International Journal of Modern Computation, Information and Communication Technology,
2018;1(2):51-56.
ISSN: 2581-5954
http://ijmcict.gjpublications.com

Research Article

Soft b-Separation Axioms between Two Soft b-Open Sets

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Abstract
In the present article the author introduces the concept of Soft b W-D_2 structure in soft topological spaces as well as in soft bi topological spaces. Heriditary properties and product of the spaces is also discussed. There are many structures in soft topology but soft b W-D_2 topological structure is interesting and more practical because it is obtained from the mixture of two structures that why it arrested our attention.

Keywords: **S**oft Topology Soft set; Soft b-open set; Soft b closed set; Soft b W-D_2 space.

Introduction

General topology play attractive role in space time geometry as well as different branches of pure and applied mathematics. In Separation Axioms we discuss points of the space. It shows the points are separated by neighbour-hood**.** When we are interested to know the distance among the points that are separated from each other, then in that case the concept of separation axioms will come in play**.** Most of the real life problems have various uncertainties. Kelly [1] discussed some results in Bi topological Spaces with respect to ordinary points. Reilly [2] discussed different separation axioms and their properties in bi topology. Soft set theory is one of the young topics achieving importance in finding balanced and reasonable way out in day to day life activities problems which involves uncertainty and ambiguity.

In 1999, Molodtsov [3] originated a new concept of soft set theory, which is absolutely a new method for modelling vagueness and uncertainty Shabir and Naz [4] are the founder of soft topology. They discussed the basic and some properties and separation axioms in soft topology. Ittanagi [5] discussed Soft Bi topological Spaces pairwiselly with respect to ordinary points. Sruthi et al [6] studied Soft W-Hausdorff Spaces. B. Chen, [7] studied Soft semi-open sets and related properties in soft topological spaces.

Preliminary

Throughout this paper, $\overset{\frown}{X}$ denotes the master set and $\breve{E}$ denotes the set of parameters for the master set $\overset{\frown}{X}$.

Definition1: [4]
Let $\overset{\frown}{X}$ be the master and $\breve{E}$ be a set of parameters. Let $P(\overset{\frown}{X})$ denotes the power set of $\overset{\frown}{X}$ and $\breve{A}$ be a nonempty subset of $\breve{E}$. A pair $(F, \breve{A})$ denoted by $F_{\breve{A}}$ is named a soft set over $\overset{\frown}{X}$, where F is a mapping given by $F: \breve{A} \rightarrow P(\overset{\frown}{X})$.In other words, a soft set over $\overset{\frown}{X}$ is a parameterized family of subsets of the master X. For a specific $e \in \breve{A}$, $F(e)$ may be considered the set of e-approximate elements of the soft set $(F, \breve{A})$ and if $e \notin \breve{A}$, then $F(e) = \phi$
$i.e. F_A = \{F(e): e \in \breve{A} \subseteq \breve{E} \; ; F: \breve{A} \rightarrow P(\overset{\frown}{X})\}$

Received: 26.06.2018; Received after Revision: 12.07.2018; Accepted: 12.07.2018; Published: 02.08.2018

The family of all these soft sets over X with respect to the restriction set $\breve{E}$ is signified by $SS(X)_{\breve{E}}$.

Definition 2: [4]

Let $F_{\breve{A}}, G_{\breve{B}} \in SS(X)_{\breve{E}}$. Then $F_{\breve{A}}$ is soft sub set of $G_{\breve{B}}$, denoted by $F_{\breve{A}} \tilde{\subseteq} G_{\breve{B}}$, if

(1) $\breve{A} \tilde{\subseteq} \breve{B}$. , and

(2) $F(e) \tilde{\subseteq} G(e), \forall e \in \breve{A}$.

In this case, $F_{\breve{A}}$ is supposed to be a soft subset of $G_{\breve{B}}$ and $G_{\breve{B}}$ is said to be a soft super set of $F_{\breve{A}}, G_{\breve{B}} \tilde{\subseteq} F_{\breve{A}}$

Definition 3: [4]

Two soft subsets F_A and G_B over a common universe X are said to be soft equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A .

Definition 4: [4]

The complement of a soft set (F, A) denoted by $(F, A)'$ is defined by $(F, A)' = (F', A), F' : A \longrightarrow P(X)$ is a mapping given by $F'(e) = X - F(e); \forall e \in A$ and F' is called the soft complement function of F. Clearly $(F')'$ is the same as F and $((F, A)')' = (F, A)$.

Definition 5: [4]

A soft set (F, A) over X is said to be a Null soft set denoted by $\tilde{\Phi}$ or ϕ_A if for all $e \in A$, $F(e) = \phi$ $(vacuous\ set)$.

Definition6: [4]

A soft set (F, A) over X is said to be an absolute soft set denoted by $\tilde{A}$ or X_A if for all $e \in A$, $F(e) = X$. obviously we have $X'_A = \phi_A$ and $\phi'_A = X_A$.

