Impulse Noise Mitigation for OFDM Using Decision Directed Noise Estimation

Jean Armstrong and Himal A. Suraweera
Department of Electronic Engineering, La Trobe University,
Melbourne, Victoria 3086, Australia.
j.armstrong@latrobe.edu.au

Abstract-- **This paper describes a new technique for mitigating the effects of impulse noise in OFDM. An estimate is made of the noise component in each received input sample. The estimates are based on the transmitted data. No pilot tones are required. When the estimate is large enough to indicate that impulse noise is present in the sample, the estimated noise component is subtracted from the input sample before final demodulation. Estimates of the noise are obtained from preliminary decisions based on the noisy signal. The technique is effective because the energy from each noise impulse is spread across the received spectrum. The technique can also be applied to multicarrier CDMA. Simulations show that in cases of practical importance the symbol error rate can be reduced by several orders of magnitude.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) technology is used in many digital broadband communication systems. One of the advantages of OFDM compared to single carrier systems is that it is more resistant to the effects of impulse noise. However impulse noise can still cause significant problems in OFDM systems.

The effects of impulse noise in multicarrier systems have previously been analyzed [1], and a number of techniques for mitigating the effect of impulse noise have been described. One approach is to identify peaks in the received time domain signal and reduce these by either clipping or nulling the sample [2-4]. This is effective only for impulse noise which has peaks which are larger than the wanted OFDM signal. This will be true only in very extreme cases. In high signal to noise environments such as broadcast television, the impulse noise can be well above the background Gaussian noise, yet well below the OFDM signal.

Several authors have used techniques that operate on the signal in the frequency domain [5-7]. Häring and Han Vinck [5] describe an iterative process in which information is exchanged between estimators operating in the time and frequency domains. The simulation results they present are for extreme cases with very large noise impulses. In [6], impulses are detected in the frequency domain by identifying subcarriers with extreme values. In [7] the positions of noise impulses are identified using pilot tones. This allows the corrupted samples to be nulled but no estimate of the actual value of the noise is made.

In this paper we use a new technique for estimating and mitigating impulse noise in OFDM. Preliminary decisions are made about the transmitted data. Based on these an estimate of the noise is made. This is subtracted from the input signal before final demodulation. When the input noise is impulsive, the technique substantially reduces the noise power. The technique depends on the fact that the signal appears random in the time domain and highly structured in the discrete frequency domain whereas for the impulse noise the converse is true.

The technique is analyzed by considering the decision process as a non-linearity operating on the noise and applying Bussgang's theorem. The technique is shown to significantly reduce the symbol error rate (SER), is applicable to systems that have already been standardized, and is well suited to digital signal processing (DSP) implementation.

II. New Impulse mitigation technique

Fig. 1 shows the block diagram of a receiver with the new mitigation technique. The received OFDM baseband signal samples are given by

$$x(l) = r(l) + n_g(l) + n_i(l) = r(l) + n_t(l) \qquad (1)$$

where $r(l)$ is the wanted OFDM signal, $n_g(l)$ is the Gaussian noise and $n_i(l)$ is the impulse noise. $n_t(l) = n_g(l) + n_i(l)$ is the total noise at the input. The samples $x(l)$ are optionally passed through a non-linearity that clips or nulls large samples. The samples at the output of the non-linearity, $z(l)$, are serial-to-parallel converted to form the vector of *N* complex samples that are input to the *N*-point discrete Fourier transform (DFT). The output of the DFT is the *N*-point vector $\mathbf{Z}$. Preliminary decisions, $\hat{D}_p(k)$, about the transmitted data are made based on $Z(k)$. The noise component of $Z(k)$ is $N_t(k)$. The observed noise, $N_p(k)$, is calculated using

$$N_p(k) = Z(k) - \hat{D}_p(k) \qquad (2)$$

Except for extreme cases, most of the received subcarriers are correctly decoded and the observed noise is exactly equal to the received noise in that subcarrier. In the cases where the subcarrier is incorrectly decoded, 'decision noise' will be added to the observed value.

