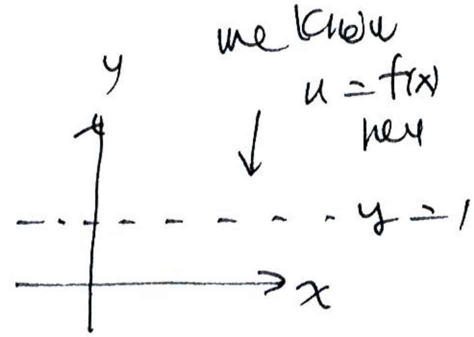
Math 4315 – PDE's

Mapping Boundaries

Consider



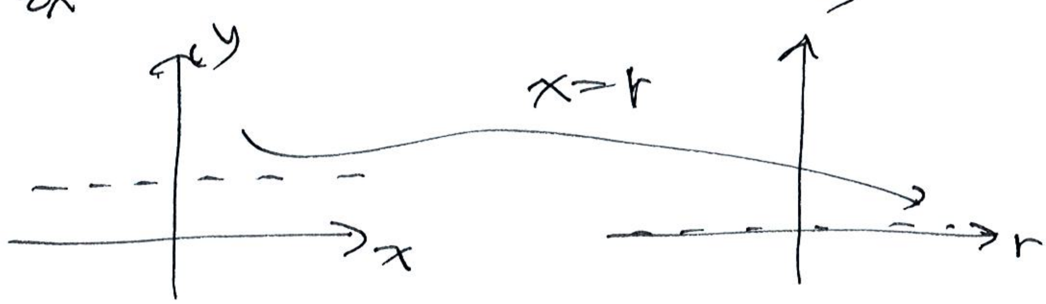
$$u_x + u_y = 0 \qquad u(x,1) = f(x)$$

if $x_s = 1,\ y_s = 1$ then $u_s = 0$

we choose a new boundary in the (r,s) plane.

I'm saying I have a lot of freedom here

For Ex



$$y = 1 \Rightarrow s = 0$$

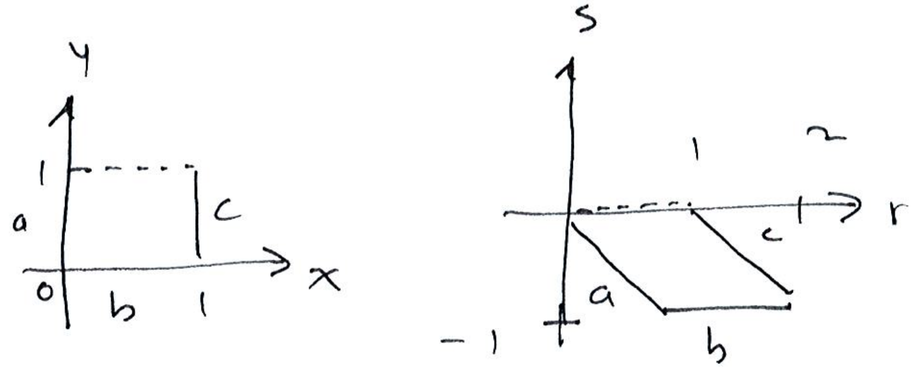
so $x_s = 1$ then $x = s + a(r)$ $\qquad s = 0\ x = r \Rightarrow a(r) = r$

$$x = s + r$$

$y_s = 1 \Rightarrow y = s + b(r)$ $\qquad s = 0\ y = 1 \Rightarrow b(r) = 1$

$$y = s + 1$$

so $r = x - y + 1, \quad s = y - 1$



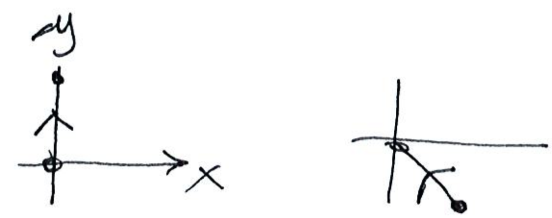
Consider boundary a where $x = 0$

so $r = 1 - y, \quad s = y - 1$

$\Rightarrow \quad s = -r$ which is a in the 2nd pic

Also $y : 0 \to 1$ (in pic 1)

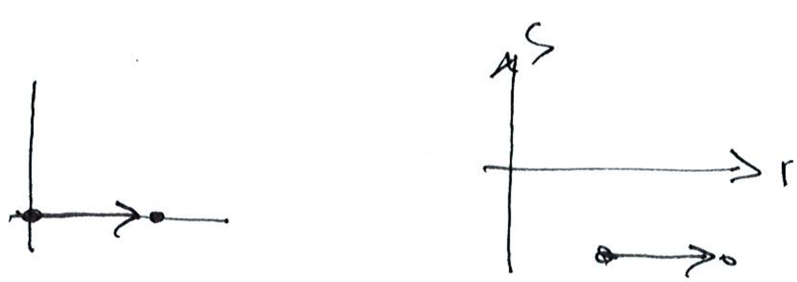
then $r : 1 \to 0 \quad s : -1 \to 0$



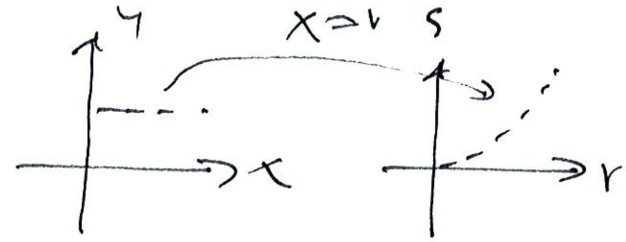
the boundary b ($y = 0$)

$r = x + 1 \qquad s = -1$

and $\quad x : 0 \to 1 \qquad r : 1 \to 2$



instead let's choose $s=r^2$ as the new boundary



So $x_s = 1$

So $x = s + a(r)$

$s = r^2 \quad x = r \Rightarrow a(r) = r - r^2$ so $x = s + r - r^2$

$y_s = 1 \Rightarrow y = s + b(r)$

$s = r^2 \quad y = 1 \Rightarrow 1 = r^2 + b(r) \Rightarrow b(r) = 1 - r^2$

$$y = s + 1 - r^2$$

so $x = s + r - r^2, \; y = s + 1 - r^2$

or $r = x - y + 1, \; s = y - 1 + (x - y + 1)^2$