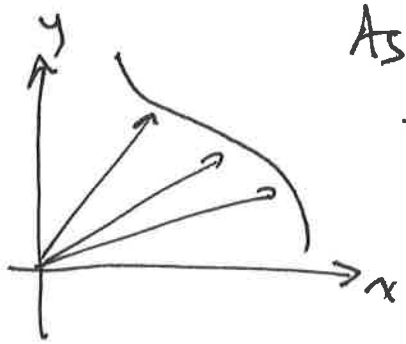
Math 1497 Calc 2

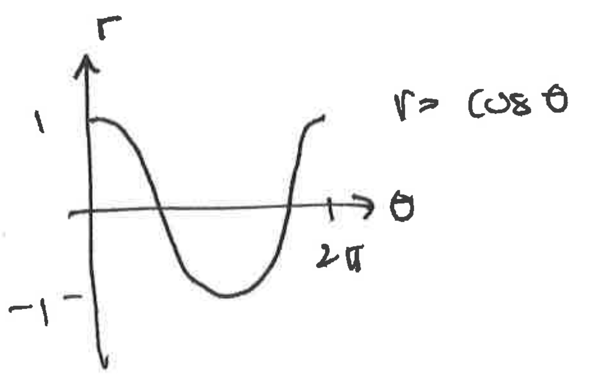
Polar Graphs $r = f(\theta)$

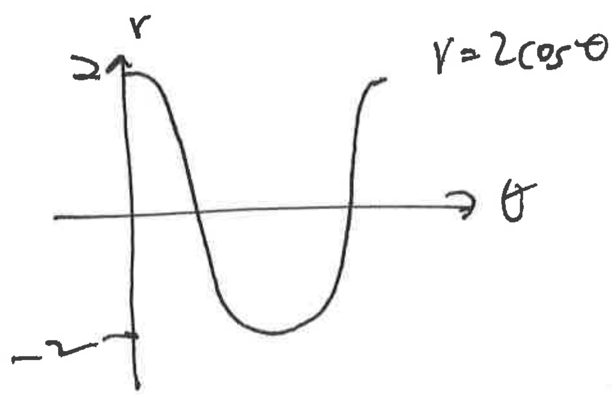


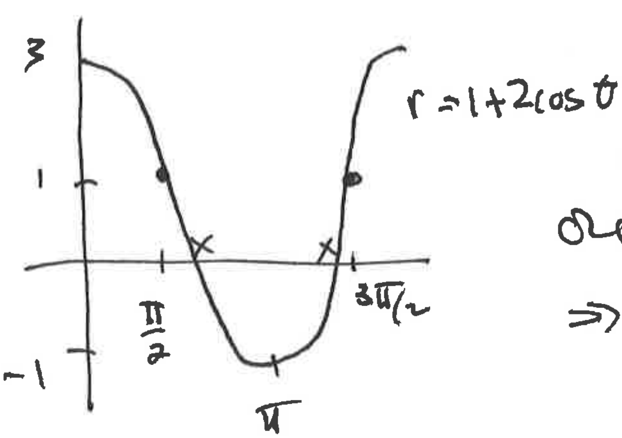
As we rotate θ, r moves from the origin along a ray (line) from the origin to $f(\theta)$.

