Math 3331 – ODEs

we now start to find sol^n's of ODE.

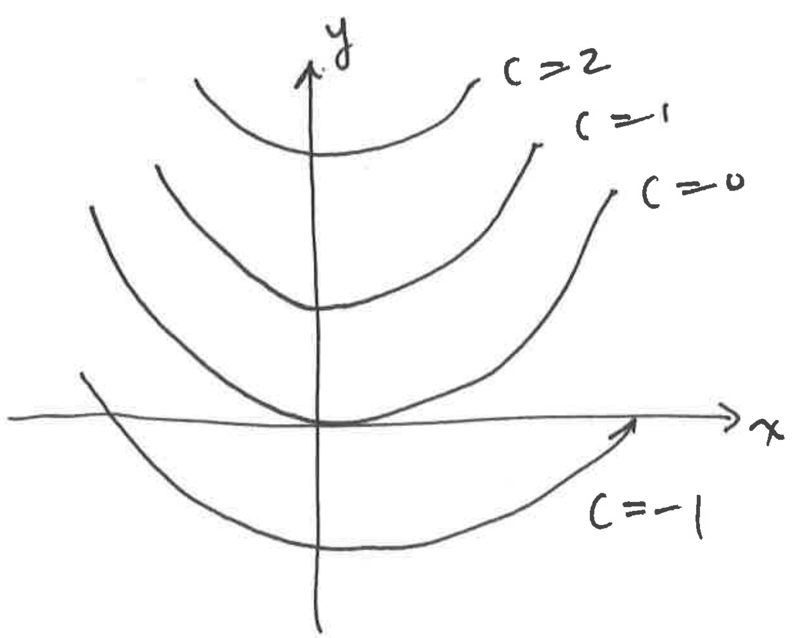
we will start with 1^{st} order

$$\frac{dy}{dx} = f(x,y)$$

Consider $y = x^2$ so $\frac{dy}{dx} = 2x$

Given $\frac{dy}{dx} = 2x$ then $dy = 2x\,dx$

& $y = x^2 + c$ ← this sol^n is a family of parabola's and depending on c gives

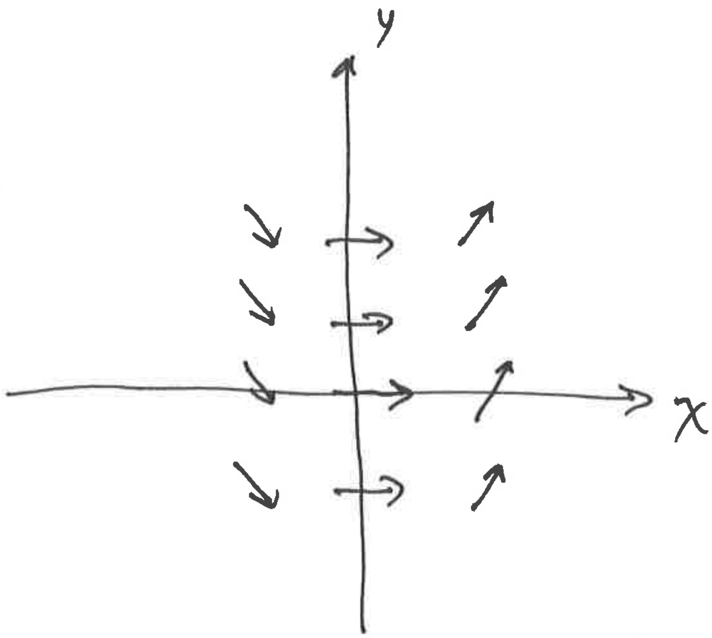


As the parabola's shift up (or) down depending on c

Given $\frac{dy}{dx} = 2x$

this is a slope and as we change x, $\frac{dy}{dx}$ changes. So at $(0,0)$ $\frac{dy}{dx} = 0$

also at $(0,1)$ $(0,2)$ $(0,-1)$ $\frac{dy}{dx} = 0$



At

$(1,-1)$, $(1,0)$ & $(1,+1)$

$\frac{dy}{dx} = 2$

and at

$(-1,-1)$ $(-1,0)$ $(-1,1)$

$\frac{dy}{dx} = -2$

If we keep going like this, at all pts (x,y) there is a slope and a direction the sol^n will travel. This is called a "direction field" and once we have a starting pt we follow the arrows.

