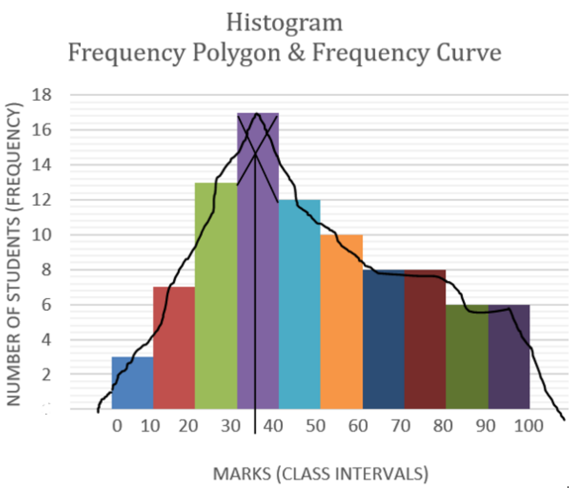
Mean – Mode = 3 (Mean - Median)

(i.e.,) 48.61 – 34.94 = 3 (48.61 – 44.67)

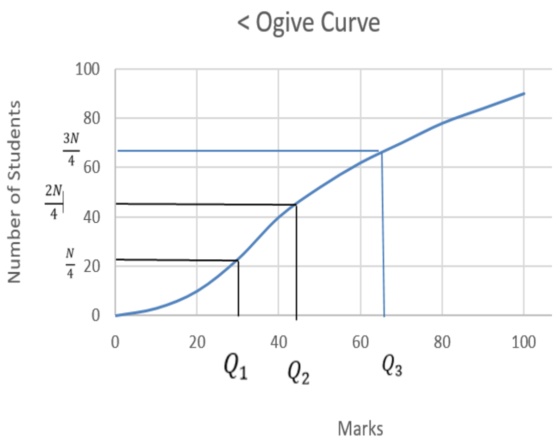
13.67 = 3 (4.006)

13.67 = 12.18, **which is not true**

Graphical Method



$M_o = 35$ (*Graphical Method*)



$Q_1 = 30, Q_3 = 45$ & $Q_3 = 67$

Measures of Dispersion				
		Formula	**Calculation**	**Answer**
1	**Range (R)**	$L - S$	$100 - 1$	99
	Other-way		$100.5 - 0.5$	100
	Co – efficient of Range	$\frac{L-S}{L+S}$	$\frac{100-1}{100+1}$	0.98
2	**Quartile Deviation (QD)**	$\frac{Q_3 - Q_1}{2}$	$\frac{67.38 - 30.12}{2}$	18.63
	Coefficient of Quartile Deviation	$\frac{Q_3 - Q_1}{Q_3 + Q_1}$	$\frac{67.38 - 30.12}{67.38 + 30.12}$	0.38
3	**Mean Deviation ($MD_{\bar{X}}$)**	$\frac{1}{N}\sum \lvert m - \bar{X} \rvert$	$\frac{1843.54}{90}$	20.48
	Co – efficient of MD	$\frac{MD_{\bar{X}}}{Mean}$	$\frac{20.48}{48.61}$	0.42
4	**Standard Deviation (s)**	$\sqrt{\frac{\sum f(m - \bar{X})^2}{N}}$	$\sqrt{\frac{53128.89}{90}}$	24.29
	$Var\ (X)$	S^2	$(24.29)^2$	590.49
	$Coefficient\ of\ variation, var(X)$	$\frac{s}{\bar{X}} \times 100$	$\frac{24.29}{48.61} \times 100$	25.29%

Working Notes						
Marks (Class boundaries)	m	f	$\lvert m - \bar{X} \rvert$	$f\lvert m - \bar{x} \rvert$	$(X - \bar{X})^2$	$f\ (X - \bar{X})^2$
0.5 – 10.5	5.5	3	**43.11**	129.33	1858.4701	5575.4163
10.5 – 20.5	15.5	7	**33.11**	231.77	1096.2721	7673.9047
20.5 – 30.5	25.5	13	**23.11**	300.43	534.0721	6942.9373
30.5 – 40. 5	35.5	17	**13.11**	222.87	171.8721	2921.8252
40.5 – 50.5	45.5	12	**3.11**	37.32	9.6721	116.0652
50.5 – 60.5	55.5	10	**6.89**	68.90	47.4733	474.7210
60.5 – 70.5	65.5	8	**16.89**	135.12	285.2721	2282.1768
70.5 – 80.5	75.5	8	**26.89**	215.12	723.0721	5784.5768
80.5 – 90.5	85.5	6	**36.89**	221.34	1360.8721	8165.2326
90.5 – 100.5	95.5	6	**46.89**	281.34	2198.6721	13192.0326
Total		90		1843.54		53128.889

PROPERTIES: (A) MEASURES OF AVERAGES / CENTRAL TENDENCY

Arithmetic Mean

Property 1: If all the observations assumed by a variable are constants, say k, then the AM is also k.