We consider the Cartesian-Polar Method

Ex $r = 1 + 2\cos\theta$



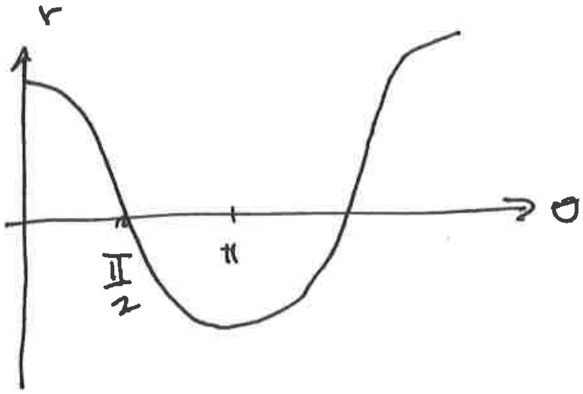




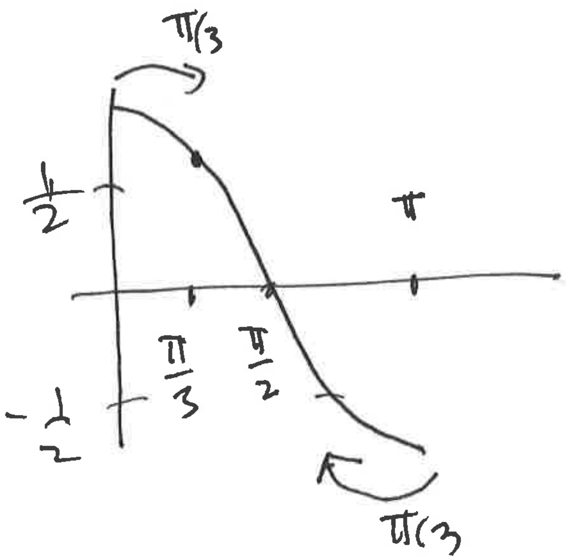
So there are 2 places when we go to the origin $r = 0$

$\Rightarrow \quad 1 + 2\cos\theta = 0 \quad \cos\theta = -\frac{1}{2}$

We now use the cosine graph to determine these

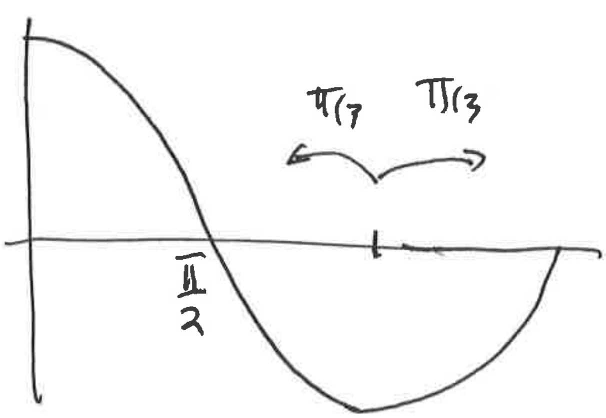


θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\pi/2$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