This work was supported by an Australian Research Council (ARC) Large Grant.

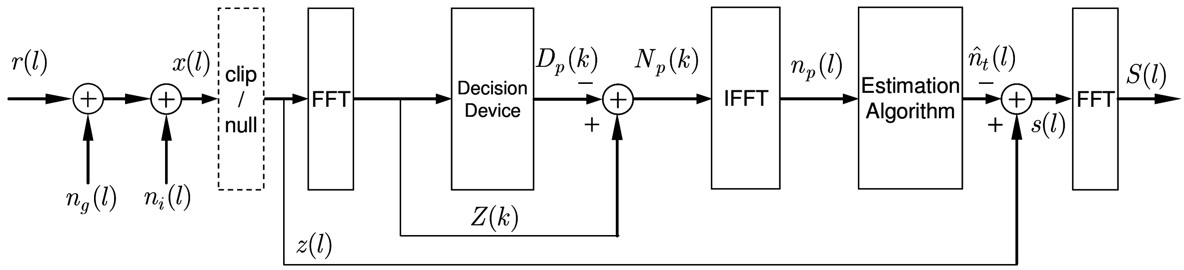


Fig. 1. Block diagram of receiver with impulse mitigation.

OFDM is more resistant to the effects of impulse noise than single carrier systems because of the spreading effect of the receiver DFT operation. The energy of each impulse is spread evenly across all of the subcarriers in that symbol. When there is more than one impulse in a received symbol period T, the contributions combine linearly in each subcarrier. When there are enough impulses during T for the central limit theorem to apply, $N_t(k)$ has a Gaussian distribution.

The vector $\mathbf{N}_p$ is then converted back into the discrete time domain using an inverse FFT to give the vector $\mathbf{n}_p$. If there are no decision errors, $n_p(l) = n_t(l)$. However even in the presence of decision errors $n_p(l)$ contains some information about $n_t(l)$. $n_p(l)$ is then input to an estimation device to generate an estimate $\hat{n}_t(l)$ of the total input noise. This is subtracted from $z(l)$ to generate $s(l)$. The rest of the receiver is a standard OFDM receiver consisting of DFT etc.

For the technique to be effective in reducing the overall bit error rate (BER) of the system, $n_t(l)$ must be impulsive (not stationary Gaussian) and the estimation algorithm must be non-linear. If the estimation process is linear, it will appear to improve the received constellation as each point moves towards the value $\hat{D}_p(k)$, but the points will move closer to both incorrect and correct decision points. However, the technique is very effective if non-linear processing is used and the noise is impulsive. This depends on the fact that for large values of $n_t(l)$, $n_p(l) \approx n_t(l)$.

Fig. 1 shows an optional clipping or nulling function operating on the received baseband samples. This reduces the effect of very large noise impulses that are above the envelope of the OFDM signal. However the simulations show that this improves the performance only in very extreme cases.

The analysis and simulations in this paper are for OFDM in a flat fading channel. However the technique can also be applied to OFDM in frequency selective fading channels and to multicarrier code division multiple access (MC-CDMA) systems. For a frequency selective channel a single tap equalizer must be placed before the decision device and the inverse of this equalizer between the decision device and the adder which outputs $N_p(k)$. For MC-CDMA the signal must be despread before the decision device and respread before the adder. The key factor is that noise which is impulsive in the time domain must be spread out in the 'decision' domain.

III. Impulse Noise Models

A number of models for impulse noise have been presented in the literature [1], [8], [9]. Some characterize only the probability density function of the amplitude of the noise, whereas others also consider the time correlation of impulse events. Very recent research by the BBC, which measured a variety of impulse noise sources, has shown that many of the impulse noise sources of practical importance in OFDM applications can be modeled as gated Gaussian noise [10,11].

