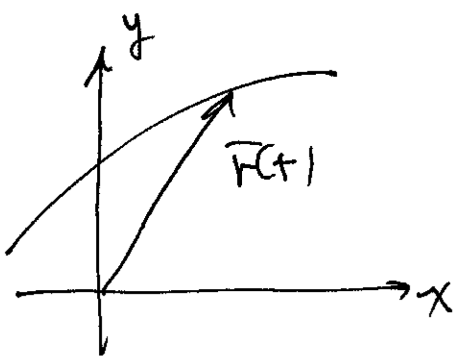
Math 2471 - Calc 3

Vector Function's



$\bar{r}(t)$ - position vector

$\bar{T} = \dfrac{\bar{r}'}{\|\bar{r}'\|}$ tangent vector

$\bar{N} = \dfrac{\bar{T}'}{\|\bar{T}'\|}$ Normal vector

Last class

$\bar{v} = \dfrac{d\bar{r}}{dt}$ velocity vector

$\bar{a} = \dfrac{d\bar{v}}{dt}$ acceleration vector

Ex $\bar{r} = \langle t, \frac{1}{2}t^2 \rangle$

$\bar{r}'(t) = \langle 1, t \rangle \quad \|\bar{r}'\| = \sqrt{1+t^2}$

so $\bar{T} = \left\langle \dfrac{1}{\sqrt{1+t^2}}, \dfrac{t}{\sqrt{1+t^2}} \right\rangle$

next $\bar{T}' = \left\langle \dfrac{-t}{(1+t^2)^{3/2}}, \dfrac{1}{(1+t^2)^{3/2}} \right\rangle$

so $\|\bar{T}'\| = \sqrt{\dfrac{t^2}{(1+t^2)^{3}} + \dfrac{1}{(1+t^2)^3}} = \dfrac{1}{1+t^2}$

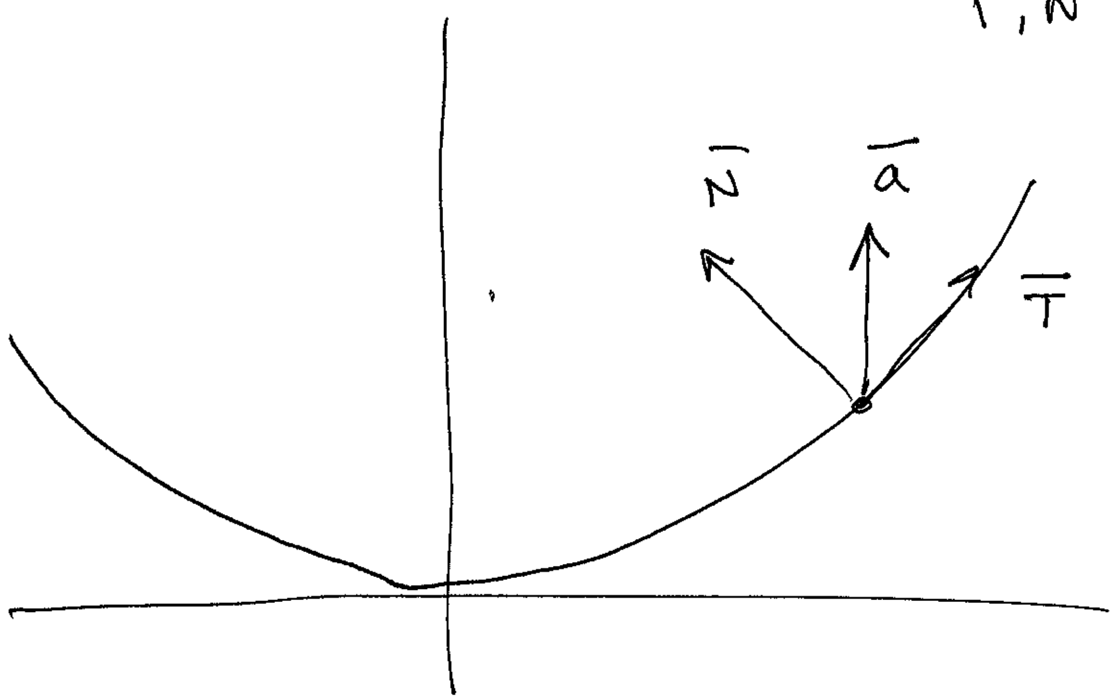
& $\bar{N} = \left\langle \dfrac{-t}{\sqrt{1+t^2}}, \dfrac{1}{\sqrt{1+t^2}} \right\rangle$

