Math 1496 – Cole 1

Consider $y = \sqrt{x^2+1}$

To find $\frac{dy}{dx}$ we use the chain rule

Let $u = x^2+1$ so $y = \sqrt{u} = u^{1/2}$

so $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{\cancel{2}} u^{-1/2} \cdot \cancel{2}x = x(x^2+1)^{-1/2}$

If $y = \sin(x^2)$

let $u = x^2$ so $y = \sin(u)$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \cos(u) \cdot 2x = 2x\cos(x^2)$

or $y = \frac{1}{3}(x^2+1)^3$ we could expand but

if $u = x^2+1$, $y = \frac{u^3}{3}$ & $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 2x\left(\frac{\cancel{3}}{\cancel{3}}(x^2+1)^2\right)$

$= 2x(x^2+1)^2$

we now ask how does one integral

$$\int x(x^2+1)^{-1/2}\,dx$$

$$\int 2x\cos(x^2)\,dx$$

$$\int 2x(x^2+1)^2\,dx$$

As the secret to differentiating via the Chain rule was to let $u = x^2+1$ or $u = x^2$

we do the same for integrals

Ex 1 $\int 2x(x^2+1)^2\,dx$

Recall differentials

$u = f(x)$

$du = f'(x)\,dx$

let $u = x^2+1$ $\quad du = 2x\,dx$

So $\int (x^2+1)^2\,2x\,dx$

$$\int u^2\,du = \frac{u^3}{3} + C = \frac{1}{3}(x^2+1)^3 + C$$

standard $\int$.

This technique is called "u substitution"

Ex 2 $\int e^{2x+1}\,dx$

Here, the integrand composes of 2 steps

(i) $2x+1$

(2) $e^{\,\text{this}}$

let $u = 2x+1$ (the 1st step)

$du = 2\,dx$ so $\frac{du}{2} = dx$

$$\int e^u \frac{du}{2} = \frac{1}{2}e^u + c = \frac{1}{2}e^{2x+1}$$

Ex 3 $\int \sqrt{3x-4}\,dx$

$u = 3x-4$ 1st step

$du = 3\,dx \Rightarrow \frac{du}{3} = dx$

$$\int u^{1/2}\frac{du}{3} = \frac{u^{3/2}}{3/2 \cdot 3} + c = \frac{2}{9}(3x-4)^{3/2} + c$$

ex4 $\displaystyle\int \frac{x^3\,dx}{(1+x^4)^2}$

Hard part of integral is bottom

let $u = 1+x^4$

$du = 4x^3\,dx$ or $\dfrac{du}{4} = x^3\,dx$

$$\int \frac{\frac{1}{4}\,du}{u^2} = \frac{1}{4}\int u^{-2}\,du = \frac{1}{4}\,\frac{u^{-1}}{-1} + c$$

$$= \frac{-1}{4u} + C = \frac{-1}{4(1+x^4)} + C$$

Ex5 $\displaystyle\int \sin x \cos^2 x\,dx$

hard part

let $u = \cos x$

$du = -\sin x\,dx$

$$-\int u^2\,du = \frac{-u^3}{3} + c$$

$$= -\frac{1}{3}\cos^3 x + C$$

Ex 6 $\int_0^{\sqrt{\pi/2}} x\cos(x^2)\,dx$ Now we have limits

let $u = x^2$ $\quad x=0 \quad u = x^2 = 0$

$du = 2x\,dx$ $\quad x = \sqrt{\pi/2} \quad u = \pi/2$

so new prob $\int_0^{\pi/2} \frac{\cos(u)\,du}{2} = \frac{\sin(u)}{2}\Big|_0^{\pi/2} = \frac{1}{2}$

Ex 7 $\int_0^3 t(1+t^2)^4\,dt$

let $u = 1+t^2$ $\quad t=0 \quad u=1$

$du = 2t\,dt$ $\quad t=3 \quad u = 1+3^2 = 10$

So $\int_1^{10} u^4 \frac{du}{2} = \frac{u^5}{25}\Big|_1^{10} = \frac{10^5-1^5}{10}$

Ex 8 $\int_1^{e^2} \frac{\ln x}{x}\,dx$ $\quad u = \ln x \quad u = \ln 1 = 0$

$du = \frac{dx}{x}$ $\quad u = \ln e^2 = 2$

$\int_0^2 u\,du = \frac{u^2}{2}\Big|_0^2 = \frac{4}{2} - 0 = 2$

HW pg 391

17, 19, 25, 28

33, 39, 41, 43, 45

49, 51,