In this paper we use a particular form of gated Gaussian noise, where the noise is the sum of additive white Gaussian noise (AWGN) of variance σ_n^2 and a second higher variance Gaussian noise component which lasts for a fraction, μ of the time duration of each OFDM symbol and which has variance σ_i^2 during this time. (i.e. the variance is calculated over only μT not over T). In general $\sigma_i^2 >> \sigma_n^2$. The total noise power is then $\sigma^2 = \mu\sigma_i^2 + \sigma_n^2$. Each of these variances is for the real and imaginary components taken separately. The impulsive samples are spread randomly throughout each OFDM symbol.

The gated Gaussian model is used because it gives a good indication of the performance of OFDM systems. Here the critical factor is whether the BER for each symbol is above the threshold at which the error correcting coding will reduce the final BER to an acceptable level, rather than the BER averaged over the entire received signal. It also allows the length and power of the impulse noise to be varied in a way that makes clear the practical implications of the technique. For example, in the context of digital video broadcasting (DVB), it indicates how the resistance to impulse noise can be improved by increasing the transmitter power or choosing the 8k rather than 2k mode.

IV. Analysis

In the following we will analyze the operation of the decision device and the noise estimation algorithm. The optional input non-linearity will not be considered. First consider the decision process. To derive an expression for the observed noise $N_p(k)$ we must consider the operation of the decision device on the noise. This is different from the usual analysis of demodulation where the focus is on the data. The decision process can be considered as a non-linear process with input $N_t(k)$ and output $N_p(k)$. The non-linearity operates on the real and imaginary components independently. Fig. 2 shows a 16QAM constellation and the non-linear process operating on $\Re\{N_t(k)\}$, the real component of $N_t(k)$. The non-linear process is different for inner and outer constellation points. For an inner point

$$\Re/\Im\{N_p\} = \begin{cases} \Re/\Im\{N_t\} & -d \le \Re/\Im\{N_t\} \le d \\ \Re/\Im\{N_t - 2d\} & d < \Re/\Im\{N_t\} \le \infty \\ \Re/\Im\{N_t + 2d\} & -3d < \Re/\Im\{N_t\} \le -d \\ \Re/\Im\{N_t + 4d\} & -\infty < \Re/\Im\{N_t\} \le -3d \end{cases} \tag{3}$$

If $N_t(k)$ has a Gaussian distribution, The process can be analyzed using Bussgang's theorem, which states that a zero mean stationary Gaussian process distorted by a memoryless non-linearity can be expressed as the sum of an attenuated input replica and an uncorrelated distortion component [12]. Thus

$$\Re\{N_p\} = \lambda \Re\{N_t\} + \Re\{N_d\} \tag{4}$$

where λ is a constant which depends on the non-linearity and $\Re\{N_d\}$ is the real part of the uncorrelated 'distortion' component.

Expressions for λ in terms of the d/σ are derived in Appendix A. The derivation is applicable to any impulse noise model, to OFDM and MC-CDMA and to both flat and frequency selective fading channels, as long as the central limit theorem applies so that the noise at the decision device has a Gaussian distribution. Fig. 3 shows the results for an inner and an outer constellation point and for the value of λ measured in computer simulations for 4QAM, 16QAM and 64QAM. 4QAM has only outer constellation points and the measured value of λ is very close to the theoretical result. 16QAM and 64QAM, which have both inner and outer constellation points, result in λ between the outer and inner point calculations. Using the same approach an expression can be calculated for $E[\Re\{N_d\}^2]$.