Definition 7: [4]

The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ & $\forall e \in C$,

$$H(e) = \begin{cases} F(e), e \in A - B \\ G(e), e \in B - A \\ F(e) \tilde{\cap} G(e), e \in A \tilde{\cap} B \end{cases}$$

Definition 8: [4]

The intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \tilde{\cap} B$ and for all $e \in C, H(e) = F(e) \tilde{\cap} G(e)$.

Definition 9: [4]

Let $\tilde{\tau}$ be the collection of soft sets over $\widehat{X}$, then $\tilde{\tau}$ is said to be a soft topology on $\widehat{X}$, if

(1) $\phi, \widehat{X} \in \tilde{\tau}$ `

(2) Union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

(3) Intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

Definition10: [4].

Let $(\widehat{X}, \tilde{\tau}, E)$ be a soft topological space, $(F, E) \in SS(X)_E$ and $\tilde{Y}$ be a non-vacuous subset of $\widehat{X}$. Then the soft subset of (F, E) over $\tilde{Y}$ signified by $(F_{\tilde{Y}}, E)$ is defined as follows:

$F_Y(e) = \tilde{Y} \tilde{\cap} F(e), \forall e \in E$

In other words, $(F_{\tilde{Y}}, E) = \tilde{Y}_E \tilde{\cap} (F, E)$.

Definition11: [4].

Let $(X, \tilde{\tau}, E)$ be a soft topological space and $\tilde{Y}$ be a non-vacuous subset of X. Then $\tilde{\tau}_{\tilde{Y}} = \{(F_{\tilde{Y}}, E) : (F, E) \in \tilde{\tau}\}$ is said to be the relative soft topology on $\tilde{Y}$ and $(Y, \tilde{\tau}_Y, E)$ is called a soft subspace of $(\widehat{X}, \tilde{\tau}, E)$.

Definition12: [6].

Let $F_A \in SS(X)_E$ & $G_B \in SS(Y)_K$. The Cartesian product $F_A \circledast G_B$ is defined by $(F_A \circledast G_B)(e, k) = F_A(e) \times G_B(k), \forall (e, k) \in A \times B$

According to this definition $F_A \circledast G_B$ is a soft set over $X \times Y$ and its parameter set is $E \times K$.

Definition13: [6].

Let $(\widehat{X}, \tilde{\tau}_X, E)$ and $(\tilde{Y}, \tilde{\tau}_{\tilde{Y}}, K)$ be two soft topological spaces. The soft product topology $\tilde{\tau}_X \circledast \tau_Y$ over $X \times Y$ with respect to $E \times K$ is the soft topology having the collection $\{F_E \circledast G_K / F_E \in \tilde{\tau}_{\tilde{X}}, G_K \in \tilde{\tau}_{\tilde{Y}}\}$ as the basis.

Definition14: [6].

Let $\left(\widehat{X}, \tilde{\tau}_{1\widehat{X}}, E\right)$ and $\left(\widehat{X}, \tilde{\tau}_{2\widehat{X}}, E\right)$ be two not the same soft topological spaces on $\widehat{X}$. Then $(X, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$ is called a Soft bi topological space if the two soft topologies $\tilde{\tau}_{1\widehat{X}}$ and $\tilde{\tau}_{2\widehat{X}}$ individually gratify the axioms of soft topology. The participants of $\tilde{\tau}_{1\widehat{X}}$ are called $\tilde{\tau}_{1\widehat{X}}$ soft open sets and the complements of $\tilde{\tau}_{1\widehat{X}}$ soft open sets are named $\tilde{\tau}_{1\widehat{X}}$ soft closed sets. Similarly, The participants of $\tilde{\tau}_{2\widehat{X}}$ are called $\tilde{\tau}_{2\widehat{X}}$ soft open sets and the complements of $\tilde{\tau}_{2\widehat{X}}$ soft open sets are named $\tilde{\tau}_{2\widehat{X}}$ soft closed sets.

Definition 15: [6].

Let $(X, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$ be a soft bi topological space over X and Y be a non-empty subset of X. Then $\tau_{1\tilde{Y}} = \{(F_Y, E) : (F, E) \in \tilde{\tau}_{1\widehat{X}}\}$ and $\tilde{\tau}_{2\tilde{Y}} = \{(G_{\tilde{Y}}, E) : (G, E) \in \tilde{\tau}_{2\widehat{X}}\}$ are said to be the relative topologies on Y and $\{Y, \tilde{\tau}_{1\tilde{Y}}, \tilde{\tau}_{2\tilde{Y}}, E\}$ is named as soft sub space of $(X, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$.

