## Petersburg Mathematics Cohort

## Day 2

## Sarah R. Powell, Ph.D.

Associate Professor
The University of Texas at Austin


srpowell@utexas.edu


## Introduce yourself.

Describe your role as an educator.
Describe the mathematics you support.

Share your Twitter handle!

## Schedule for Today



$x A+H$

## Trajectories in Mathematics

An important subset of the major work in grades K-8 is the progression that leads toward middle school algebra.

| K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Know number names and the count sequence <br> Count to tell the number of objects <br> Compare numbers <br> Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from <br> Work with numbers 1119 to gain foundations for place value | Represent and solve problems involving addition and subtraction <br> Understand and apply properties of operations and the relationship between addition and subtraction <br> Add and subtract within 20 <br> Work with addition and subtraction equations <br> Extend the counting sequence <br> Understand place value <br> Use place value understanding and properties of operations to add and subtract <br> Measure lengths indirectly and by iterating length units | Represent and solve problems involving addition and subtraction <br> Add and subtract within 20 <br> Understand place value <br> Use place value understanding and properties of operations to add and subtract <br> Measure and estimate lengths in standard units <br> Relate addition and subtraction to length | Represent \& solve problems involving multiplication and division <br> Understand properties of multiplication and the relationship between multiplication and division <br> Multiply \& divide within 100 <br> Solve problems involving the four operations, and identify \& explain patterns in arithmetic <br> Develop understanding of fractions as numbers <br> Solve problems involving measurement and estimation of intervals of time, liquid volumes, \& masses of objects <br> Geometric measurement: understand concepts of area and relate area to multiplication and to addition | Use the four operations with whole numbers to solve problems <br> Generalize place value understanding for multi-digit whole numbers <br> Use place value understanding and properties of operations to perform multidigit arithmetic <br> Extend understanding of fraction equivalence and ordering <br> Build fractions from unit fractions by applying and extending previous understandings of operations <br> Understand decimal notation for fractions, and compare decimal fractions | Understand the place value system <br> Perform operations with multi-digit whole numbers and decimals to hundredths <br> Use equivalent fractions as a strategy to add and subtract fractions <br> Apply and extend previous understandings of multiplication and division to multiply and divide fractions <br> Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition <br> Graph points in the coordinate plane to solve real-world and mathematical problems* | Apply and extend previous understandings of multiplication and division to divide fractions by fractions <br> Apply and extend previous understandings of numbers to the system of rational numbers <br> Understand ratio concepts and use ratio reasoning to solve problems <br> Apply and extend previous <br> understandings of arithmetic to algebraic expressions <br> Reason about and solve one-variable equations and inequalities <br> Represent and analyze quantitative relationships between dependent and independent variables | Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers <br> Analyze proportional relationships and use them to solve real-world and mathematical problems <br> Use properties of operations to generate equivalent expressions <br> Solve real-life and mathematical problems using numerical and algebraic expressions and equations | Work with radical and integer exponents <br> Understand the connections between proportional relationships, lines, and linear equations** <br> Analyze and solve linear equations and pairs of simultaneous linear equations <br> Define, evaluate, and compare functions <br> Use functions to model relationships between quantities |

 listed here are a subset of those designated as major in the assessment consortia's draft documents.
${ }_{* *}$ Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

## $\square$ <br> What's the critical math content for your students?



MA+ $:$


Two-dimensional images


https://bit.ly/srpowell


Manipulatives: Early Numeracy

## Five Counting Principles

## Stable order

One-to-one correspondence

## Cardinality

## Abstraction

## Order irrelevance

## Stable order

Saying the number words in order
"One, two, three, four, five $\cdots$ "


## Stable order

Teacher modeling with echoing
One-minute timings

- Count to 12 as many times as you can

Songs
 repetitive, not based on music, and focused on lyrics.

## One-to-One Correspondence

Ability to match number words to objects


## One-to-One Correspondence

Ability to match number words to objects


## One-to-One Correspondence

## Partitioning and tagging

- Transferred from the "to-be-counted" category to the "already-counted" category
- A distinct numeral word is assigned and not to be used again in the counting sequence


Stable Order AND One-to-One Correspondence

$x A+1 \cdot 1$

## Stable Order AND One-to-One Correspondence

Count to 8
Count to 5


## Cardinality

The number tag used for the last object in a count symbolizes the total number of objects in a set

- Students must coordinate the stable order and one-toone correspondence


## Cardinality

Teacher asking, "How many?"