Illustration: Consider 2, 2, 2

Property	Calculation	Answer
$\bar{X} = \frac{k + k + \cdots + k}{n} = k$	$\frac{2+2+2}{3}$	2

Property 2: (a) The algebraic sum of deviations of a set of observations from their AM is zero. And (b) the sum of the square of the deviation taken from the Mean ($\bar{X}$) is always minimum compared to the deviations taken from any other Assumed Mean (A)

Illustration: Consider (X): 2, 3, 4

	Property	Formula	Calculation	Answer
		$\bar{X} = \sum X$	$\frac{2+3+4}{3}$	3
(a)	$\sum (X - \bar{X}) = 0$ $\sum f(X - \bar{X}) = 0$	$\sum (X - \bar{X})$	$(2-3) + (3-3) + (4-3)$	0
(b)	$\sum (X - \bar{X})^2 \le \sum (X - A)^2$	$\sum (X - \bar{X})^2$	$(2-3)^2 + (3-3)^2 + (4-3)^2$	2
		$\sum (X - A)^2$ $Where\ A = 4$	$(2-4)^2 + (3-4)^2 + (4-4)^2$	5

Property 3: AM is affected due to a change of origin $(+/-)$ **and / or scale** $(\times/\div)$

i.e., If $y = a + bx$, then the AM of y is given by $\bar{y} = a + b\bar{x}$ (where a is change of origin and b is change of scale)

Illustration: Consider $(X) = 2, 3, 4,$

		Formula	Calculation	Answer	$\bar{Y} = \frac{\sum Y}{n} = a + b\bar{x}$
1	$X = 2, 3, 4,$	$\bar{X} = \frac{\sum X}{n}$	$\frac{2+3+4}{3}$	**3**	
2	$Y = 4, 5, 6,$	$\bar{Y} = \frac{\sum Y}{n}$	$\frac{4+5+6}{3}$	5	Change of Origin ($a = 2$)
	$Being\ Y = X + 2$	$\bar{y} = a + b\bar{x}$	$2 + 1 \times 3$	5	
3	$Y = 0, 1, 2,$	$\bar{Y} = \frac{\sum Y}{n}$	$\frac{0+1+2}{3}$	1	Change of Origin ($a = -2$)
	$Being\ Y = X - 2$	$\bar{y} = a + b\bar{x}$	$-2 + 1 \times 3$	1	
4	$Y = 4, 6, 8,$	$\bar{Y} = \frac{\sum Y}{n}$	$\frac{4+6+8}{3}$	6	Change of Scale ($b = 2$)
	$Being\ Y = X \times 2$	$\bar{y} = a + b\bar{x}$	$0 + 2 \times 3$	6	

5	$Y = 1, 1.5, 2,$	$\bar{Y} = \frac{\sum Y}{n}$	$\frac{1 + 1.5 + 2}{3}$	1.5	Change of Scale $\left(b = \frac{1}{2}\right)$
	$Being\ Y = X \times \frac{1}{2}$	$\bar{y} = a + b\bar{x}$	$0 + \frac{1}{2} \times 3$	1.5	
6	$Y = 7, 9, 11,$	$\bar{Y} = \frac{\sum Y}{n}$	$\frac{7 + 9 + 11}{3}$	9	Change of Origin and change of scale $(a = 3) \& (b = 2)$
	$Being\ Y = 3 + 2 \times X$	$\bar{y} = a + b\bar{x}$	$3 + 2 \times 3$	9	

Property 4: If there are two groups containing n_1 and n_2 observations and $\bar{x}_1$ and $\bar{x}_2$ as the respective arithmetic means, then the combined AM is given by $(\bar{x}_{12}) = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

Illustration		Combined mean	Calculation	Answer
Group 1	**Group II**	$\bar{x}_{12} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$	$\frac{(5 \times 9) + (15 \times 5)}{5 + 15}$	6
$n_1 = 5$	$n_2 = 15$			
$\bar{x}_1 = 9$	$\bar{x}_2 = 5$			

Points to Ponder:

1. In the case of "n" number of groups, Combined mean $(\bar{x}_{1\ldots n}) = \frac{\sum n_i \bar{x}_i}{\sum n_i}$

2. If sizes of the group are same, then the combined Mean is the average of the group means

   Explanation: If $n_1 = n_2 = n$, then $\bar{X}_{1+2} = \frac{n\bar{x}_1 + n\bar{x}_2}{n+n} = \frac{n(\bar{x}_1 + \bar{x}_2)}{2n} = \frac{\bar{x}_1 + \bar{x}_2}{2}$

   Illustration

		Formula	Calculation	Answer
1	$X_1 = 2, 3, 4,$	$\bar{X}_1 = \frac{\sum X_1}{n}$	$\frac{2 + 3 + 4}{3}$	**3**
2	$X_2 = 4, 5, 6,$	$\bar{X}_2 = \frac{\sum X_2}{n}$	$\frac{4 + 5 + 6}{3}$	5
3		$\bar{X}_{1+2} = \frac{n\bar{x}_1 + n\bar{x}_2}{n + n}$	$\frac{3 \times 3 + 3 \times 5}{3 + 3}$	4
		$\bar{X}_{1+2} = \frac{n(\bar{x}_1 + \bar{x}_2)}{2n}$	$\frac{3(3 + 5)}{2 \times 3}$	4
		$\bar{X}_{1+2} = \frac{\bar{x}_1 + \bar{x}_2}{2}$	$\frac{3 + 5}{2}$	4