Definition 16: [7]
Let (X,τ,E) be a soft topological space and $(F,E) \subseteq SS(X)_A$ then (F,E) is called a b open soft set. $((F,E) \subseteq Cl(int(F,E) U int(Cl(F,E))$
The set of all *b open soft* set is denoted by $BOS(X,\tau,E)$ or $BOS(X)$ and the set of all *b closed soft* set is denoted by $BCS(X,\tau,E)$ or $BCS(X)$

New Results in Soft Topological Space

Definition 17: Let $(\widehat{X},\tilde{\tau},E)$ be a soft topological space and a soft set (P,E) of soft topological space is called soft b-difference set (in short hand bD-soft set) if there exists two soft b-open sets (P_1,E),(P_2,E) such that $(B,E) \neq (\widehat{X},\tilde{\tau},E)$ and set $(B,E) = (B_1,E) \backslash (B_2,E)$. Here clearly the soft (B,E) is the difference of two soft b-open sets.
Definition 18:
A soft topological space $(\widehat{X},\tilde{\tau},E)$ is said to be Soft b W-D_2 space of type 1 signified by $pre(DW-H)_1$ if for every $e_1,e_2 \in E, e_1 \neq e_2$ there exists soft bD set which is difference of soft pre-open sets $(B_1,\breve{A})$, $(B_2,\breve{G})$ such that
$B_{1_{\breve{A}}}(e_1) = \widehat{X}, B_{2_{\breve{G}}}(e_2) = \widehat{X}$ and $(B_1,\breve{A}) \tilde{\cap} (B_2,\breve{G}) = \tilde{\phi}$
Definition 19:
Let $(\widehat{X},\tilde{\tau},E)$ be a soft topological space and $H \subseteq E$. Then $(X,\tilde{\tau}_H,H)$ is called soft p-subspace of $(X,\tilde{\tau},E)$ relative to the parameter set H where
$\tilde{\tau}_H = \{(B_{\breve{A}}) / H : H \subseteq \breve{A} \subseteq E, bD\ soft\ pre\ open\ set\ B_{\breve{A}} \in \tilde{\tau}\}$
and $(B_{\breve{A}}) / H$ is the restriction map on H.
Theorem 1. [1]
Soft subspace of a b $(DW-H)_1$ space is soft b$(DW-H)_1$.
(2) Soft p-subspace of a b $(DW-H)_1$ space is soft b$(DW-H)_1$.
(3) Product of two soft b $(DW-H)_1$ space is soft b$(DW-H)_1$.
(1) Proof:
Let $(\widehat{X},\tilde{\tau}_1,\tilde{\tau}_2)$ be soft b $(DW-H)_1$ space. Let Y be a non-vacuous sub set of $\widehat{X}$. Let $(\widehat{Y},\tilde{\tau}_Y,E)$ be a soft sub space of $(\widehat{X},\tau,E)$ where $\tilde{\tau}_Y = \{(\tilde{F}_Y,E)$:(S, E) $\in \tilde{\tau}\}$ is the relative soft topology on $\widehat{Y}$. Consider $e_1,e_2 \in E, e_1 \neq e_2$ there exists bD set which is difference of soft b-open sets (B_1,A), (B_2,G) such that
$B_{1_{\breve{A}}}(e_1) = X, B_{2_{\breve{G}}}(e_2) = X$ and $(B_1,\breve{A}) \tilde{\cap} (B_2,\breve{G}) = \tilde{\phi}$
Therefore $((B_{\breve{A}})_Y,E),((B_{2_{\breve{G}}})_Y,E) \in \tilde{\tau}_Y$. Also
$(B_{1_{\breve{A}}})_Y(e_1) = Y \cap B_{1_{\breve{A}}}(e_1) = Y \tilde{\cap} X = Y$.
$(B_{1_{\breve{G}}})_Y(e_2) = Y \cap B_{2_{\breve{G}}}(e_2) = Y \tilde{\cap} X = Y$.
$((B_{1_{\breve{A}}})Y \tilde{\cap} B_{2_G})\tilde{Y})(e) = ((B_{1_{\breve{A}}} \tilde{\cap} B_{2_{\breve{G}}})\tilde{Y})(e)$
$= \tilde{Y} \tilde{\cap} (B_{2_{\breve{G}}} \tilde{\cap} c)(e)$
$= \tilde{Y} \cap \tilde{\phi}(e)$
$= \tilde{Y} \cap \phi$ `
$= \phi$
$(B_{1_{\breve{A}}})_Y \cap (B_{2_{\breve{G}}})_Y = \phi$. Hence $(\widehat{Y},\tilde{\tau}_Y,E)$ is b $(DW-H)_1$.
(2) Proof: Let (X,τ, E) be a b $(DW-H)_1$ space. Let $H \subseteq E$. Let $(\widehat{X},\tilde{\tau}_H,H)$ be a soft p-subspace of $(\widehat{X},\tau,E)$ relative to the parameter set H. $\tilde{\tau}_H = \{(B_{1_{\breve{A}}})/H : H \subseteq A \subseteq E, B_{1_{\breve{A}}} \in \tau$.
Suppose $\tilde{h}_1,\tilde{h}_2 \in H, h_1 \neq h_2$. Then $\tilde{h}_1,\tilde{h}_2 \in E$. Therefore, there happens soft bD set which is difference of soft b-open sets $(B_1,A),(B_2,G)$ such that $B_{1_{\breve{A}}}(h_1)$=X, $B_{2_{\breve{G}}}(h_2) = \widehat{X}$ & $(B_1,\breve{A}) \cap (B_2,\breve{G}) = \tilde{\phi}$.
Therefore $(B_{1_{\breve{A}}})/H$ $(B_{2_{\breve{G}}})/H \in \tilde{\tau}_H$. Also
$((B_{1_{\breve{A}}})/H)(h_1) = B_{1_{\breve{A}}}(h_1) = \widehat{X}$
$((B_{2_{\breve{G}}})/H)(h_2) = B_{2_G}(h_2) = \widehat{X}$ &
$((B_{1_{\breve{A}}})/H) \cap ((B_{2_{\breve{G}}})/H) = (B_{1_{\breve{A}}} \cap B_{2_{\breve{G}}})/H$
$= \tilde{\phi} / H$
$= \tilde{\phi}$
Hence b $(\widehat{X},\tilde{\tau}_H,H)$ is b$(DW-H)_1$.
(3) Proof: Let $(\widehat{X},\tilde{\tau}_x,E)$ and $\{\widehat{Y},\tilde{\tau}_Y,K\}$ be two $(BDW-H)_1$ spaces. Consider two distinct points (e_1,k_1), $(e_2,k_2 \in E \times K$ either $e_1 \neq e_2$ or $k_1 \neq k_2$.Suppose $e_1 \neq e_2$. Since $(\widehat{X},\tilde{\tau}_x,E)$ is $(BDW-H)_1$, there exist bD set which is difference of soft b-open sets $(B_1,\breve{A}) \cap (B_2,\breve{G})$ such that $B_{1_{\breve{A}}}(h_1)$=X, $B_{2_{\breve{G}}}(h_2) = \widehat{X}$ & $(B_1,\breve{A}) \cap (B_2,\breve{G}) = \tilde{\phi}$.
Therefore
$B_{1_A} \circledast Y_k \in \tilde{\tau}_{1X} \circledast \tilde{\tau}_{1Y}, B_{2_{\breve{G}}} \circledast Y_K \in \tilde{\tau}_X \circledast \tilde{\tau}_Y$
$(B_{1_{\breve{A}}} \circledast \tilde{Y}_K)(e_1,k_1) = B_{1_{\breve{A}}}(e_1) \times \tilde{Y}_K(k_1) = \widehat{X} \times \tilde{Y}$
$(B_{2_{\breve{G}}} \circledast Y_K)(e_2,k_2) = B_{2_{\breve{G}}}(e_2) \times Y_K(k_2)$
$= \widehat{X} \times \tilde{Y}$

