Image Reconstruction Using Compressed Sampling Matching Pursuit

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***Abstract*—** Reconstruction of images with incoherent subsampling is making immense progress in advancement of image reconstruction and processing domsdain. By taking random samples and using Compressed Sensing or Compressive Sampling (CS) algorithms enables the reconstruction of the actual image with least number of samples and less error when compared to the conventional Nyquist Sampling theorem. In the current work, the sparse approximation algorithm Compressive Sampling Matching Pursuit (CoSaMP) is being implemented which is an extension of the Orthogonal Matching Pursuit (OMP) algorithm. This algorithm belongs to class of greedy algorithms. This paper also compares the CoSaMP algorithm with the OMP algorithm and proves that it provides better results.

Keywords*—Compressive Sampling Matching Pursuit (CoSaMP); Orthogonal Matching Pursuit (OMP); Compressed Sensing (CS); Sensing matrix*

I. INTRODUCTION

Compressive Sensing is a novel sampling method that samples signals in a much more efficient way. CS has recently gained a lot of attention due to its exploitation of signal sparsity [1]. CS combines both the sampling and compression into one step by measuring minimum samples that contain maximum information about the signal. This eliminates the need to acquire and store large number of samples only to be eliminated because of their minimal value.

In CS, the image is first represented in vector form and it should be sparse in some domain which means that the vector should have least number of non-zero integers to compress very efficiently which is nothing but l_0*-norm* of that vector. The sparse vector of size $N \times 1$ is multiplied with sensing matrix A of size $M \times N$, which results in $M \times 1$ vector [6]. This resultant vector is called compressed vector B and the value of M will be typically in the order of $klog(N)$, where k is the sparsity of the image. The Discrete Cosine Transform (DCT), Discrete Wavelet Transform (DWT) etc., are used to transform the image to other basis to be more sparse [2] [3]. Similarly, sensing matrix should also satisfy some properties to recover the image from its compressed form to original form: Null space condition and Restricted Isometric Property (RIP) [4] [5] explained in section II.

The interesting thing about CoSaMP algorithm is that every iteration increases the signal to noise ratio (SNR) of the reconstructing signal by 3 dB. Thus for precise reconstruction more number of iterations can be used. It is most suitable over OMP because iteration counts can be much lesser than the sparsity of the target signal. The proof for this major advancement over OMP is in [7].

In the current work, CoSaMP reconstruction algorithm is implemented by using Gaussian distribution for the sensing matrix, the DWT with Deubachies filter for the making the signal sparse. In our previous research paper OMP was implemented [15], so in this paper we have compared CoSaMP with OMP algorithm and the mathematical deductions mentioned in [7] are proved. The qualitative and quantitative comparison results of the experiment are as mentioned in section IV.

II. PROPERTIES

1. Null space condition:

For any pair of distinct vectors $x, \hat{x} \in \Sigma_k$, we must have $Ax = A\hat{x}$, otherwise it will be impossible to distinguish x from $\hat{x}$ based solely on the measurements, then we can recover all sparse signals x from the measurements Ax. If $Ax = A\hat{x}$ then $A(x - \hat{x}) = 0$ with $x - \hat{x}, \in \Sigma_{2k}$, we see that A uniquely represents all $x \in \Sigma_k$ if and only if $N(A)$ contains no vectors in Σ_{2k} where, null space of A, denoted $N(A) = \{z: Az = 0\}$.

2. The Restricted Isometry Property:

Matrix A satisfies the Restricted Isometry Property (RIP) of order k if there exists a $\delta k \in (0,1)$ such that

$$(1-\delta_k)||x||_2^2 \le ||Ax||_2^2 \le (1+\delta_k)\,||x||_2^2$$

holds for all $x \in \Sigma_k$

If a matrix A satisfies the RIP of order $2k$, then we can interpret above equation as saying that A approximately preserves the distance between any pair of k-sparse vectors [5].

III. RECONSTRUCTION USING COSAMP ALGORITHM

A. *Compressive Sampling Matching Pursuit*

CoSaMP (Compressive Sampling Matching Pursuit) algorithm belongs to family of Matching Pursuit algorithms which are iterative in nature, chooses dictionary elements in a greedy fashion that best approximate the signal [7]. Estimating the correct value with its exact index of a sparse signal is the main objective of the CS algorithms. With the help of RIP and the matching pursuit algorithm [13] one can achieve the above objective. In CoSaMP algorithm the k-sparse signal is recovered by using signal proxies, since proxy signal values corresponds to the original signal [8]-[12].

In this algorithm the signal proxy is Y a vector formed by the equation $Y = A^*Ax$. Here the proxy signal is Y for the sparse signal x, hence corresponding k components in Y and x are approximate. Since B is the compressed signal, the proxy signal can be obtained from it just by multiplying B *and* A^*. During each iteration the residual will be calculated to check how much estimation is done. In successive iteration the estimate of x gets updated, hence the residual, which hints when the iteration can be terminated.