3. If the averages are same, then the combined mean is the average itself

   Explanation: If $\bar{X}_1 = \bar{X}_2 = \bar{X}_{12}$

$$\bar{X}_{12} = \frac{n_1\bar{X} + n_2\bar{X}}{n_1 + n_2} = \frac{\bar{X}(n_1 + n_2)}{n_1 + n_2}$$

   Illustration

		Formula	Calculation	Answer
1	$X_1 = 2, 3, 4,$	$\bar{X}_1 = \frac{\sum X_1}{n}$	$\frac{2 + 3 + 4}{3}$	**3**
2	$X_2 = 4, 2,$	$\bar{X}_2 = \frac{\sum X_2}{n}$	$\frac{4 + 2}{2}$	3

		3		$\bar{X}_{1+2} = \frac{n\bar{x}_1 + n\bar{x}_2}{n+n}$	$\frac{3 \times 3 + 2 \times 3}{3+2}$	3	
				$\frac{\bar{X}(n_1 + n_2)}{n_1 + n_2}$	$\frac{3(3+2)}{2+3}$	3	
				$\bar{X}_{1+2} = \bar{X}_1 = \bar{X}_2$		3	

Geometric Mean

Property 1: Transformation in terms of log function

$$GM = Antilog\left(\frac{1}{n}\sum log\, x\right) Or\ log\ GM = \frac{1}{n}\sum log\, x$$

Property 2: If all the observations assumed by a variable are constants, say $k > 0$, then the GM of the observations is also K.

Property	Illustration	Calculation	Answer
$(k \times k \times \ldots . \times k)^{1/n} = k$	Consider: 2, 2, 2	$= (2 \times 2 \times 2)^{1/3}$	2

Property 3: GM of the product of two variables is the product of their GM's.

Property 4: GM of the ratio of two variables is the ratio of the GM's of the two variables.

	Illustration	Formula	Calculation	Answer
	$X = 3, 6, 12$	$\text{GM} = (X_1 \times X_2 \times \ldots \times X)^{\frac{1}{n}}$	$(3 \times 6 \times 12)^{1/3}$	6
	$Y = 1, 2, 4$		$(1 \times 2 \times 4)^{1/3}$	2
	$Z = 3, 12, 48$		$(3 \times 12 \times 48)^{1/3}$	12
Property 3	Being $Z = X \times Y$	$\text{GM}_Z = \text{GM}_X \times \text{GM}_Y$	6×2	12
	$Z = \frac{3}{1}, \frac{6}{2}, \frac{12}{4}$		$(3 \times 3 \times 3)^{1/3}$	3
Property 4	Being $Z = \frac{X}{Y}$	$GM_z = \frac{\text{GM}_X}{\text{GM}_Y}$	$\frac{6}{2}$	3

Harmonic Mean:

Property 1: If all the observations taken by a variable are constants, say k, then the HM of the observations is also k.

Property	Illustration	Calculation	Answer
$\frac{n}{\frac{1}{k} + \frac{1}{k} + \ldots . + \frac{1}{k}} = k$	$X = 2, 2, 2$	$\frac{3}{\frac{1}{2} + \frac{1}{2} + \frac{1}{2}}$	2

Property 2: If there are two groups containing n_1 and n_2 observations and X_1 and X_2 as the respective Harmonic Means, then the combined HM is given by $(\bar{X}_{12}) = \frac{n_1 + n_2}{\frac{n_1}{X_1} + \frac{n_2}{X_2}}$

Illustration		Combined H.M.	Calculation	Answer
Group 1	**Group II**	$\bar{X}_{12} = \frac{n_1 + n_2}{\frac{n_1}{\bar{x}_1} + \frac{n_2}{\bar{x}_2}}$	$\frac{15 + 10}{\frac{15}{3} + \frac{10}{2}}$	3.125

$n_1 = 15$	$n_2 = 10$			
$\bar{x}_1 = 3$	$\bar{x}_2 = 2$			

Median:

Property 1: If x and y are two variables, to be related by $Y = a + bX$ for any two constants a and b, then the median of y is given by $Y_{M_e} = a + bX_{M_e}$

(i.e., Median is affected due to a change of origin $(+/-)$ and / or scale $(\times/\div)$)