Thus the frequency domain noise observations are described by

$$N_p = \lambda N_t + N_d \tag{5}$$

N_p is then input to the IFFT. Assuming that second order effects, such as correlation between the decision noise components are negligible then the noise observation after the IFFT can be expressed as

$$n_p(l) = \lambda n_t(l) + n_d(l) \tag{6}$$

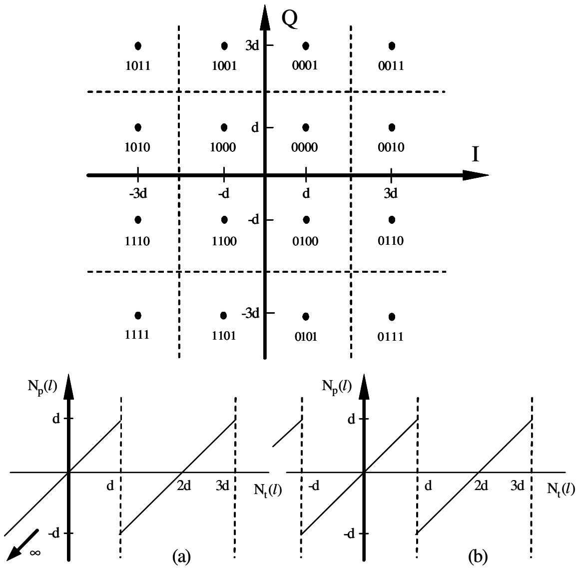


Fig. 2. 16QAM constellation and non linear effect of decision process on noise.

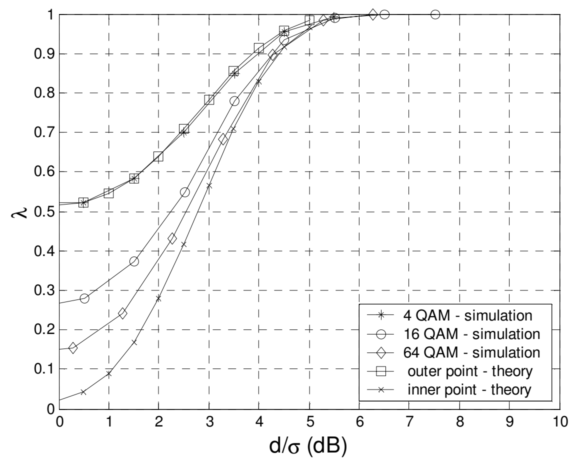


Fig. 3. λ versus d/σ.

The task of the estimation algorithm is to estimate the presence and size of noise impulses. A number of algorithms are possible. In this paper a threshold operating on the real and imaginary components separately was used for most of the simulations. Noise components above a certain threshold are subtracted out and those below a certain threshold are ignored. Represent $\Re(n_p(l))$ as r_p, $\Re(n_t(l))$ as r_t and $\Re(n_d(l))$ as r_d. Thus the estimate of r_t is given by

$$\begin{aligned} \hat{r}_t &= a r_p \quad \text{for } |r_p| > \alpha \\ &= 0 \quad \text{for } |r_p| < \alpha \end{aligned} \tag{7}$$

The error in the estimation process is given by

$$r_t - \hat{r}_t = (1-a)\, r_p - a r_d \quad \text{for } |r_p| > \alpha$$
$$= r_t \quad \text{for } |r_p| < \alpha \qquad (8)$$

Now assuming that minimum mean square error (MMSE) estimation is to be used then substituting for the value of r_p, taking the expectation of the squared value and noting that r_p and r_d are uncorrelated gives

$$E\left[(r_t - \hat{r}_t)^2\right] = \left((1-a\lambda)^2 E\left[(r_t)^2 \,\middle|\, |r_p| > \alpha\right] + a^2 E\left[(r_d)^2\right]\right) \Pr\left(|r_p| > \alpha\right) + E\left[(r_t)^2 \,\middle|\, |r_p| \le \alpha\right] \Pr\left(|r_p| \le \alpha\right) \qquad (9)$$

For the case of gated Gaussian noise, this can be considered as the sum of the expectations in the impulsive and non-impulsive states.

$$E\left[(r_t - \hat{r}_t)^2\right] = \mu E\left[(r_t - \hat{r}_t)^2 \,|\, \text{impulse}\right] + (1-\mu) E\left[(r_t - \hat{r}_t)^2 \,|\, \text{Non impulse}\right] \qquad (10)$$