If for any
$(e,k) \in (E \times K), (B_{1_A} \circledast \tilde{Y}_K)(e,k) \neq \phi \Rightarrow$
$B_{1_A}(e) \times \tilde{Y}_K(k) \neq \phi$
$\Rightarrow B_{1_{\breve{A}}}(e) \times \tilde{Y} \neq \phi \Rightarrow B_{1_{\breve{A}}}(e) \neq \phi \Rightarrow$
$B_{2_{\breve{G}}}(e) = \phi$
Since
(
$B_{1_A} \cap B_{2_{\breve{G}}} = \phi \Rightarrow B_{1_A}(e) \cap B_{2_{\breve{G}}}(e) =$
ϕ)
$\Rightarrow B_{2_{\breve{G}}}(e) \times \tilde{Y}_K(k) = \phi \Rightarrow (B_{2_G} \otimes$
$\tilde{Y}_K)(e,k) = \phi$
$\Rightarrow (B_{1_{\breve{A}}} \circledast \tilde{Y}_K) \cap (B_{2_{\breve{G}}} \circledast \tilde{Y}_K) = \tilde{\phi}$
Assume $k_1 \neq k_2. Since \{Y, \tilde{\tau}_{1Y}, \tilde{\tau}_{2Y}, K\}$ is b $(DW-H)_1$ so there exist soft bD set which is difference of soft set b-open sets $(((B_1,\breve{A}) \cap (B_2,\breve{G}))$ such that $B_{1_{\breve{A}}}(k_1) =, \tilde{Y}\ B_{2_{\breve{G}}}(k_2) = \tilde{Y}$ &
$B_{1_{\breve{A}}} \cap B_{2_{\breve{G}}} = \tilde{\phi}$. `
Therefore $X_E \circledast B_{1_A} \in \tilde{\tau}_Y, \hat{X}_E \circledast B_{2_{\breve{B}}} \in \tilde{\tau}_Y$
$(\hat{X}_E \circledast B_{1_{\breve{A}}})(e_1,k_1) = \hat{X}_E(e_1) \times B_{1_{\breve{A}}}(k_1)$
$= \hat{X} \times \tilde{Y}$
$(X_E \circledast B_{2_{\breve{G}}})(e_2,k_2) = \hat{X}_E(e_2) \times B_{1_{\breve{A}}}(k_1)$
$= \hat{X} \times \tilde{Y}$
If for any
$(e,k) \in E \times K, (\hat{X}_E \circledast B_{1_{\breve{A}}})(e,k) \neq \phi \Rightarrow$
$\hat{X}_E(e) \times B_{1_{\breve{A}}}(k) \neq \phi$
$\Rightarrow \hat{X} \times B_{1_{\breve{A}}}(k) \neq \phi \Rightarrow B_{1_{\breve{A}}}(k) \neq \phi \Rightarrow$
$B_{1_A}(k) \neq \phi$
So $B_{2_B}(k) = \phi$
$(Since B_{1_A} \cap B_{2_B} = \tilde{\phi} \Rightarrow B_{1_{\breve{A}}}(k) \cap B_{2_G}(k) =$
$\phi) \Rightarrow X_E(e) \times B_{2_G}(k) = \phi \Rightarrow (X_E \circledast$
$B_{2_G})(e,k) = \phi$