This algorithm has five main procedures which approximates the target signal with least residual. The first proxy signal is created by initiating the compressed signal as the initial value of the residual. Next, the support of 2k largest values of that proxy signal vector are identified and merged with the previous support. Thirdly the estimation of target signal is achieved by solving least squares. Only the largest k values of estimated target samples are kept for next iteration. Finally, the residual is updated. The same steps are coded and shown mathematically in below section.

B. *CoSaMP Algorithm*

Input:

Sensing matrix A, compressed vector B and sparsity k.

1. $x^0 = 0$ (initiating target signal to zero)
2. $r = B$ (initiating residual to compressed signal)

Algorithm:

Iteration: during ith iteration, do

1. $Y = A^*R$ (creating proxy signal)
2. $\Lambda = supp(y_{2s})$ (finding supports)
3. $S = \Lambda \cup supp(x^{i-1})$ (merging the supports)
4. $V_S = A_S^{\dagger}B$ (solving least squares)
5. $x^i = \hat{V}_s$ (selecting largest k values only (pruning))
6. $r = B - Ax^i$ (updating the residue with estimated signal)

Output:

1. Estimated target signal x

The iteration can be stopped by fixing the iteration counts. Also, the iterations can be stopped by using the condition $\|r\|_2 \le \varepsilon$ where ε is the tolerable error value [7].

On every iteration the target signal gets improved as given in the equation

$$||x-\hat{x}||_2 \le 2^{-k}||x||_2 + 20\varepsilon$$

The improvement is given by SNR_{rec} value represented by the equation,

$$SNR_{rec} \ge \min\{3k, SNR - 13\} - 3$$

until the noise level reaches the noise margin where,

$$\text{SNR} = 10\log_{10}\left(\frac{||x||_2}{\varepsilon}\right)$$

ε is the error $||e||_2$.

IV. IMPLEMENTATION RESULTS

The CoSaMP algorithm was implemented and executed using Matlab R2016a software. Tests were executed on Windows operating system machine with an Intel processor with core i5-7300HQ CPUs @ 2.50 GHz, NVIDIA 1050Ti GPUs. The standard barbara_gray.jpg and lena_gray.jpg images of size 512 X 512 were used for testing purpose. Discrete wavelet transform with Daubechies wavelet was used for converting image into its sparse signal vector. Initially the images were introduced to the Gaussian noise of 0.02 variance and zero mean before creating sparse vector. Gaussian random variables with independent and identically distributed (i.i.d.) entries having dimension $M \times N$ was used for the sensing matrix A. M value was defined as the order of k*log(N). The experiment showed significant improvement in the quality of the reconstructed image as shown in Fig. 1 and Fig.2. For performance analysis Peak Signal to Noise Ratio (PSNR) and Mean Square error (MSE) metrics were used. The results are as shown in Table I:

TABLE I. PSNR AND MSE RESULTS

Image used	MSE	PSNR
Lena_gray	80.79	35.11 dB
Barbara_gray	85.46	34.86 dB

In this paper, image reconstruction using CoSaMP algorithm was examined. Comparison of PSNR output of the CoSaMP with OMP algorithm is tabulated in the Table II for different sampling rates (M/N ratio) for lena_gray.jpg image. The results of OMP algorithm with different sparse basis transform are from reference [11] & [15]. From Fig. 1 & 2 better visual quality can also be observed.

TABLE II. COMPARISON OF PSNR RESULTS

M/N ratio	CoSaMP	OMP-DCT	OMP-DWT
0.6	36.53	29.47	28.558
0.65	35.11	30.95	29.432
0.7	40.96	32.42	30.306

(a) (b) (c)

Figure 1: Results of reconstruction using CoSaMP for lena_gray image (a) Original Image, (b) Noisy Image, (c) Reconstructed Image

(a) (b) (c)

Figure 2: Results of reconstruction using CoSaMP for barbara_gray image (a) Original Image, (b) Noisy Image, (c) Reconstructed Image

V. CONCLUSION

CoSaMP being a Greedy algorithm, approaches signal proxy of residual keeping largest components to create estimation of target signal. Creation of signal proxy comes with multiplication of matrix with a vector which requires significantly large amount of time in comparison with other steps. Due to the usage of RIP, this algorithm ensures that the above procedure is successful. It is also showed that the CoSaMP is far better than OMP algorithm because CoSaMP identifies multiple values of target signal per iteration, whereas OMP identifies one values per iteration [13]. Hence there is a greater advantage over OMP that CoSaMP has fixed number of iteration and converging rate is far better than OMP. The reconstructed image also has better visual quality as shown in figures. This algorithm provides tighter bounds on its convergence and performance compared to OMP algorithm.

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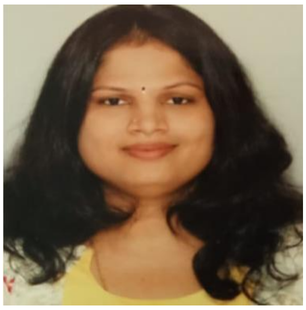

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