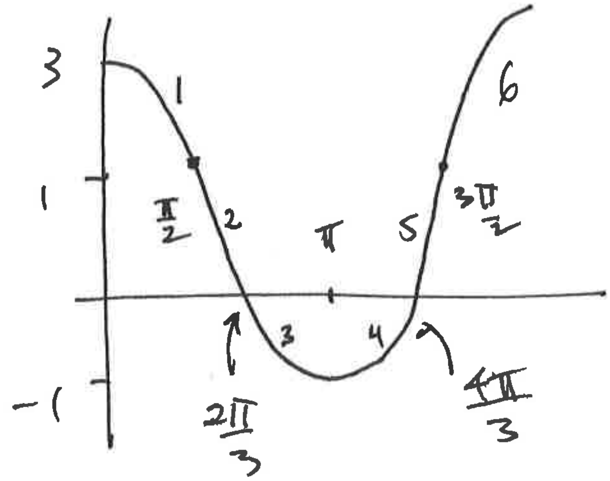


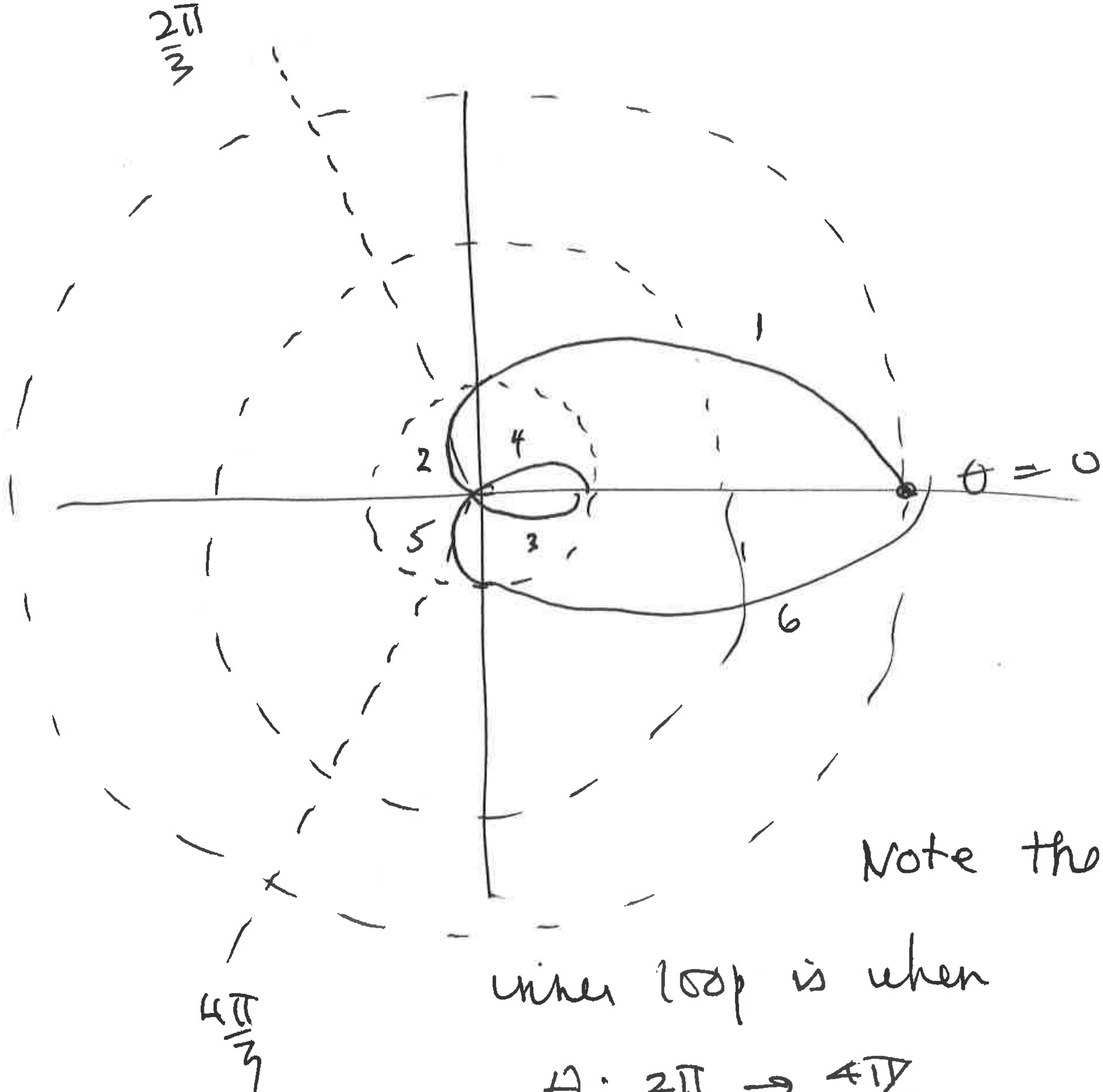
So 1^{st} value is

$$\theta = \pi - \pi/3 = \frac{2\pi}{3}$$



$$\theta = \pi + \pi/3 = \frac{4\pi}{3}$$





Note the inner loop is when

$\theta: \frac{2\pi}{3} \to 4\pi/3$

Calculus in Polar Coordinates

Derivatives

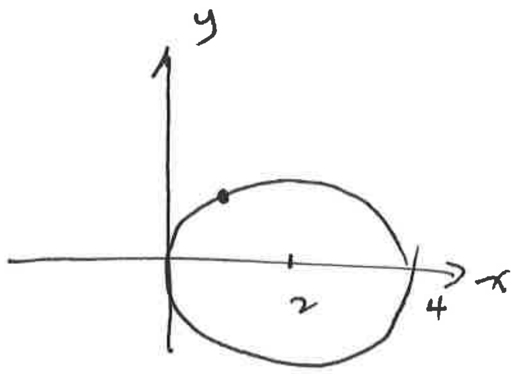
$x = r\cos\theta$ if $r = f(\theta)$ $x = f(\theta)\cos\theta$

$y = r\sin\theta$ then $y = f(\theta)\sin\theta$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{f'\cos\theta - f\sin\theta}{f'\sin\theta + f\cos\theta}$$

<u>Ex</u> pg 738 #6

$r = 4\cos\theta \quad (2, \pi/3)$



$x = r\cos\theta = 2\cos\pi/3 = 2\cdot\frac{1}{2} = 1$

$y = r\sin\theta = 2\sin\pi/3 = 2\frac{\sqrt{3}}{2} = 1.7$

Now on curve $x = 4\cos\theta\cdot\cos\theta = 4\cos^2\theta$

$y = 4\cos\theta\sin\theta$

$\frac{dy}{d\theta} = 4\cos^2\theta - 4\sin^2\theta$

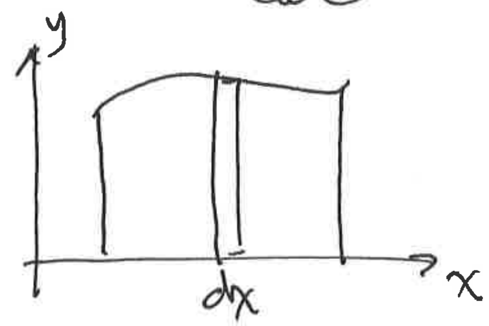
$\frac{dx}{d\theta} = -8\cos\theta\sin\theta$

