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Research Article

On Certain Properties of Identification Topological Subspaces

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Abstract

In the present work, T_0-identification spaces are used to define weakly *Po* spaces and properties, and T_0-identification *P* properties. We also give example for the characterization of T_0-identification *P* properties.

Keywords: Identification; Topology; Subspaces; T_0-identification; Property *P*.

Introduction

In [1], it was shown that T_0-identification spaces satisfy the T_0 separation axiom. Thus, for a topological property to be a T_0-identification space property, (P and T_0), denoted by *Po*, would have to exist. In [2], it was shown that a space is T_0 if and only if the natural map N from the space onto its T_0-identification space is a homeomorphism. Thus, for each topological property P for which *Po* exists, *Po* is a T_0-identification space property. Hence $\{P \mid P$ is a topological property and a T_0-identification space property$\}$ = $\{P \mid P$ is a topological property and *Po* exists$\}$. In [3], several topological properties, including R_1, were shown to be simultaneously shared by a space and its T_0-identification space. Thus R_1 is a T_0-identification space property that is not $(R_1)o = T_2$ [1], raising questions about other topological properties that are T_0-identification space properties P for which $P \neq Po$. In [4], the use of T_0-identifications space to characterize each of metrizable and T_2, as given above, motivated the introduction and investigation of weakly *Po* spaces and properties.

In [5], T_0-identification spaces were used to jointly characterize pseudometrizable and metrizable: A space is pseudometrizable iff its T_0-identification space is metrizable. Similarly, in [6], the R_1 separation axiom and T_0-identification spaces were used to further characterize the T_2 property. Since the T_0-identification space of each space is T_0, then for a topological property Q for which weakly *Qo* exists, a space (X, T) is weakly *Qo* iff $(X_0, Q(X, T))$ has property *Qo*, and, within $(X_0, Q(X, T))$, Q and *Qo* are equivalent [7-10]. By the results above, R_1 = weakly (R_1 *o* = weakly T_2, which will be used later. Hence R_1 is weakly *Po*, and R_2 is a weakly *Po* property. Also, in the [11], it was shown that for a topological property Q for which weakly *Qo* exists, weakly *Qo* is simultaneously shared by both a space and its T_0 -identification space, which when combined with the results above, led to the introduction and investigation of T_0 -identification P properties.

In [12], the search for topological properties that fail to be weakly *Po* properties led to the need and use of T_0 and "not-T_0 " revealing T_0 and "not-T_0 " as useful topological properties, motivating the addition of the long-neglected topological property "not-P" into the study of topology, where P is a topological property for which "not-P" exists. Thus far, the addition and use of "not-P" in the study of topology has led to the discovery of the never before imagined least of all topological properties L = (T_0 or "not-T_0") [13] and that there is no strongest topological property [7]. As is expected, the existence of the never before imagined topological property L revealed needed changes in classical topology, including both product [8] and subspace properties [9] leading to new, meaningful, never before imagined properties and examples for each of those two properties, expanding and changing the study of topology forever. Initially, the search for properties that are weakly *Po* or equivalently T_0-identification P was by trial and error. As

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established above, for a topological property Q for which Qo exists, a topological property W was sought such that for a space with property W its T_0 -identification space has property Qo, which, in turn, implies the initial space has property W.

Since the trial and error search process was tedious, time consuming, uncertain, and never ending, there was a need to completely characterize each of weakly Po spaces and properties. In [10], when it was not realized that weakly Po and T_0 -identification P are equivalent topological properties, weakly Po was characterized and T_0-identification P was thought to be characterized. Below a counterexample is given for the once believed characterization of T_0-identification P and necessary changes are made.

Preliminaries

Definition 2.1

Let (X, T) be a space, R be the equivalence relation on X defined by xRy if and only if $Cl(\{x\}) = Cl(\{y\})$, X_0 be the set of R equivalence classes of X, let $N : X \to X_0$ be the natural map, and $Q(X, T)$ be the decomposition topology on X_0 determined by (X, T) and the natural map N. Then $(X_0, Q(X, T))$ is the T_0-identification space of (X, T).

Definition 2.2

A space (X, T) is R_1 if and only if for x and y in X such that $Cl(\{x\}) \neq Cl(\{y\})$, there exist disjoint open sets U and V such that $x \in U$ and $y \in V$ [1].