Illustration: Consider $(X) = 2, 3, 4,$

		Formula	Calculation	Answer	$Y_{M_e} = a + bX_{M_e}$
1	$X = 2, 3, 4,$	$\bar{X}_{M_e} = \left(\frac{n+1}{2}\right)^{th}$ obs	$\left(\frac{3+1}{2}\right)^{th}$ obs	3	
2	$Y = 4, 5, 6,$	$\bar{X}_{M_e} = \left(\frac{n+1}{2}\right)^{th}$ obs	$\left(\frac{3+1}{2}\right)^{th}$ obs	5	Change of Origin $(a = 2)$
	$Being\ Y = X + 2$	$Y_{M_e} = a + bX_{M_e}$	$2 + 1 \times 3$	5	
3	$Y = 0, 1, 2,$	$\bar{X}_{M_e} = \left(\frac{n+1}{2}\right)^{th}$ obs	$\left(\frac{3+1}{2}\right)^{th}$ obs	1	Change of Origin $(a = -2)$
	$Being\ Y = X - 2$	$Y_{M_e} = a + bX_{M_e}$	$-2 + 1 \times 3$	1	
4	$Y = 4, 6, 8,$	$\bar{X}_{M_e} = \left(\frac{n+1}{2}\right)^{th}$ obs	$\left(\frac{3+1}{2}\right)^{th}$ obs	6	Change of Scale $(b = 2)$
	$Being\ Y = X \times 2$	$Y_{M_e} = a + bX_{M_e}$	$0 + 2 \times 3$	6	
5	$Y = 1, 1.5, 2,$	$\bar{X}_{M_e} = \left(\frac{n+1}{2}\right)^{th}$ obs	$\left(\frac{3+1}{2}\right)^{th}$ obs	1.5	Change of Scale $\left(b = \frac{1}{2}\right)$
	$Being\ Y = X \times \frac{1}{2}$	$Y_{M_e} = a + bX_{M_e}$	$0 + \frac{1}{2} \times 3$	1.5	
6	$Y = 7, 9, 11,$	$\bar{X}_{M_e} = \left(\frac{n+1}{2}\right)^{th}$ obs	$\left(\frac{3+1}{2}\right)^{th}$ obs	9	Change of Origin and change of scale $(a = 3)\&(b = 2)$
	$Being\ Y = 3 + 2 \times X$	$Y_{M_e} = a + bX_{M_e}$	$3 + 2 \times 3$	9	

Property 2: For a set of observations, the sum of absolute deviations is minimum when the deviations are taken from the median.

Illustration: Consider (X): 0.5, 3, 4

		Calculation	Answer	Property
	$M_e = \left(\frac{n+1}{2}\right)^{th}$ obs	$\left(\frac{3+1}{2}\right)^{th}$ obs	3	
	$\bar{X} = \frac{\sum X}{n}$	$\frac{0.5 + 3 + 4}{3}$	2.5	
(a)	$\sum \lvert X - \bar{X} \rvert$	$\lvert 0.5 - 2.5 \rvert + \lvert 3 - 2.5 \rvert + \lvert 4 - 2.5 \rvert$	4	$(b) < (a)$
(b)	$\sum \lvert X - M_e \rvert$	$\lvert 0.5 - 3 \rvert + \lvert 3 - 3 \rvert + \lvert 4 - 3 \rvert$	3.5	

Mode:

Property 1: If $Y = a + bX$, then $Y_{M_o} = a + bX_{M_o}$

(i.e., Mode is affected due to a change of origin (+/−) and / or scale (×/÷))

Illustration: Consider $(X) = 2, 3, 3, 4$

		Formula	Calculation	Answer	$Y_{M_o} = a + bX_{M_o}$
1	$X = 2, 3, 3, 4,$	Most usual		3	
2	$Y = 4, 5, 5, 6,$			5	Change of Origin $(a = 2)$
	$Being\ Y = X + 2$	$Y_{M_o} = a + bX_{M_o}$	$2 + 1 \times 3$	5	
3	$Y = 0, 1, 1, 2,$	Most usual		1	Change of Origin $(a = -2)$
	$Being\ Y = X - 2$	$Y_{M_o} = a + bX_{M_o}$	$-2 + 1 \times 3$	1	
4	$Y = 4, 6, 6, 8,$	Most usual		6	Change of Scale $(b = 2)$
	$Being\ Y = X \times 2$	$Y_{M_e} = a + bX_{M_e}$	$0 + 2 \times 3$	6	
5	$Y = 1, 1.5, 1.5, 2,$	Most usual		1.5	Change of Scale $\left(b = \frac{1}{2}\right)$
	$Being\ Y = X \times \frac{1}{2}$	$Y_{M_o} = a + bX_{M_o}$	$0 + \frac{1}{2} \times 3$	1.5	
6	$Y = 7, 9, 9, 11,$	Most usual		9	Change of Origin and change of scale $(a = 3) \& (b = 2)$
	$Being\ Y = 3 + 2 \times X$	$Y_{M_o} = a + bX_{M_o}$	$3 + 2 \times 3$	9	

(B) MEASURES OF DISPERSION: PROPERTY

	Property	Measure / Explanation
1	All the observations assumed by a variable are constant, then measure of dispersion = 0	Range (R) = 0 Mean Deviation (MD) = 0 Standard Deviation (s) = 0
2	Affected due to change of Scale, but not of origin	$Ry = 0 + \lvert b \rvert \times Rx$ $MD_{\bar{y}} = 0 + \lvert b \rvert \times MD_{\bar{x}}$ $s_y = 0 + \lvert b \rvert \times s_x$
3	Mean deviation takes its minimum value	$MD_{M_e} = \frac{1}{n}\sum \lvert X - M_e \rvert$ is minimum