Also, if $\bar{r} = \langle t, \tfrac{1}{2}t^2 \rangle$

$\bar{v} = \bar{r}' = \langle 1, t \rangle$

& $\bar{a} = \bar{v}' = \langle 0, 1 \rangle$

So there is a connection between $\bar{T}, \bar{N}$ & $\bar{a}$



Since $\bar{T} = \dfrac{\bar{r}'}{\|\bar{r}'\|} \Rightarrow \bar{r}' = \|\bar{r}'\|\,\bar{T}$

and $\bar{v} = \bar{r}'$ so $\bar{v} = \|\bar{r}'\|\,\bar{T}$

$\bar{a} = \bar{v}' = \|\bar{r}'\|'\,\bar{T} + \|\bar{r}'\|\,\bar{T}'$

Now $\bar{N} = \dfrac{\bar{T}'}{\|\bar{T}'\|} \Rightarrow \bar{T}' = \|\bar{T}'\|\,\bar{N}$

$\bar{a} = \|\bar{r}'\|'\,\bar{T} + \|\bar{r}'\|\,\|\bar{T}'\|\,\bar{N}$ ← the connection

Ex 1 con't

$\|\bar{r}'\| = \sqrt{1+t^2}$ so $\|\bar{r}'\|' = \frac{1}{2}(1+t^2)^{-1/2}\,2t = \dfrac{t}{\sqrt{1+t^2}}$

$\bar{T}' = \left\langle \dfrac{-t}{(1+t^2)^{3/2}}, \dfrac{1}{(1+t^2)^{3/2}} \right\rangle$

$\|\bar{T}'\| = \dfrac{1}{1+t^2}$ so $\|\bar{T}'\|' = \dfrac{-2t}{(1+t^2)^2}$

Now combine

$$\bar{a} = \frac{t}{\sqrt{1+t^2}} \left\langle \frac{1}{\sqrt{1+t^2}}, \frac{t}{\sqrt{1+t^2}} \right\rangle + \sqrt{1+t^2} \cdot \frac{1}{1+t^2} \left\langle \frac{-t}{\sqrt{1+t^2}}, \frac{1}{\sqrt{1+t^2}} \right\rangle$$

$$= \left\langle \frac{t}{1+t^2}, \frac{t^2}{1+t^2} \right\rangle + \left\langle \frac{-t}{1+t^2}, \frac{1}{1+t^2} \right\rangle$$

$$= \left\langle 0, \frac{t^2+1}{t^2+1} \right\rangle = \langle 0, 1 \rangle$$

ex 2 $\quad \bar{r} = \langle \cos t, \sin t, t \rangle$

$$\bar{r}' = \langle -\sin t, \cos t, 1 \rangle$$

$$\| \bar{r}' \| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}$$

$$\bar{T} = \left\langle \frac{-\sin t}{\sqrt{2}}, \frac{\cos t}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$\bar{T}' = \left\langle -\frac{\cos t}{\sqrt{2}}, \frac{-\sin t}{\sqrt{2}}, 0 \right\rangle$$

$$\| \bar{T}' \| = \sqrt{\frac{\cos^2 t + \sin^2 t}{2}} = \frac{1}{\sqrt{2}}$$

$\bar{N} = \frac{\bar{T}'}{\|\bar{T}'\|} = \langle -\cos t, -\sin t, 0 \rangle$

Now $\bar{v} = \bar{r}' = \langle -\sin t, \cos t, 1 \rangle$

$\bar{a} = \bar{v}' = \langle -\cos t, -\sin t, 0 \rangle$

$\bar{a} = \|\bar{r}'\|' \bar{T} + \|\bar{r}'\| \|\bar{T}'\| \bar{N}$

$= 0 \bar{T} + \sqrt{2} \cdot \frac{1}{\sqrt{2}} \langle -\cos t, -\sin t, 0 \rangle$

$= \langle -\cos t, -\sin t, 0 \rangle = a$ ✓

HW verify $\bar{a} = a_T \bar{T} + a_N \bar{N}$

when $a_T = \|\bar{r}'\|'$ $a_N = \|\bar{r}'\| \|\bar{T}'\|$

for

$$\left.\begin{array}{l} \text{(i)} \quad \bar{r} = \langle t, 2t, -3t \rangle \\ \text{(ii)} \quad \bar{r} = \langle e^t, 2t, e^{-t} \rangle \\ \text{Note} \quad (e^t)^2 + 2^2 + (e^{-t})^2 = (e^t + e^{-t})^2 \end{array}\right\}$$

I will change these.