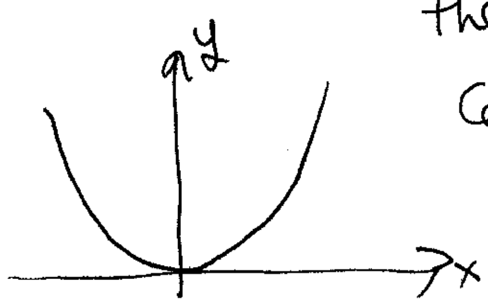
With all the tools we have learned so far we now use these to explore the behavior of functions.

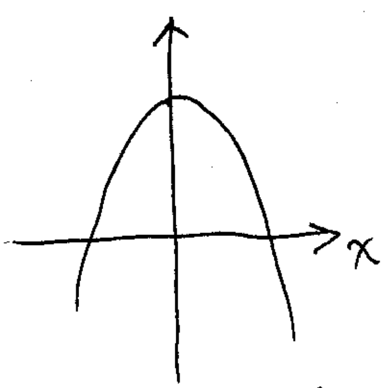
Consider $f(x) = x^2$. In the graph we see



there is a low point which we call a "minimum" at it location is at $x = 0$. We see that

$f(x) \geq f(0)$ for all x.

Consider $f(x) = 1 - x^2$. Here we see a "maximum" and that



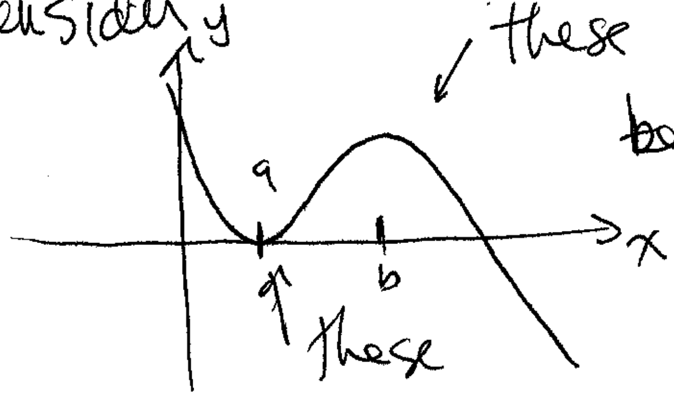
$f(x) \leq f(0)$

In general $f(c)$ is a minimum when $f(c) \leq f(x)$

$f(c)$ " maximum " $f(c) \geq f(x)$

min's & max's are called extrema

Consider



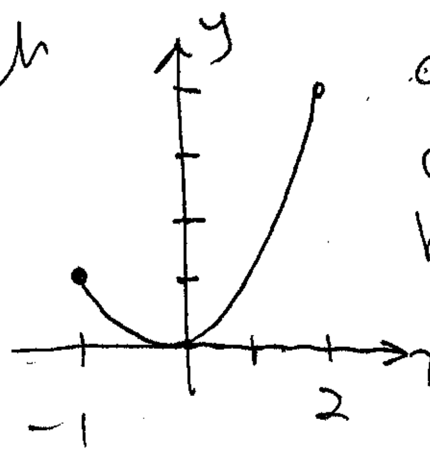
these are not min & max because the graph is greater than & less than these values.

However, near $x=a$ we have a min & near $x=b$ we have a max. These are called relative or local mins/max. In the previous 2 graphs we have what is called global min & max's

Consider now only part of an interval

say $f(x)=x^2$ on $[-1,2]$

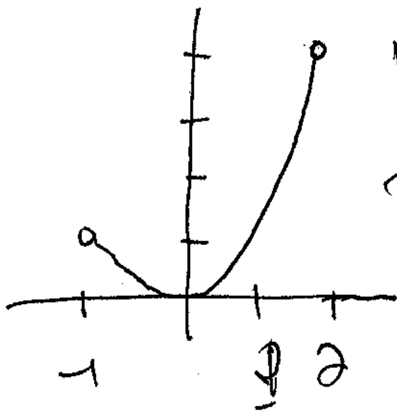
Graph



on this interval we have a min at $x=0$ $(f(0)=0)$ but we also have a max at $x=2$ when $f(2)=4$.

Now consider $f(x) = x^2$ on $(-1, 2)$

so



we still have a min at $x=0$ however we do not have a max. b/c for every value you give me near $x=2$ I can always get a value closer.