## Abstraction

Any types of objects can be counted together in a set


## Order Irrelevance

The order in which objects are counted does not matter as long as none of the other counting principles are violated

When teaching counting to "inefficient" counters, however, you should teach a strategy - like partitioning and tagging, working left to right, or using a work mat.

## Five Counting Principles

1. Stable order (rote counting)

## BREAKOUT

2. One-to-one correspondence
3. Cardinality
4. Abstraction
5. Order irrelevance

Combine all five counting principles.

Explicit<br>instruction

Precise language

Multiple
representations

## Counting Instruction

Counting Boards


## Place Value



## Place Value



MA+ $:$

Three Representations of Number
7
seven


## Quantity Comparison

## Compare to 7



Quantity Comparison

favorite virtual manipulatives for early numeracy.
3. Considerations for using these tools with students?

1. Share some of your favorite hands-on materials for early numeracy.
2. Share some of your

BREAKOUT

Manipulatives: Whole Numbers


Two-dimensional images


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## Addition: Part-Part-Whole (Total)

Count one set, count another set, put sets together, count sum


# ■ 



앙

$2+3=5$

## Addition: Part-Part-Whole (Total)



## Addition: Join (Change Increase)

 Start with a set, add the other set, count sum

## 

$2+3=5$

## Addition: Join (Change Increase)



# Subtraction：Separate（Change Decrease） 

 Start with a set，take away from that set，count difference$$
\begin{aligned}
& \text { 昌量畳 }
\end{aligned}
$$

$$
\begin{aligned}
& 5-3=2
\end{aligned}
$$

Subtraction: Separate (Change Decrease)


## Subtraction: Compare (Difference)

Compare two sets, count difference

$\square$
$5-3=2$

## Subtraction: Compare (Difference)



## Addition and Subtraction

Focus on place value


## Addition and Subtraction

Introduce regrouping/trading/exchanging after mastery of problems without regrouping

Addition<br>10 ones for 1 ten<br>10 tens for 1 hundred

1 ten for 10 ones
1 hundred for 10 tens

> Regroup
> Trade
> Exchange

Regroup
Trade
Exchange

## Autumn

Traditional

- Work right to left (start in ones column)


Addition and Subtraction Computation
$227+185=$
$232-164=$
$\times A+H$

## Julie

Traditional Method

- Work right to left (start in ones column)

$$
\text { А. } \begin{array}{r}
5 \\
62 \\
-\quad 17 \\
\hline 45
\end{array}
$$

8. 289
$-\quad 96$
-209


## Multiplication: Equal Groups

Show the groups, show the amount for each group, count product


$$
3 \times 2=6
$$

## Multiplication: Equal Groups


$x A+H$

## Multiplication: Array/Area

Make the array, count product

$3 \times 2=6$

## Multiplication: Array/Area


$1 x A+H$

## Multiplication: Comparison

## Show a set, then multiply the set

## ↔-1

$3 \times 2=6$

## Multiplication: Comparison


|x $\mathrm{A}+\mathrm{H}$

## Division: Equal Groups (Partitive Division)

 Show the dividend, divide equally among divisor, count quotient
$x A+E$

## Division: Partitive Division




## Division: Equal Groups (Measurement

 Shiowisthera) vidend, make groups of the divisor, count groups

$$
6 \div 3=2
$$

## Division: Measurement Division



## Angie

Traditional

- Work right to left (start in ones column)



Pam

Traditional Method

- Work Left to Right



1. Share some of your favorite hands-on materials for whole numbers.
2. Share some of your favorite virtual manipulatives for whole numbers.
3. Considerations for using these tools with students?

Manipulatives:
Fraction Concepts


Two-dimensional images


https://bit.ly/srpowell


## Length/Measurement

Fractions are appropriated by length


## Length/Measurement

Fractions are appropriated by length

## $\frac{2}{3}$



Cuisenaire rods

## Length/Measurement

Fractions are appropriated by length



## Area/Region

Shapes divided into equal sections


## Area/Region

Shapes divided into equal sections

## $\frac{2}{3}$



Geoboards

## Area/Region

Shapes divided into equal sections


Pattern blocks

## Area/Region

Shapes divided into equal sections

## $\frac{2}{3}$



Legos


## Set/Discrete

Individual shapes match the fraction
$\frac{2}{3}$

Two-color counters

## Set/Discrete

Individual shapes match the fraction

$$
\frac{2}{3}
$$





1. Share some of your favorite hands-on materials for fractions.

## BREAKOUT

2. Share some of your favorite virtual manipulatives for fractions.
3. Considerations for using these tools with students?

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