$\Rightarrow (\hat{X}_E \circledast B_{1_{\breve{A}}}) \cap (X_E \circledast B_{2_G}) = \tilde{\phi}$
Hence Product of two soft b $(DW-H)_1$ space is soft b$(DW-H)_1$.

New Results in Soft Bi-Topological Space

Definition 20:
Let $\hat{X}$ be a non-empty set and $\tilde{\tau}_1$ and $\tilde{\tau}_2$ be two different topologies on $\hat{X}$. Then $(\hat{X}, \tilde{\tau}_1, \tilde{\tau}_2)$ is called a bi topological space.

Definition 21:
A soft bi topological space $(\hat{X}, \tilde{\tau}_{1X}, \tilde{\tau}_{2X}, E)$ is said to be Soft b W-D_2 space or soft b $W-T_2$ space of type 1 denoted by $(BDW-H)_1$ if it is soft b $(BDW-H)_1$ With respect to $\tau_{1\hat{X}}$ or soft b$(SDW-H)_1$ with respect to$\tilde{\tau}_{2\hat{X}}$.

Definition 22:
A soft bi topological space $(\hat{X}, \tau_{1\hat{X}}, \tau_{2\hat{X}}, E)$ is supposed to be Soft b W-D- Hausdorf space of type 2 signified by $(BDW-H)_2$ if for every $e_1, e_2 \in E, e_1 \neq e_2$ there occurs soft there exist soft bD set which is combination of soft set soft b open sets $(B_1,A),(B_2,G)$ such that $(P_1,\breve{A}) \in \tilde{\tau}_{1\hat{X}}$ soft $(P_2,\breve{G}) \in \tilde{\tau}_{2\hat{X}}$ such that $P_{1_{\breve{A}}}(e_1)$=X, $P_{2_{\breve{B}}}(e_2) = \hat{X}$
and $(\tilde{B}_1, \breve{A}) \cap (\tilde{B}_2, \breve{G}) = \tilde{\phi}$.

Theorem 2.
Soft subspace of a b $(SDW-H)_1$ space is b $(SDW-H)_1$.

Proof: Let $(\hat{X}, \tilde{\tau}_{1\hat{X}}, \tau_{2\hat{X}}, E)$ be a pre $(BDW-H)_1$ space. Then it is b $(SDW-H)_1$ with respect to $\tilde{\tau}_{1X}$ *or* $b(SDW-H)_1$ with respect to $\tilde{\tau}_{2\hat{X}}$. Let $\tilde{Y}$ be a non-vacuous subset of$\hat{X}$,. Let
$\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tilde{\tau}_{1\tilde{Y}}, E\}$ be a soft subspace of $(X, \tilde{\tau}_{1X}, \tilde{\tau}_{2X}, E)$. From Theorem1 a soft subspace of pre $(BDW-H)_1$space is $(BDW-H)_1$. Therefore, $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tilde{\tau}_{2\tilde{Y}}, E\}$ is $b(BDW-H)1$ with respect to $\tilde{\tau}_{1Y}$ or $b(BDW-H)_1$ with respect to$\tilde{\tau}_{2Y}$. Hence $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tilde{\tau}_{2\tilde{Y}}, E\}$ is
$b(BDW-H)_1$.

Theorem 3. Soft subspace of a $b(SDW-H)_2$ space is $b(SDW-H)_2$.