For the impulsive state

$$E\left[(r_t - \hat{r}_t)^2 \,|\, \text{impulse}\right] = \begin{pmatrix} (1-a\lambda)^2 E\left[(r_t)^2 \,\middle|\, |r_p| > \alpha, \text{impulse}\right] \\ + a^2 E\left[(r_d)^2\right] \end{pmatrix} \Pr\left(|r_p| > \alpha \,|\, \text{impulse}\right) + E\left[(r_t)^2 \,\middle|\, |r_p| \le \alpha, \text{impulse}\right] \Pr\left(|r_p| \le \alpha \,|\, \text{impulse}\right) \qquad (11)$$

If there are enough decision errors for the central limit theorem to apply, r_d has a Gaussian distribution. In this case all of the conditional random variables have Gaussian distributions and all of the terms in (11) can be calculated.

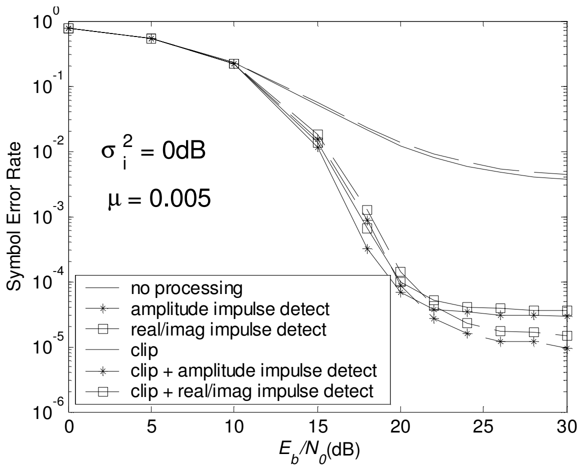
V. Simulation Results

Matlab simulations were used to examine the performance of the new technique using the gated Gaussian noise model. For each simulation, the average power in each of the real and imaginary components of the wanted OFDM signal is unity. Figs. 4-5 show the resulting SER as a function of E_b/N_0 where N_0 is the single sided spectral density of the white Gaussian (non impulse component). In other words, for each plot, the impulse noise is kept constant and the effect of varying E_b/N_0 is measured. 64QAM modulation and 1024 subcarriers were used.

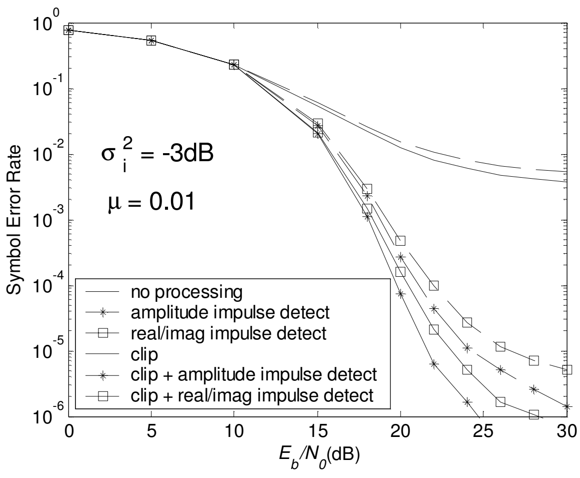
A number of possible variations on the basic concept are possible depending on whether the optional input non-linearity is used, and on whether the input non-linearity and the non-linear noise estimation algorithms operate on the amplitude of the complex signals or on the real and imaginary components separately.

Figs. 4(a) and (b) show the performance of a number of different combinations and compare them with the results for an OFDM receiver with no impulse mitigation and with a system using only an input non-linearity. Amplitude clipping with a threshold A_{clip}, set to 3.2 was used as the input non-linearity. A threshold of 0.7 was used for real and imaginary impulse detection and a threshold of 0.8 for amplitude detection. All of the thresholds are standardized in terms of the standard deviation of the wanted OFDM signal. The weighting factor was set at $a = 1$.