Illustration: Consider (X): 2, 2, 2

Formula	Calculation	Answer
$\bar{X} = \frac{\sum X}{n}$	$\frac{2+2+2}{3}$	**2**
Range = L – S	$2 - 2$	0
$MD_{\bar{X}} = \frac{1}{n}\sum \lvert X - \bar{X} \rvert$	$\frac{\lvert 2-2 \rvert + \lvert 2-2 \rvert + \lvert 2-2 \rvert}{3}$	0
$SD = \sqrt{\sum \frac{(X - \bar{X})^2}{n}}$	$\sqrt{\sum \frac{(X-2)^2}{3}}$	0

	when A = Median	
4	Combined SD	$s_{12} = \sqrt{\dfrac{n_1 S_1^2 + n_2 S_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$ where $d_1 = \bar{x}_1 - \bar{x}_{12}$ and $d_2 = \bar{x}_2 - \bar{x}_{12}$ Note: If $\bar{x}_1 = \bar{x}_2$, then $\bar{x}_1 = \bar{x}_2 = \bar{x}_{12}$ Then $d_1 = 0$ & $d_2 = 0$ $\therefore s_{12} = \sqrt{\dfrac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2}}$

Illustration		Calculation	Answer
Group I	Group II	$s_{12} = \sqrt{\dfrac{5 \times (0.8)^2 + (15 \times (0.5)^2) + (5 \times 3^2) + (15 \times (-1)^2)}{5 + 15}}$ $d_1 = \bar{x}_1 - \bar{x}_{12} = 9 - 6 = 3$ $d_2 = \bar{x}_2 - \bar{x}_{12} = 5 - 6 = -1$	1.83
$n_1 = 5$	$n_2 = 15$		
$\bar{x}_1 = 9$	$\bar{x}_2 = 5$		
$s_1 = 0.8$	$s_2 = 0.5$		
$\bar{x}_{12} = 6$			

Problem for SD under Change of scale and origin

		Formula	Calculation	Answer	$\boldsymbol{\bar{Y} = \frac{\sum Y}{n} = a + b\bar{x}}$
1	$\boldsymbol{X = 2, 3, 4,}$	$\boldsymbol{\bar{X} = \frac{\sum X}{n}}$	$\boldsymbol{\frac{2+3+4}{3}}$	**3**	
		$R_X = L - S$	$4 - 2$	2	
		$MD_{\bar{X}} = \frac{\sum \lvert X - \bar{X} \rvert}{n}$	$\frac{\sum \lvert X - 3 \rvert}{3}$	$\frac{2}{3}$	
		$s_X = \sqrt{\frac{\sum (X - \bar{X})^2}{n}}$	$\sqrt{\frac{\sum (X - 3)^2}{3}}$	0.82	
2	$Y = 4, 5, 6,$	$\bar{Y} = \frac{\sum Y}{n}$	$\frac{4+5+6}{3}$	5	Change of Origin $(a = 2)$
	Being $Y = X + 2$	$\bar{y} = a + b\bar{x}$	$2 + 1 \times 3$	5	
		$R_Y = L - S$	$6 - 4$	2	
		$MD_{\bar{Y}} = \frac{\sum \lvert Y - \bar{Y} \rvert}{n}$	$\frac{\sum \lvert Y - 5 \rvert}{3}$	$\frac{2}{3}$	
		$s_Y = \sqrt{\frac{\sum (Y - \bar{Y})^2}{n}}$	$\sqrt{\frac{\sum (Y - 3)^2}{3}}$	0.82	
3	$Y = 0, 1, 2,$	$\bar{Y} = \frac{\sum Y}{n}$	$\frac{0+1+2}{3}$	1	Change of Origin $(a = -2)$
	Being $Y = X - 2$	$\bar{y} = a + b\bar{x}$	$-2 + 1 \times 3$	1	
		$R_Y = L - S$	$2 - 0$	2	
		$MD_{\bar{Y}} = \frac{\sum \lvert Y - \bar{Y} \rvert}{n}$	$\frac{\sum \lvert Y - 1 \rvert}{3}$	$\frac{2}{3}$	

		$s_Y = \sqrt{\frac{\sum(Y-\bar{Y})^2}{n}}$	$\sqrt{\frac{\sum(Y-1)^2}{3}}$	0.82	
4	$Y = 4, 6, 8,$	$\bar{Y} = \frac{\sum Y}{n}$	$\frac{4+6+8}{3}$	6	Change of Scale ($b = 2$)
	$Being\ Y = X \times 2$	$\bar{y} = a + b\bar{x}$	$0 + 2 \times 3$	6	
		$R_Y = L - S$	$8 - 4$	4	
		$MD_{\bar{Y}} = \frac{\sum\lvert Y-\bar{Y}\rvert}{n}$	$\frac{\sum\lvert Y-6\rvert}{3}$	$\frac{4}{3}$	
		$s_Y = \sqrt{\frac{\sum(Y-\bar{Y})^2}{n}}$	$\sqrt{\frac{\sum(Y-6)^2}{3}}$	1.64	
5	$Y = 1, 1.5, 2,$	$\bar{Y} = \frac{\sum Y}{n}$	$\frac{1+1.5+2}{3}$	1.5	Change of Scale $\left(b = \frac{1}{2}\right)$
	$Being\ Y = X \times \frac{1}{2}$	$\bar{y} = a + b\bar{x}$	$0 + \frac{1}{2} \times 3$	1.5	
		$R_Y = L - S$	$2 - 1$	1	
		$MD_{\bar{Y}} = \frac{\sum\lvert Y-\bar{Y}\rvert}{n}$	$\frac{\sum\lvert Y-1.5\rvert}{3}$	$\frac{1}{3}$	
		$s_Y = \sqrt{\frac{\sum(Y-\bar{Y})^2}{n}}$	$\sqrt{\frac{\sum(Y-1.5)^2}{3}}$	0.41	
6	$Y = 7, 9, 11,$	$\bar{Y} = \frac{\sum Y}{n}$	$\frac{7+9+11}{3}$	9	Change of Origin and change of scale ($a = 3$)&($b = 2$)
	$Being\ Y = 3 + 2 \times X$	$\bar{y} = a + b\bar{x}$	$3 + 2 \times 3$	9	
		$R_X = L - S$	$11 - 7$	4	
		$MD_{\bar{x}} = \frac{\sum\lvert X-\bar{X}\rvert}{n}$	$\frac{\sum\lvert X-9\rvert}{3}$	$\frac{4}{3}$	
		$s_X = \sqrt{\frac{\sum(X-\bar{X})^2}{n}}$	$\sqrt{\frac{\sum(X-9)^2}{3}}$	0.41	