Proof: Let $(X, \tilde{\tau}_{1\hat{X}}, \tilde{\tau}_{2\hat{X}}, E)$ be a b $(BDW-H)_2$ space. Let Y be a non-vacuous subset of $\hat{X}$. Let $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tilde{\tau}_{2\tilde{Y}}, E\}$ be a soft subspace of $(X, \tilde{\tau}_{1\hat{X}}, \tilde{\tau}_{2\hat{X}}, E)$ where
$\tilde{\tau}_{1\tilde{Y}} = \{(P_{1_{\tilde{Y}}}, E) :$ bD soft b open $(B_1, E) \in \tilde{\tau}_{1\hat{X}}\}$
&
$\tilde{\tau}_{2\tilde{Y}} = \{(B_{2_{\tilde{Y}}}, E) :$ bD soft b open$(B_2, E) \in \tilde{\tau}_{2\hat{X}}\}$
are supposed to be the relative topologies on Y. Consider $e_1, e_2 \in E$ *with* $e_1 \neq e_2$ there occur bD soft open
$(B_1, E) \in \tilde{\tau}_{1\hat{X}}$, bD soft open $(P_2, E) \in \tilde{\tau}_{2\hat{X}}$ such that $B_{1_A}(e_1) = X, B_{2_G}(e_2) = X$
&$(B_1, E) \tilde{\cap} (B_2, E) = \tilde{\phi}$.
Hence $((B_{1_{\breve{A}}})\tilde{Y}, E) \in \tilde{\tau}_{1Y}, ((B_{2_G})\tilde{Y}, E) \in \tilde{\tau}_{2\tilde{Y}}$
Also $(B_{1_{\breve{A}}})\tilde{Y}(e_1) = \tilde{Y} \tilde{\cap} B_{1_{\breve{A}}}(e_1)$
$= \tilde{Y} \tilde{\cap} X = Y$
$(B_{2_G})\tilde{Y}(e_2) = \tilde{Y} \tilde{\cap} B_{2_{\breve{G}}}(e_2)$

$= \tilde{Y} \tilde{\cap} X$
$= \tilde{Y}$
$((B_{1_A})Y \tilde{\cap} B_{2_G})\tilde{Y})(e) = ((B_{1_A} \tilde{\cap} B_{2_{\breve{G}}})\tilde{Y})(e)$
$= \tilde{Y} \tilde{\cap} (B_{1_A} \tilde{\cap} B_{2_{\breve{G}}})(e)$
$= \tilde{Y} \cap \tilde{\phi}(e)$
$= \tilde{Y} \cap \phi$
$= \phi$
$(B_{1_A})Y \cap (B_{2_{\breve{G}}})Y = \phi$
Hence $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tilde{\tau}_{2\tilde{Y}}, E\}$ is $b(SDW-H)_2$.

Definition23: Let $(X, \tilde{\tau}_{1\hat{X}}, \tilde{\tau}_{2\hat{X}}, E)$ be a soft bi-topological space over $\hat{X}$ & $H \subseteq E$. Then $\{X, \tilde{\tau}_{1H}, \tilde{\tau}_{2H}, H\}$ is called Soft p-subspace of $(X, \tilde{\tau}_{1\hat{X}}, \tilde{\tau}_{2\hat{X}}, E)$ relative to the parameter set H where
$\tilde{\tau}_{1H} = \{(B_{1_A}) / H : H \subseteq A \subseteq E, B_{1_A} \in \tilde{\tau}_{1\hat{X}}$ *such that* B_{1_A} *is* bD soft open $\}$,

$\tilde{\tau}_{2}H = \{(B_{2_B}) / H : H \subseteq B \subseteq E, B_{2_{\breve{B}}} \in \tilde{\tau}_{2\hat{X}}\}$
& $(B_{1_A}) / H, (B_{2_G}) / H$ are the restriction maps on H.

Theorem 4. Soft p-subspace of a $b(SDW-H)_1$ space is b$(SDW-H)_1$.

Proof: Let $(X, \tilde{\tau}_{1\hat{X}}, \tilde{\tau}_{2\hat{X}}, E)$ be a $b(BDW-H)_1$ space. Then it is $b(BDW-H)_1$ with respect to $\tilde{\tau}_{1\hat{X}}$ *or* $pre(SDW-H)_1$ with respect to $\tilde{\tau}_2$. Let $H \subseteq E$. Let $(X, \tilde{\tau}_{1H}, \tilde{\tau}_{2H}, H)$ be a soft p-subspace of $(X, \tilde{\tau}_{1\hat{X}}, \tilde{\tau}_{2\hat{X}}, E)$ relative to the parameter set H. From the above theorem, the soft p-subspace of $b(SDW-H)_1$ space $b(SDW-H)_1$. Therefore, the soft p-subspace of $(BDW-H)_1$is b$(BDW-H)_1$with respect to $\tilde{\tau}_{1H}$ or with respect to $\tilde{\tau}_{2H}$. Hence $(X, \tilde{\tau}_{1H}, \tilde{\tau}_{2H}, H)$ is $b(BDW-H)_1$.

Theorem 5. Soft p-subspace of a $b(BDW-H)_2$ space is b $(BDW-H)_2$.