$$\frac{dy}{dx} = \frac{4(\cos^2\theta - \sin^2\theta)}{-8\cos\theta\sin\theta} = -\frac{\cos 2\theta}{\sin 2\theta}$$

$$\left.\frac{dy}{dx}\right|_p = -\frac{\cos 2\pi/3}{\sin 2\pi/3} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

Areas

Calc 1



area of rectangle

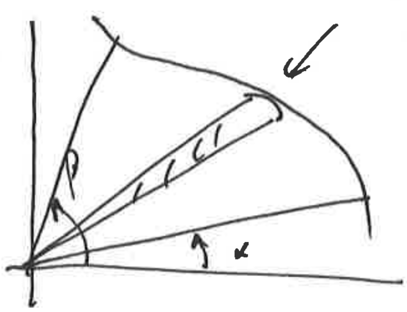
$f(x)\,dx$

height

thickness

$$A = \int_a^b f(x)\,dx$$ add up rectangles

Polar



thin slice approx. like a part of a circle

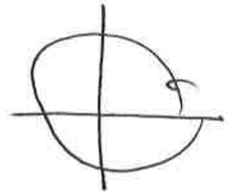
ratio $\frac{dA}{\pi r^2} = \frac{d\theta}{2\pi}$ dA - area of slice

$\Rightarrow dA = \frac{1}{2} r^2 d\theta$

Now add up the slices

$$A = \int_\alpha^\beta \frac{1}{2} r^2 d\theta$$

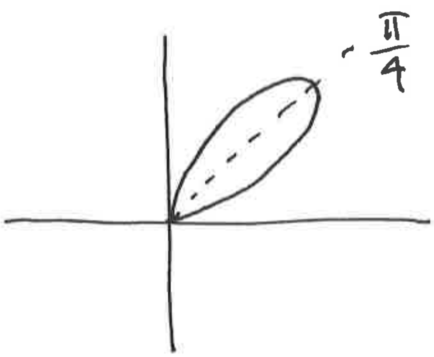
Ex $r = a$ (circle)



$$A = \frac{1}{2}\int_0^{2\pi} a^2\,d\theta$$

$$= \frac{1}{2}a^2\theta\Big|_0^{2\pi} = \frac{1}{2}a^2\,2\pi = \pi a^2 \quad (\text{like } \pi r^2)$$

Ex $r = \sin 2\theta$ 1 leaf



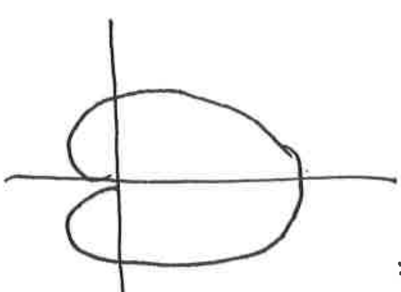
$$A = 2\cdot\frac{1}{2}\int_0^{\pi/4}\sin^2 2\theta\,d\theta$$

$$= \int_0^{\pi/4}\frac{1-\cos 4\theta}{2}\,d\theta$$

$$= \frac{\theta}{2} - \frac{\sin 4\theta}{8}\Big|_0^{\pi/4} = \left(\frac{\pi}{8} - \frac{\sin\pi}{8}\right) - (0 - 0)$$

$$= \frac{\pi}{8}$$

Ex $r = 1 + \cos\theta$



$$A = 2\cdot\frac{1}{2}\int_0^{\pi}(1+\cos\theta)^2\,d\theta$$

$$= \int_0^{\pi}(1 + 2\cos\theta + \cos^2\theta)\,d\theta$$

$$= \int_0^{\pi}\left(1 + 2\cos\theta + \frac{1+\cos 2\theta}{2}\right)d\theta = \frac{3}{2}\theta + 2\sin\theta + \frac{\sin 2\theta}{4}\Big|_0^{\pi}$$

$$= \frac{3\pi}{2}$$