Coefficient of Variation (CV): $CV = \frac{s}{\bar{x}} \times 100$

<table>
<tr><th>Illustration</th><th>Calculation</th><th>Comparison</th></tr>
<tr>
<td>
<table>
<tr><th>Group 1</th><th>Group II</th></tr>
<tr><td>$\bar{x}_1 = 9$</td><td>$\bar{x}_2 = 5$</td></tr>
<tr><td>$s_1 = 0.8$</td><td>$s_2 = 0.5$</td></tr>
<tr><td colspan="2">$\bar{x}_{12} = 6$</td></tr>
</table>
</td>
<td>$CV(I) = \frac{0.8}{9} \times 100 = 8.88\%$
$CV(II) = \frac{0.5}{5} \times 100 = 10\%$</td>
<td>
<table>
<tr><td>$CV(I) = 8.88\%$</td><td>$<$</td><td>$CV(II) = 10\%$</td></tr>
<tr><td>More Stable
More Consistent
Less Variable
Less Dispersed</td><td></td><td>Less Stable
Less Consistent
More Variable
More Dispersed</td></tr>
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EXTRA PROBLEMS

Comparison between Arithmetic Mean and Geometric Mean

Question 1: Find the average rate of return.

Year	1	2	3
Rate of Return (r %)	10%	60%	20%

Answer: The average rate of return

	Formula	Calculation	Answer
GM	$G = (X_1 \times X_2 \times ... \times X_n)^{\frac{1}{n}}$	$(1.10 \times 1.60 \times 1.20)^{1/3}$	1.283 *or* 128.3% *or* 28.3%
AM	$\bar{X} = \frac{\sum X}{n}$	$\frac{1.10 + 1.60 + 1.20}{3}$	1.3 *or* 130% *or* 30% which is not possible

Comparison between Arithmetic Mean and Harmonic Mean

Question 2: An aeroplane covered a distance of 800 miles with four different speeds of 100, 200, 300 and 400 m/p.h for the first, second, third and fourth quarter of the distance. Find the average speed in miles per hour.

Answer: The average speed is given by the H.M. of the given set of data.

	Formula	Calculation	Answer
H M	$HM = \frac{n}{\sum \frac{1}{X}}$	$\frac{4}{\frac{1}{100} + \frac{1}{200} + \frac{1}{300} + \frac{1}{400}}$	192 m/p.h
AM	$\bar{X} = \frac{\sum X}{n}$	$\frac{100 + 200 + 300 + 400}{4}$	250 m/p.h, which is not true

Combined Mean

Question 3: Two groups of students reported mean weights of 160 kg and 150 kg respectively. Find out, when the weight of both the groups together be 155 kg?

Answer:

Given Data			Formula	Calculation	Answer
	Group I	Group II	$\bar{X}_{12} = \frac{N_1\bar{X}_1 + N_2\bar{x}_2}{N_1 + N_2}$	$155 = \frac{160N_1 + 150N_2}{N_1 + N_2}$	$N_1 = N_2$
Number	N_1	N_2			
Mean (kg.)	$\bar{X}_1 = 160$	$\bar{X}_2 = 150$		$155N_1 + 155N_2 = 160N_1 + 150N_2$	
Combined Mean: $\bar{X}_{12} = 155$kg					

Question 4: Show that for any two numbers a and b, standard deviation is given by $\frac{|a-b|}{2}$

Answer: For two numbers a and b, AM is given by $\bar{X} = \frac{a+b}{2}$

$$\text{The variance is} = \frac{\sum(X_i - \bar{X})^2}{2}$$

$$= \frac{\left(a - \frac{a+b}{2}\right)^2 + \left(b - \frac{a+b}{2}\right)^2}{2} = \frac{\frac{(a-b)^2}{4} + \frac{(a-b)^2}{4}}{2} = \frac{(a-b)^2}{4} \Longrightarrow s = \frac{|a-b|}{4}$$

(The absolute sign is taken, as SD cannot be negative).

Question 5: Prove that for the first n natural numbers, *is* $\sqrt{\frac{n^2 - 1}{12}}$.