Remark 2.3

Since for any topological property P and any space with property P, its T_0-identification space exists, then there are no restrictions on spaces for which its T_0-identification space exists. Thus attention shifted from properties of spaces (X, T) for which its T_0-identification space $(X_0 , Q(X, T))$ exists to the properties of the T_0-identification spaces $(X_0, Q(X, T))$, motivating the definition and work below.

Definition 2.4

A topological property P is a T_0-identification space property if and only if there exists a space (X, T), whose T_0-identification space has property P.

Definition 2.5

Let P be a topological property for which Po exists. Then a space (X, T) is weakly Po if and only if its T_0-identification space $(X_0, Q(X, T))$ has property P. A topological property Qo for which weakly Qo exists is called a weakly Po property.

Definition 2.6

A topological property S is a T_0-identification P property if and only if S is simultaneously shared by both a space and its T_0-identification space [5]. Then, by definition, for a topological property Q, Q is weakly Po if and only if Q is a T_0-identification P property and weakly Po and T_0-identification P are equivalent properties.

Definition 2.7

Let Q be a topological property such that Qo exists. A space (X, T) has property QNO if and only if (X, T) is "not-T_0" and $(X_0 , Q(X, T))$ has property Qo.

Results and discussion

In [10], for a topological property Q for which Qo exists, a property QNO was defined whereby it was shown that for a topological property for which Qo exists, QNO exists and is a topological property, and a space has property (Qo or QNO) if and only if its T_0-identification space has property (Qo or QNO). Thus for a topological property Q for which Qo exists, (Qo or QNO) is a T_0-identification P property and (Qo or QNO) = weakly (Qo or QNO)o. Since QNO is "not-T_0", then (Qo and QNO)o = Qo. Thus $\{Uo|\ U$ is a topological property for which Uo exists$\} \subseteq \{Uo|\ U$ is a topological property and Uo is a weakly Po property$\} = \{Uo|\ U$ is a topological property and T_0-identification $P\}$ and since $\{Uo|\ U$ is a topological property and Uo is a weakly Po property$\} \subseteq \{Uo|\ U$ is a topological property for which Uo exists$\}$, then the three sets are equal and the weakly Po properties are completely characterized replacing the uncertainty of selecting Qo in the trial and error search process by certainty.

Claim 3.1.

For a topological property Q for which both Qo and (Q and "not-T_0") exist, Q is a T_0 -identification P property, QNO = (Q and "not-T_0"), and Q = weakly Qo = (Qo or (Q and "not-T_0")). The following example shows.

Example 3.2

Let $W = R_1$. Then $Wo = (R_1$ and $T_0) = T_2$ [1] exists. Since R_1 is a T_0 -identification P property, then $WNO = (R_1$ and "not-T_0"). Let $Q = (T_0$ or $(R_1$ and "not-T_0"). Then $Qo = T_0$ and (Q and "not-T_0") = (R_1 and "not-T_0" exist and by Claim 3.1, Q is a T_0 -identification P property. Hence Q is weakly Po and Q = weakly Qo = weakly (T_0 or (R_1 and "not-T_0")) $o = T_0$, but, since L = weakly Lo = weakly T_0 [11], then L = (T_0 or (R_1 and "not-T_0")), which is a contradiction. Thus Claim 3.1 is not true.

Theorem 3.3

Let Q be a topological property. Then the following are equivalent:
(a) Q is a T_0 -identification P property,
(b) Q is weakly Po,
(c) both Qo and (Q and "not-T_0 ") exists, and (Q and "not-T_0") = QNO,
(d) Q = (Qo and QNO).

Proof: Clearly, by the results above, (a) and (b) are equivalent.

(b) implies (c): Let (X, T) be a space with property Q. Then (X, T) has property Q = weakly Qo, Qo exists, and $(X0, Q(X, T))$ has property Qo, which implies (X, T) has property (Qo or QNO), where Qo and QNO are distinct topological properties. Thus Q is a T_0 -identification P property and Q = (Qo or (Q and "not-T_0")) [12], which implies (Q and "not-T_0") = QNO.

(c) implies (d): Since both Qo and (Q and "not-T_0 ") exist, then Q = (Qo or (Q and "not-T_0 ")) = (Qo or QNO), which is a T_0-identification P property.

Conclusions

From the findings in the present study, it is concluded that the uncertainty in the trial and error search process for selecting the starting place Qo is resolved leaving only the uncertainty of determining weakly Qo. Each of the T_0-identification space and weakly Po processes have been internalized greatly simplifying the search for weakly Qo.

Conflicts of interest

The authors declare no conflict of interest.

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