Fig. 4(a) shows the results for the case where $\sigma_i^2 = 0\,\text{dB}$ and $\mu = 0.005$ and Fig. 4(b) for $\sigma_i^2 = -3\,\text{dB}$ and $\mu = 0.01$. The energy of the impulse noise was the same, but in (a) results are for higher levels of impulse noise for a shorter fraction of the symbol. Noise mitigation is clearly very effective. It reduces the SER by about two orders of magnitude for practical values of E_b/N_0. For the shorter, higher power, bursts of impulse noise, clipping at the input improves the performance, but not for the case where the impulse noise power is 3 dB below the signal power.



(a) $\sigma_i^2 = 0$ dB and $\mu = 0.005$.



(b) $\sigma_i^2 = -3$ dB and $\mu = 0.01$.

Fig. 4. SER versus E_b/N_0 for varying forms of the noise mitigation technique.

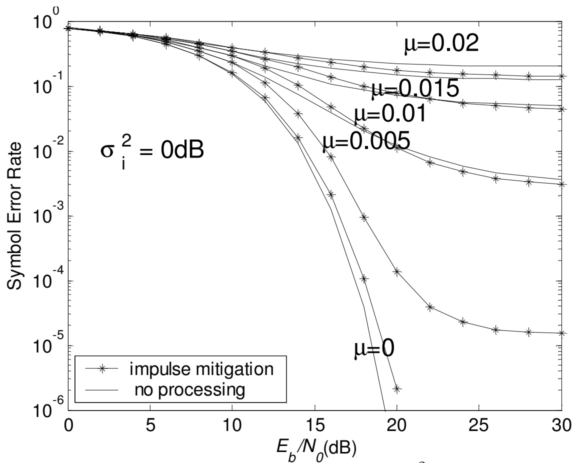


Fig. 5. SER versus E_b/N_0 for varying . μ , $\sigma_i^2 = 0$ dB.

Fig. 5 shows the effect of varying μ. The impulse mitigation scheme is most effective for intermediate values of impulse power at which the SER without mitigation is around 0.01. For higher error rates there is too much 'decision noise', for lower error rates the power of the impulses is not enough above the background.

Further simulations show that the technique is also effective for a wide range of other impulse noise parameters and for 4QAM and 16QAM. It is less effective for smaller value of N because the central limit theorem does not hold.

VI. Conclusions

A new decision directed noise estimation technique for mitigating the effects of impulse noise in OFDM has been presented. Noise impulses are estimated based on preliminary decisions on the data. Simulation results demonstrate that the technique can reduce the symbol error rate by two or three orders of magnitude for cases of practical importance. The technique has been analyzed by considering the decision process as a non-linear process operating on the noise and applying Bussgang's theorem.

Appendix A
Effect of the Decision Process on Noise

Let $u = \Re\{N_p\}$, $v = \Re\{N_t\}$ and $w = \Re\{N_d\}$ then,

$$\lambda = \frac{E[v.u)]}{E[v^2]} = \frac{E[v.u]}{\sigma^2} \tag{A.1}$$

where σ^2 is the total noise power for the real component. For gated Gaussian model, $\sigma^2 = \sigma_n^2 + \mu\sigma_i^2$.

Using the fact that v is normally distributed with variance σ^2 we can obtain an expression for λ_{inner}.

$$\lambda_{\text{inner}} = \frac{1}{\sqrt{2\pi\sigma^6}}\left(\begin{array}{l}\int\limits_{-d}^{d} v^2 \exp\left(\frac{-v^2}{2\sigma^2}\right)dv + \int\limits_{d}^{\infty} v(v-2d)\exp\left(\frac{-v^2}{2\sigma^2}\right)dv \\ + \int\limits_{-3d}^{-d} v(v+2d)\exp\left(\frac{-v^2}{2\sigma^2}\right)dv + \int\limits_{-\infty}^{-3d} v(v+4d)\exp\left(\frac{-v^2}{2\sigma^2}\right)dv\end{array}\right)$$