See Schaums Outline pg 157-

We now consider the ODEs

(1) $\dfrac{dy}{dx} = -\dfrac{x}{y}$

(2) $\dfrac{dy}{dx} = x^2 + y^2$

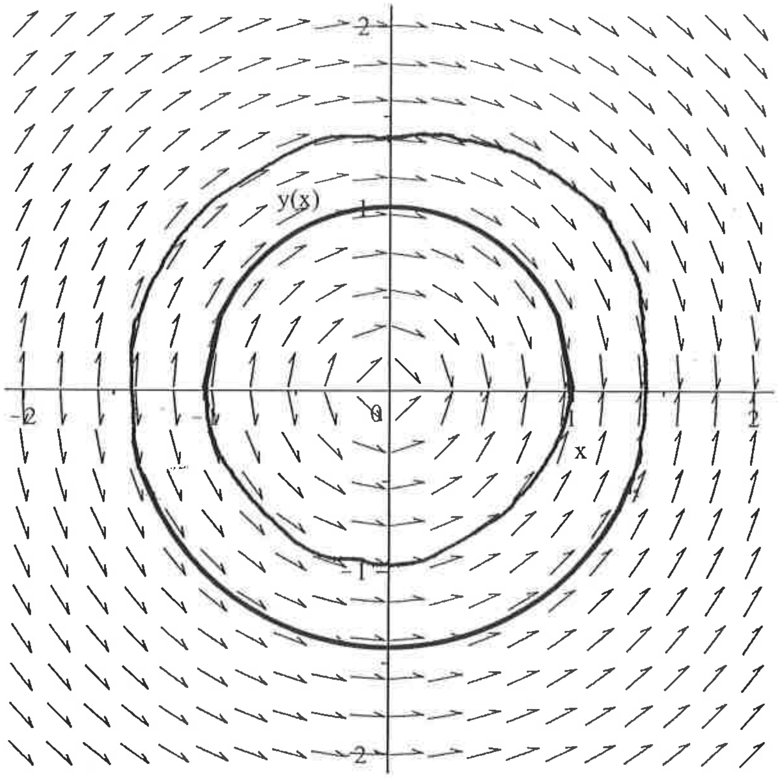
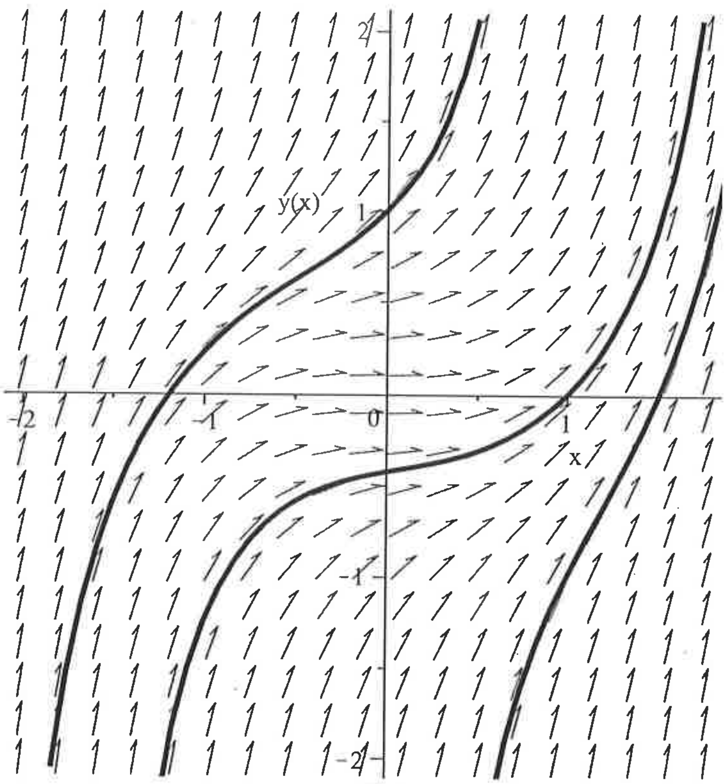
(3) $\dfrac{dy}{dx} = y(1-y)$

and consider the ICs (initial conditions)

(i) $y(1) = 0$

(ii) $y(0) = 1$

(iii) $y(1) = -1$


y(x)
x

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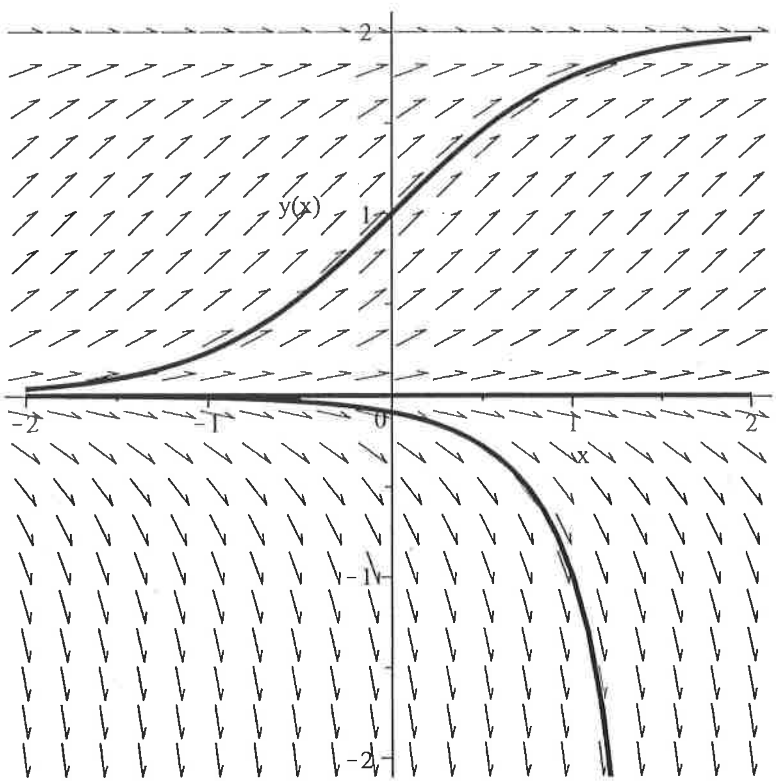
y(x)
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