Answer: for the first n natural numbers AM is given by

$$\overline{X} = \frac{1 + 2 + \ldots\ldots\ldots + n}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

$$\therefore SD = \sqrt{\frac{\sum X_i^2}{n} - \overline{X}^2} = \sqrt{\frac{1^2 + 2^2 + 3^2 \ldots\ldots.. + n^2}{n} - \left(\frac{n+1}{2}\right)^2}$$

$$\sqrt{\frac{n(n+1)(2n+1)}{6n} - \left(\frac{n+1}{2}\right)^2} = \sqrt{\frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2}$$

$$\sqrt{\frac{(n+1)(2n+1)}{6} - \frac{n+1}{2} \times \frac{n+1}{2}} = \sqrt{(n+1)\left(\frac{(2n+1)}{6} - \frac{n+1}{4}\right)}$$

$$\sqrt{\frac{(n+1)(4n+2-3n-3)}{12}} = \sqrt{\frac{n^2-1}{12}}$$

Thus, SD of first n natural numbers is $SD = \sqrt{\frac{n^2-1}{12}}$

COMPARISON BETWEEN MEASURES OF CENTRAL TENDENCY

No	Measures	Arithmetic Mean	Geometric Mean	Harmonic Mean	Median	Mode	Range	Quartile Deviation	Mean Deviation	Standard Deviation
1	Well defined	**Yes**	**Yes**	**Yes**	**Yes**	**No** (when the number of observations is small, then use Empirical Relationship**)**	**Yes**	**Yes**	**A may be** $\overline{\text{X}}, M_e, M_o$	**Yes**
2	Easy to calculate & simple to understand	Yes	No	No	Yes	Location Method, but not Grouping method	Yes	Yes	Yes	No
3	Based on all the items	Yes	Yes (but able to find only for Positive Values)	Yes (ONLY positive values and no "0")	No	No	No	No	Yes	Yes
4	capable of further mathematical treatment	Yes	Yes (Useful for calculation of Index Numbers)	Yes	Yes (but only in Mean Deviation, no combined Median)	No	No (But in case of Quality control and stock market fluctuations)	No	No (Useful for Economists and Businessmen and in public reports)	Yes
5	Good basis for comparison	Yes	Not much							Yes
6	Necessary for arrange of data	No	No	No	Yes	No	------Not on Discussion-----			
7	Affected by extreme values	Yes	Yes (Not much compared to	Yes	No	No	Yes	No	Less than SD	Yes

			AM)							
8	Not Precise – Mis-leading impressions (E.g. Average number of persons is 1.5 which is not possible)	No	No	No	Yes (except when Median lies in between two values)	Yes (except on continuous series)	------Not on Discussion-----			
9	Location (Inspection) Method	No	No	No	Yes (on arrangement)	Yes	------Not on Discussion-----			
10	Graphical Method				Yes (using Ogive Curves)		------Not on Discussion-----			
11	Calculated in the case of open end class intervals	No	No	No	Yes	Yes	No	Yes	Based on "A"	No
12	Affected by sampling fluctuations	No (least)	No	No	Yes	Yes	Yes	Yes	Yes	Less affected
13	Affected by Change of origin	Yes	Yes	Yes	Yes	Yes	No	No	No	No
	Affected by Change of Scale	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Explanations to Formulae:

1. Geometric Mean

Logarithmic formulae of Geometric Mean

Individual Observation

GM	=	$\sqrt[n]{x_1 \times x_2 \times \dots . \times x_n}$
$\log G.M$	=	$\log \sqrt[n]{x_1 \times x_2 \times \dots . \times x_n}$
	=	$\frac{1}{n}\log(x_1 \times x_2 \times \dots \times x_n)$
	=	$\frac{1}{n}(\log x_1 + \log x_2 + \dots . \log x_n)$
	=	$\frac{1}{n}\sum \log x$
GM	=	Anti log $\left(\frac{1}{n}\sum \log x\right)$

Discrete

GM	=	$\sqrt[N]{x_1^{f_1} \times x_2^{f_2} \times \dots . x_n^{f_n}}$
$\log G.M$	=	$\log \sqrt[N]{x_1^{f_1} \times x_2^{f_2} \times \dots . x_n^{f_n}}$
	=	$\frac{1}{N}[(\log x_1^{f_1} \times x_2^{f_2} \times \dots . x_n^{f_n})]$
	=	$\frac{1}{N}[\log x_1^{f_1} + \log x_2^{f_2} + \dots . \log x_n^{f_n}]$
	=	$\frac{1}{N}[f_1 \log x_1 + f_2 \log x_2 + \cdots f_n \log x_n]$
	=	$\frac{1}{N}\sum f \log x$
GM	=	Antilog $\frac{1}{N}\sum f \log x$

Continuous

GM	=	$\sqrt[N]{m_1^{f_1} \times m_2^{f_2} \times \dots . \times m_n^{f_n}}$
$\log G.M$	=	$\log \sqrt[N]{m_1^{f_1} \times m_2^{f_2} \times \dots \times m_n^{f_n}}$
	=	$\frac{1}{N}[(\log m_1^{f_1} \times m_2^{f_2} \times \dots \times m_n^{f_n})]$
	=	$\frac{1}{N}[\log m_1^{f_1} + \log m_2^{f_2} + \dots . \log m_n^{f_n}]$
	=	$\frac{1}{N}[f_1 \log m_1 + f_2 \log m_2 + \cdots f_n \log m_n]$
	=	$\frac{1}{N}\sum f \log m$
GM	=	Antilog $\frac{1}{N}\sum f \log m$