After simplifying these integrals and cancelling terms it can be shown that

$$\lambda_{\text{inner}} = \frac{1}{\sqrt{2\pi\sigma^2}}\int\limits_{-\infty}^{\infty}\exp\left(\frac{-v^2}{2\sigma^2}\right)dv - \frac{2d}{\sqrt{2\pi\sigma^2}}\left(\exp\left(\frac{-9d^2}{2\sigma^2}\right) + 2\exp\left(\frac{-d^2}{2\sigma^2}\right)\right) \tag{A.3}$$

Normalizing by σ so that the dependence on the ratio $k = d/\sigma$ is clear gives

$$\lambda_{\text{inner}} = \frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty}\exp\left(\frac{-v^2}{2}\right)dv - \sqrt{\frac{2k^2}{\pi}}\left(\exp\left(\frac{-9k^2}{2}\right) + 2\exp\left(\frac{-k^2}{2}\right)\right) \tag{A.4}$$

Similarly for an outer point

$$\lambda_{\text{outer}} = \frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty}\exp\left(\frac{-v^2}{2}\right)dv - \sqrt{\frac{2k^2}{\pi}}\left(\exp\left(\frac{-k^2}{2}\right)\right) \tag{A.5}$$

References

[1] M. Ghosh, "Analysis of the effect of impulse noise on multicarrier and single carrier QAM systems," *IEEE Trans. Commun.,* vol. 44, pp. 145-147, Feb. 1996.

[2] T. N. Zogakis, P. S. Chow, J. T. Aslanis and J. M. Cioffi, "Impulse noise mitigation strategies for multicarrier modulation," in *Proc. IEEE ICC93*, Geneva, Switzerland, May 1993, pp. 784 – 788.

[3] J. H. Stott, "Detection and removal of clipping in multi-carrier receivers," European Patent Office, EP 1043874, Oct. 2000.

[4] H. A. Suraweera, C. Chai, J. Shentu and J. Armstrong, "Analysis of impulse noise mitigation techniques for digital television systems," in *Proc. 8th International OFDM Workshop (InOWo'03)*, Hamburg, Germany, Sept. 2003. pp. 172-176.

[5] J. Häring and A. J. Han Vinck, "Iterative decoding of codes over complex number for impulsive noise channels," *IEEE Trans. Inform. Theory,* vol. 49, pp. 1251-1260, May 2003.

[6] M. Sliskovic, "Signal processing algorithm for OFDM channel with impulse noise," *in Proc. 7th IEEE Conference on Electronics, Circuits and Systems (ICECS 2000)*, Lebanon, Dec. 2000, pp 222-225.

[7] F. Abdelkefi, P. Duhamel and F. Alberge, "On the use of pilot tones for impulse noise cancellation in Hiperlan2," in Proc. *ISSPA,* Kuala Lumpur, Malaysia, Aug. 2001, pp. 591-594.

[8] S. Y. Tan, Y. H. Chew, T. T. Tjhung and M. W. Wu, "A two-dimensional impulse noise model and error probability of QAM in DSL," *IEEE Trans. Commun.*, vol. 51, pp. 129-134, Feb. 2003.

[9] A. D. Spaulding and D. Middleton, "Optimum reception in an impulsive interference environment-Part I: Coherent detection," *IEEE Trans. Commun.*, vol. COM-25, pp. 910-923, Sept. 1977.

[10] J. Salter and J. Lago-Fernández, "DTT comparison of 64-QAM with 16QAM – Co-channel interference from PAL, echoes and impulsive interference," EBU Technical Review, Oct. 2003.

[11] BBC Research and Development Section, private communication.

[12] D. Dardari, V. Tralli and A. Vaccari, "A theoretical characterization of nonlinear distortion effects in OFDM systems," *IEEE Trans. Commun.*, vol. 48, pp. 1755-1764, Oct. 2000.