Extreme Value Thm

If $f(x)$ is continuous on $[a,b]$ the $f(x)$ has both a min & a max. They will be located at

(1) inside interval

or (2) the endpoints

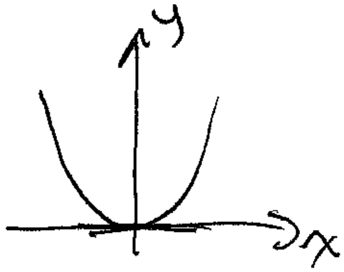
Derivatives & Relative Extrema

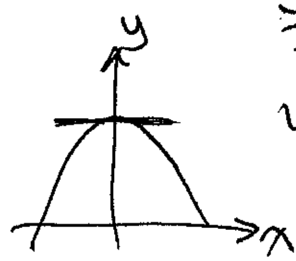
Consider $f(x) = x^2$ & $f(x) = 1-x^2$

$f'(x) = 2x$ $\qquad$ $f'(x) = -2x$

we see that in both

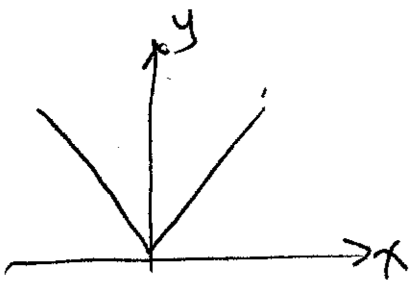
$$f'(0) = 0$$





$f'(c) = 0$ means we have horizontal tan

Consider $f(x) = |x|$ Derivative from the def^n



$$\lim_{x \to 0^-} \frac{-x}{x} = -1 \neq \lim_{x \to 0^+} \frac{x}{x} = 1$$

so the derivative DNE

Def^n Critical Number

Let $f(x)$ be defined at $x = c$. If $f'(c) = 0$ or DNE

If $f(x)$ has a rel min/max it will occur at a critical number

Ex Find the absolute min & max of $y = x^3 - 3x$ on $[0, 2]$

Sol^n

1st find y' so $y' = 3x^2 - 3$

$y' = 0$ when $3(x^2 - 1) = 0$ or $3(x-1)(x+1) = 0$

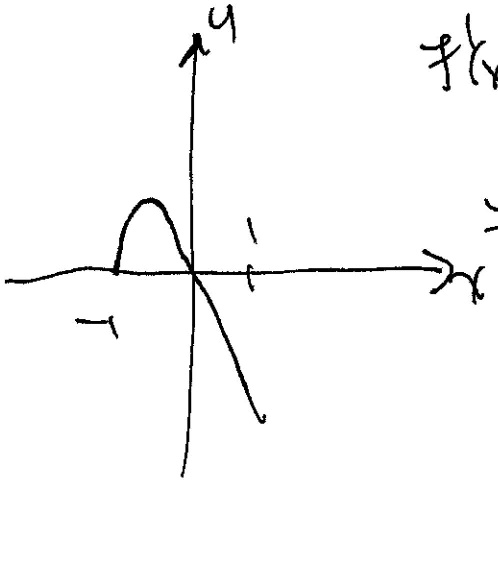
so $x = -1$, or 1 (Note $x = -1$ is not in the interval)

so look at the c# $y|_{x=1} = 1 - 3 = -2$

look at the endpoint $y|_{x=0} = 0$ $y|_{x=2} = 8 - 6 = 2$

so the absolute min is -2 & max is 2

Ex pg 211 # 8 $f(x) = -3x\sqrt{x+1}$ on $[-1, 1]$



$f'(x) = -3\sqrt{x+1} - 3x \cdot \frac{1}{2\sqrt{x+1}}$

$f'(x) = 0$ when

$\cancel{3}\sqrt{x+1} + \frac{\cancel{3}\ x}{2\sqrt{x+1}} = 0$

so $2(x+1) + x = 0$

$$3x+2=0 \Rightarrow x = -2/3$$

$$f(-2/3) = -3\left(+\frac{2}{3}\right)\sqrt{-\frac{2}{3}+1}$$

$$= 2\sqrt{\frac{1}{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Also we see that if

$$f'(x) = -3\sqrt{x+1} - \frac{3x}{2\sqrt{x+1}}$$

$f'(-1)$ DNE

so we have 2 critical numbers

$x = -2/3$ & $x = -1$