Standard Deviation:

$$s = \sqrt{\frac{\sum(X - \overline{X})^2}{n}}$$

$$\sum(X - \overline{X})^2 = \sum[X^2 - 2X\overline{X} + \overline{X}^2]$$

$$\sum(X - \overline{X})^2 = \sum X^2 - \sum(2X\overline{X}) + \sum \overline{X}^2$$

$$\sum(X - \overline{X})^2 = \sum X^2 - 2\overline{X}\sum X + n\overline{X}^2$$

$$\sum(X - \overline{X})^2 = \sum X^2 - 2\frac{\sum X}{n}\sum X + n.\frac{\sum X}{n}.\frac{\sum X}{n}$$

$$\sum(X - \overline{X})^2 = \sum X^2 - 2\frac{(\sum X)^2}{n} + \frac{(\sum X)^2}{n}$$

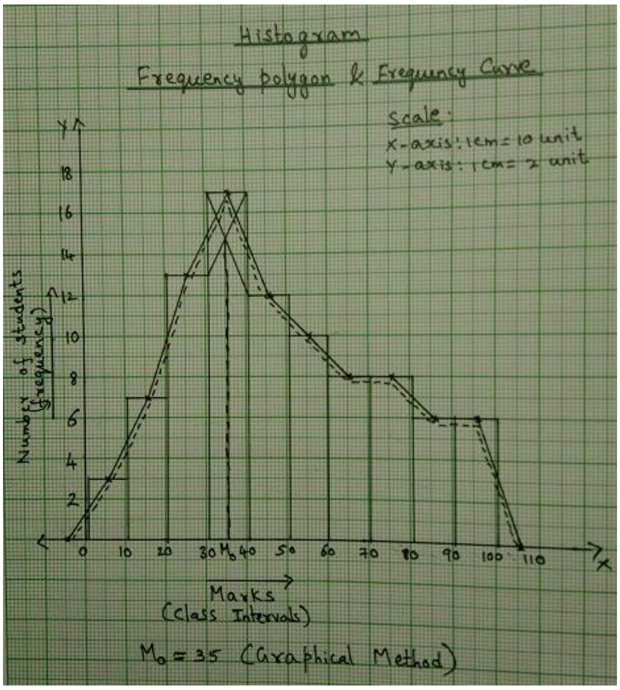
$$\sum(X - \overline{X})^2 = \sum X^2 - 2\frac{(\sum X)^2}{n} + \frac{(\sum X)^2}{n}$$

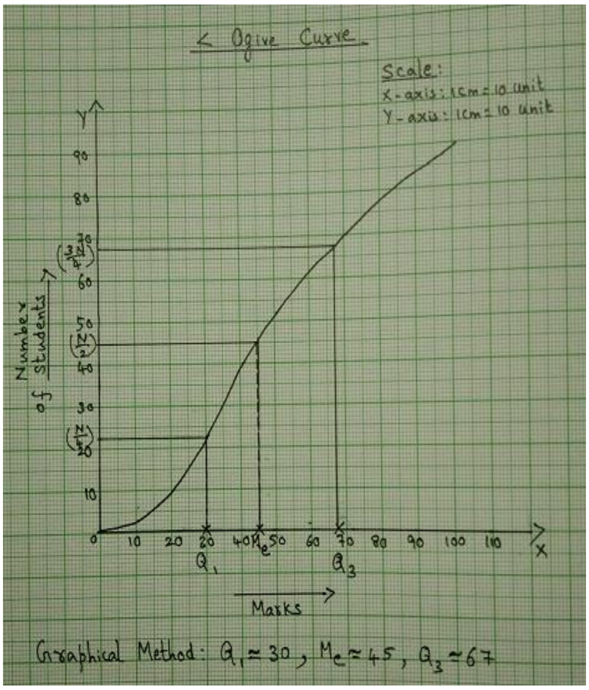
$$\sum(X - \overline{X})^2 = \sum X^2 - \frac{(\sum X)^2}{n}(2 - 1)$$

$$\frac{\sum(X - \overline{X})^2}{n} = \frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n}$$

$$\frac{\sum(X - \overline{X})^2}{n} = \frac{\frac{n\sum X^2 - (\sum X)^2}{n}}{n} = \frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2 = \frac{\sum X^2}{n} - \overline{X}^2$$

Graphical Method





Weighted Average:

1. Calculate goodwill using weighted average method:

Profit	20,000	10,000	(7000)
Weight	3	2	1

Missing Frequency:

1. Given N = 581 and Mean = 15. Find the missing frequencies.

x	10	11	12	13	14	15	16	17	18	19
f	8	15	x	100	98	95	y	75	50	30

2. Given Mean = 47, Median = 45, Mode = 35 and N= 90. Find the missing frequencies.

Marks	01-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Number of Students	3	7	x	17	12	y	8	8	6	6