Proof: Let $(X, \tilde{\tau}_{1X}, \tilde{\tau}_{2X}, E)$ be a $b(BDW-H)_2$ space. Let
$H \subseteq E$. Let $(X, \tilde{\tau}_{1H}, \tilde{\tau}_{2H}, H)$ be a soft p-subspace of $(X, \tilde{\tau}_{1\hat{X}}, \tilde{\tau}_{2\hat{X}}, E)$ relative to the parameter set H where
$\tilde{\tau}_{1H} = \{(B_{1_{\breve{A}}}) / H : H \subseteq \breve{A} \subseteq$
E, bD soft b open $B_{1_{\breve{A}}} \in \tilde{\tau}_{1\hat{X}}\}$,
$\tilde{\tau}_{2H} = \{(B_{1_{\breve{G}}}) / H : H \subseteq \breve{G} \subseteq$
E, bD soft b open $B_{2_{\breve{G}}} \in \tilde{\tau}_{2\hat{X}}\}$.
Consider $h_1, h_2 \in H$, $h_1 \neq h_2$.Then $h_1, h_2 \in E$.
Therefore,
there exists bD soft b open $B_{1_{\breve{A}}} \in \tilde{\tau}_{1\hat{X}}$,
(there exists bD soft b open $B_{2_{\breve{A}}} \in \tilde{\tau}_{2\hat{X}}$ such that
$B_{1_{\breve{A}}}(e_1) = X, B_{2_{\breve{G}}}(e_2) = \hat{X}$ *and* $(B_1, \breve{E}) \cap (B_2, \breve{B}) = \tilde{\phi}$
Therefore
$(B_{1_{\breve{A}}}) /) / H \in \tilde{\tau}_{1H},, (B_{2_G}) / H \in \tilde{\tau}_{2H}$
Also $((B_{1_{\breve{A}}}) / H)(h_1) = B_{1_{\breve{A}}}(h_1) = \hat{X}$
$((B_{2_G}) / H)(h_2) = B_{2_G}(h_2) = \hat{X}$ and
$((B_{1_{\breve{A}}}) / H) \cap ((B_{2_{\breve{G}}}) / H) = (B_{1_{\breve{A}}} \cap B_{2_{\breve{G}}}) / H$
$= \tilde{\phi} / H$
$= \tilde{\phi}$
Hence $b(X, \tilde{\tau}_{1H}, \tilde{\tau}_{2H}, H) is\ b(BDW-H)_2$.

Theorem 6. Product of two pre $(SDW-H)_1$ spaces is $pre(SDW-H)_1$.

Proof: Let $(X, \tilde{\tau}_{1\hat{X}}, \tilde{\tau}_{2\hat{X}}, E)$ and $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tau_{2\tilde{Y}}, K\}$ be two $b(BDW-H)_1$ spaces. Then $(\hat{X}, \tilde{\tau}_{1\hat{X}}, \tilde{\tau}_{2\hat{X}}, E)$ is $b(BDW-H)_1$ with respect to $\tilde{\tau}_{1\hat{X}}$ or $b(BDW-H)_1$ with respect to $\tilde{\tau}_{2\hat{X}}$ and $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tilde{\tau}_{2\tilde{Y}}, K\}$ is $pre(SDW-H)_1$ with respect to $\tilde{\tau}_{1\tilde{Y}}$ or $pre(SDW-H)_1$ with respect to $\tilde{\tau}_{2\tilde{Y}}$. From theorem 18, the product of two $b(BDW-H)_1$ spaces is $b(BDW-H)_1$. Hence the product of two $b(BDW-H)_1$spaces is $b(BDW-H)_1$.

Theorem 7. Product of two $b(BDW-H)_2$ spaces is b$(BDW-H)_2$

Proof: Let $(\hat{X}, \tilde{\tau}_{1\hat{X}}, \tilde{\tau}_{2\hat{X}}, E)$ and $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tau_{2\tilde{Y}}, K\}$ be two $b(BDW-H)_2$ spaces. Consider two distinct points (e_1, k_1), $(e_2, k_2 \in E \times K$ either $e_1 \neq e_2$ or $k_1 \neq k_2$.
Suppose $e_1 \neq e_2$. Since $(X, \tilde{\tau}_{1X}, \tilde{\tau}_{2X}, E)$ is $b(BDW-H)_2$, there exist
there exists bD soft open B_{1_A}
$\in \tilde{\tau}_{1\hat{X}}$, there exists bD soft open $B_{2_G} \in \tilde{\tau}_{2\hat{X}}$
such that $B_{1_{\breve{A}}}(e_1) = \hat{X}, B_{2_G}(e_2) = \hat{X}$
&$B_{\breve{A}} \cap B_{2_{\breve{G}}} = \tilde{\phi}$.
Therefore
$B_{1_A} \otimes B_{2_k} \in \tilde{\tau}_{1X} \otimes \tilde{\tau}_{1Y}, B_{2_{\breve{G}}} \otimes Y_K \in \tilde{\tau}_{2\hat{X}} \otimes \tilde{\tau}_{2Y}$
$(B_{1_{\breve{A}}} \otimes \tilde{Y}_K)(e_1, k_1) = B_{1_{\breve{A}}}(e_1) \times \tilde{Y}_K(k_1) = \hat{X} \times \tilde{Y}$
$(B_{2_{\breve{B}}} \otimes Y_K)(e_2, k_2) = B_{2_{\breve{B}}}(e_2) \times Y_K(k_2)$
$= \hat{X} \times \tilde{Y}$
If for any
$(e, k) \in (E \times K), (B_{1_A} \otimes \tilde{Y}_K)(e, k) \neq \phi \Rightarrow$
$B_{1_A}(e) \times \tilde{Y}_K(k) \neq \phi$

