Petersburg Mathematics Cohort

Day 2



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Introduce yourself. Describe your role as an educator. Describe the mathematics you support.





Schedule for Today

9:00-9:05	- Trajectories in mathematics
9:05-9:45	- Manipulatives: Early Numeracy
9:45-10:30	- Manipulatives: Whole Numbers and Place Value
10:30-10:40	BREAK
10:40-11:45	- Manipulatives: Fractions
11:45-12:00	- Wrap-up

1:00-1:05	- Trajectories in mathematics
1:05-1:40	- Manipulatives: Fraction Concepts
1:40-2:20	- Manipulatives: Fraction Computation
2:20-2:30	BREAK
2:30-2:45	- Manipulatives: Decimals
2:45-3:45	- Manipulatives: Algebra
3:45-4:00	- Wrap-up



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GRADES K-5

GRADES 6-12

Trajectories in Mathematics



An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

К	1	2	3	4	5	6	7	8
Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11- 19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent & solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply & divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and	Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multidigit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and division to multiply and division to multiply and divide fractions Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems*	Apply and extend previous understandings of multiplication and division to divide fractions by fractionsApply and extend previous understandings of numbers to the system of rational numbersUnderstand ratio concepts and use ratio reasoning to solve problemsApply and extend previous understandings of arithmetic to algebraic expressionsReason about and solve one-variable equalitiesRepresent and analyze quantitative relationships between dependent and independent variables	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationships and use them to solve real-world and mathematical problems coperations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations** Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities
			to addition					

* Indicates a cluster that is well thought of as a part of a student's progress to algebra, but that is currently not designated as major by the assessment consortia in their draft materials. Apart from the one asterisked exception, the clusters listed here are a subset of those designated as major in the assessment consortia's draft documents.

** Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

https://achievethecore.org/category/774/mathematics-focus-by-grade-level





What's the critical math content for your students?





Three-dimensional objects













Two-dimensional images





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Manipulatives: Early Numeracy



Five Counting Principles

Stable order

One-to-one correspondence

Cardinality

Abstraction

Order irrelevance



Stable order

Saying the number words in order "One, two, three, four, five…"





Stable order

Teacher modeling with echoing One-minute timings

• Count to 12 as many times as you can

Songs

- <u>www.youtube.com/watch?v=g9EgE_JtEAw</u>
- <u>www.youtube.com/watch?v=F5QLp9Wxrrg</u>
- <u>www.youtube.com/watch?v=dk9Yt1PqQiw</u>
- <u>www.youtube.com/watch?v=uxPfPyYp84E</u>
- <u>www.youtube.com/watch?v=tocO1HKG2Ug</u>
- www.youtube.com/watch?v=xx5GWCgklhw
- <u>www.youtube.com/watch?v=fZ9WiuJPnNA</u>

Good songs are repetitive, not based on music, and focused on lyrics.



One-to-One Correspondence

Ability to match number words to objects





One-to-One Correspondence

Ability to match number words to objects











One-to-One Correspondence

Partitioning and tagging

- Transferred from the "to-be-counted" category to the "already-counted" category
- A distinct numeral word is assigned and not to be used again in the counting sequence





Stable Order AND One-to-One Correspondence









Stable Order AND One-to-One Correspondence





Cardinality

The number tag used for the last object in a count symbolizes the total number of objects in a set

• Students **must coordinate** the **stable order** and **one-toone correspondence**



Cardinality

Teacher asking, "How many?"





Abstraction

Any types of objects can be counted together in a set





Order Irrelevance

The order in which objects are counted does not matter as long as none of the other counting principles are violated

> When teaching counting to "inefficient" counters, however, you should teach a strategy – like partitioning and tagging, working left to right, or using a work mat.



Five Counting Principles

- 1. Stable order (rote counting)
- 2. One-to-one correspondence
- 3. Cardinality
- 4. Abstraction
- 5. Order irrelevance



Combine all five counting principles.





Counting Instruction



XA+H

Place Value









Place Value









Three Representations of Number





Quantity Comparison





Quantity Comparison











1. Share some of your favorite hands-on materials for early numeracy. 2. Share some of your favorite virtual manipulatives for early numeracy. 3. Considerations for using these tools with students?



Manipulatives: Whole Numbers





Two-dimensional images





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Addition: Part-Part-Whole (Total)

Count one set, count another set, put sets together, count sum





Addition: Part-Part-Whole (Total)









Addition: Join (Change Increase) Start with a set, add the other set, count sum







Addition: Join (Change Increase)








Subtraction: Separate (Change Decrease) Start with a set, take away from that set, count difference





5 - 3 = 2



Subtraction: Separate (Change Decrease)









Subtraction: Compare (Difference) Compare two sets, count difference





5 - 3 = 2



Subtraction: Compare (Difference)









Addition and Subtraction

Focus on place value







Addition and Subtraction

Introduce regrouping/trading/exchanging after mastery of problems without regrouping

10 ones for 1 ten 10 tens for 1 hundred Regroup Trade Exchange

1 ten for 10 ones 1 hundred for 10 tens Regroup Trade Exchange



Autumn

Traditional

• Work right to left (start in ones column)







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Julie

Traditional Method

• Work right to left (start in ones column)

в. Α.





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Multiplication: Equal Groups

Show the groups, show the amount for each group, count product





Multiplication: Equal Groups









Multiplication: Array/Area Make the array, count product



$3 \times 2 = 6$



Multiplication: Array/Area









Multiplication: Comparison Show a set, then multiply the set



$3 \times 2 = 6$



Multiplication: Comparison







Division: Equal Groups (Partitive Division) Show the dividend, divide equally among divisor, count quotient





Division: Partitive Division









Division: Equal Groups (Measurement Showsherd)vidend, make groups of the divisor, count groups







 $6 \div 3 = 2$



Division: Measurement Division







Angie

Traditional

• Work right to left (start in ones column)











Traditional Method

• Work Left to Right







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1. Share some of your favorite hands-on materials for whole numbers. 2. Share some of your favorite virtual manipulatives for whole numbers. 3. Considerations for using these tools with

students?



Manipulatives: Fraction Concepts





Two-dimensional images





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Length/Measurement

Fractions are appropriated by length



Fraction tiles/ba



Length/Measurement

Fractions are appropriated by length

2 3



Cuisenaire rods



Length/Measurement

Fractions are appropriated by length



Number line





Fraction Concepts				
Fraction	Length	Area	Set	
<u>2</u> 3				
<u>1</u> 4				
$1\frac{1}{2}$				
<u>3</u> 7				















Geoboards





2 3



Pattern blocks









Legos





Fraction Concepts				
Fraction	Length	Area	Set	
<u>2</u> 3				
<u>1</u> 4				
$1\frac{1}{2}$				
<u>3</u> 7				




Individual shapes match the fraction









Individual shapes match the fraction









Fraction Concepts			
Fraction	Length	Area	Set
<u>2</u> 3			
<u>1</u> 4			
$1\frac{1}{2}$			
<u>3</u> 7			











 Share some of your favorite hands-on materials for fractions.
Share some of your favorite virtual manipulatives for fractions.

3. Considerations for using these tools with students?



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