$\Rightarrow B_1(e) \times \tilde{Y} \neq \phi \Rightarrow B_{1_{\breve{A}}}(e) \neq \phi \Rightarrow B_{2_{\breve{G}}}(e) = \phi$

Since

$(B_{1_A} \cap B_{2_{\breve{G}}} = \phi \Rightarrow B_{1_A}(e) \cap B_{2_{\breve{G}}}(e) = \phi)$

$\Rightarrow B_{2_{\breve{G}}}(e) \times \tilde{Y}_K(k) = \phi \Rightarrow (B_{2_G} \otimes \tilde{Y}_K)(e,k) = \phi$

$\Rightarrow (B_{1_{\breve{A}}} \otimes \tilde{Y}_K) \cap (B_{2_{\breve{G}}} \otimes \tilde{Y}_K) = \tilde{\phi}$

Assume $k_1 \neq k_2$. $Since$ $\{Y, \tilde{\tau}_{1Y}, \tilde{\tau}_{2Y}, K\}$ is b$(BDW - H)_2$, there exists bD open sets $(B_1, \breve{E}) \in \tilde{\tau}_{1\hat{Y}}, (b_2, \breve{g}) \in \tilde{\tau}_{2\hat{Y}}$ such that $B_{1_{\breve{A}}}(k_1) =, \tilde{Y}\ B_{2_{\breve{G}}}(k_2) = \tilde{Y}$ & $B_{1_{\breve{A}}} \cap B_{2_{\breve{G}}} = \tilde{\phi}$.

Therefore

$X_E \otimes B_{1_A} \in \tilde{\tau}_{1_X} \otimes \tilde{\tau}_{1_{\hat{Y}}}, \hat{X}_E \otimes B_{2_G} \in \tilde{\tau}_{2_X} \otimes \tilde{\tau}_{2_{\hat{Y}}}$

$(\hat{X}_E \otimes B_{1_{\breve{A}}})(e_1, k_1) = \hat{X}_E(e_1) \times B_{1_{\breve{A}}}(k_1) = \hat{X} \times \tilde{Y}$

$(X_E \otimes B_{2_{\breve{G}}})(e_2, k_2) = \hat{X}_E(e_2) \times B_{2_{\breve{G}}}(k_2) = \hat{X} \times \tilde{Y}$

If for any

$(e,k) \in E \times K, (\hat{X}_E \otimes B_{1_{\breve{A}}})(e,k) \neq \phi \Rightarrow \hat{X}_E(e) \times B_{1_{\breve{A}}}(k) \neq \phi$

$\Rightarrow \hat{X} \times B_{1_{\breve{A}}}(k) \neq \phi \Rightarrow B_{1_{\breve{A}}}(k) \neq \phi \Rightarrow B_{2_G}(k) = \phi$

$(Since B_{1_A} \cap B_{2_G} = \tilde{\phi} \Rightarrow B_{1_{\breve{A}}}(k) \cap B_{2_G}(k) = \phi) \Rightarrow X_E(e) \times B_{2_G}(k) = \phi \Rightarrow (X_E \otimes B_{2_G})(e,k) = \phi$

$\Rightarrow (\hat{X}_E \otimes B_{1_{\breve{A}}}) \cap (X_E \otimes B_{2_G}) = \tilde{\phi}$

Hence,

$b\left(\hat{X} \times Y, \tilde{\tau}_{1\hat{X}} \otimes \tilde{\tau}_{1Y}, \tilde{\tau}_{2\hat{X}} \otimes \tilde{\tau}_{2\hat{Y}}, E \times K\right)$ $is\ b(BDW - H)_2$

Conclusions

In the present article the concept of Soft b W-D_2 structure in soft topological spaces is introduced In soft Single topological spaces and in soft bi topological spaces is introduced and some basic properties regarding this concept are demonstrated. Topology is the most significant branch of mathematics which deals with mathematical structures. Recently, many investigators have deliberated the soft set theory which is originated by polished mathematician and carefully applied to many complications which comprise uncertainties in our social life. Some mathematician familiarized and intensely studied the notion of soft topological spaces. They also deliberate topological structures and demonstrated their several belongings with respect to ordinary points. In the present work, in this paper the concept of Soft b W-D_2 structure in soft Single topological spaces and in soft bi topological spaces and more over some basic properties regarding this concept are demonstrated. This soft structure would be useful for the growth of the theory of soft topology to bury complex problems, comprising doubts in economics, engineering, medical etc. Results in the present paper will help the researchers for reinforcement the toolbox of soft topology.

Conflicts of interest

Authors declare no conflict of interest.

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