

Radiation and Free Convection Flow through a Porous Medium with Slip Parameter

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Abstract-The present investigation is conducted to study the slip parameter's steady two dimensional free convection flow through porous medium bounded by a vertical infinite porous plate. The plate moves with a constant slip effect velocity are discussed in the velocity field. The effect of the slip parameter on this distribution is studied numerically using graphs.

Keywords: Velocity, Polar fluid, Porous medium, Radiation, Slip parameter.

I. INTRODUCTION

The concept of fluid flow and mass transfer through a porous medium plays very important role in the application of geophysics, meteorology and aeronautics. The flow through a porous medium has been analysed extensively. Typical studies can be found for example in reference [1-3].

Radiative convective flow are having much application in evaporation from large open water, solar power technology and space vehicle recently. [4,5,6], have studied the effect of slip parameter in their problem. On the other hand heat and mass transfer by natural convection flow attention. Several researchers have analysed incomprehensible variety of flow connected to rotating fluid with heat and mass transfer. Free convection flow rao et.al [7] discussed steady diffusive field with thermal radiation. Reddy et.al. [8] investigated magneto convective flow on a non-Newtonian fluid. [9,10, [11] have studied free convective flow with porous medium.

The aim of present paper is to investigate the steady flow of a viscous fluid through a porous medium with slip effect bounded by a porous plate to a constant velocity by a presence of the thermal radiation.

MATHEMATICAL FORMULATION AND SOLUTIONS OF THE PROBLEM

The fluid is assumed to be gray-emitting and absorbing radiation but nonscattering medium.

The x' -axis is taken along the vertical plate in the upward direction and the y' -axis is chosen normal to it.

The radiative heat flux in the x' -direction is considered negligible in comparison to that in the y' -direction. All the fluid properties are considered constant except the influence of the density variation with temperature is considered only in the body-force term. Hence, the equation governing the problem are :

Continuity Equation :

$$\frac{\partial u'^*}{\partial y'^*} = 0 \quad (1)$$

Momentum Equation :-

$$v' \frac{\partial u'^*}{\partial y'^*} = g\beta(T' - T'_\infty) + \frac{\partial^2 u'^*}{\partial y'^{2*}} - \frac{v}{K'} u'^* \quad (2)$$

Energy Equation :-

$$v' \frac{\partial T'^*}{\partial y'^*} = \frac{k}{\rho c_p} \frac{\partial^2 T'^*}{\partial y'^{2*}} + \frac{v}{c_p} \left(\frac{\partial u'^*}{\partial y'^*} \right)^2 - \frac{1}{c_p} \frac{\partial q_r^*}{\partial y'^*} \quad (3)$$

where u' and v' are the corresponding velocity components along and perpendicular to the plate, ν the kinematic viscosity, g the acceleration due to gravity, β the coefficient of volume expansion, T' the fluid temperature, T'_∞ the fluid temperature at infinity, k the thermal conductivity, ρ the density of the fluid, K' the permeability of the medium and q_r the radiative heat flux.

The equation of continuity (1) gives

$$v' = -v_0 \quad (4)$$

where v_0 is the constant suction velocity normal to the plate.

By using Roseland approximation for the radiation [4] we take

$$q_r = - \frac{4\sigma^* \partial T'^4}{3k^* \partial y'} \quad (5)$$

Where σ^* the Stefan - Boltzmann Constant and k^* the mean absorption coefficient.

The boundary conditions for equations (2) and (3) are:

$$U = L \frac{\partial U}{\partial y}, T' = T'_w, \text{ at } y' = 0 \quad (6)$$

$$u' \rightarrow 0, T' \rightarrow T'_\infty, \text{ as } y' \rightarrow \infty \quad (7)$$

Where T'_w is the temperature of the plate.

We assume that the temperature differences within the flow are such that T'^4 may be expressed as a linear function of temperature.

This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4T'^3_\infty 3T'^4_\infty \tag{8}$$

In view of (4), (5) and (7) equations (2) and (3) now reduce to

$$-v_0 \frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{v}{c_p} \left(\frac{\partial u'}{\partial y'} \right)^2 + \frac{16\sigma^* T'^3_\infty}{3\rho c_p k^*} \frac{\partial^2 T'}{\partial y'^2} \tag{9}$$

Introducing into equations (8) and (9) the following non dimensional parameters

$$u = \frac{u'}{v_0} (\text{velocity}) y = \frac{y' v_0}{v} (\text{distance})$$

$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty} (\text{temperature}), P = \frac{\rho v c_p}{k} (\text{Prandtl number})$$

$$G = \frac{v g \beta (T'_w - T'_\infty)}{v_0^3} (\text{Grashof number}), E = \frac{v_0^2}{c_p (T'_w - T'_\infty)} (\text{Eckert number})$$

$$K = \left(\frac{v_0^0}{v^2} \right) K' (\text{Permeability parameter}), N = \frac{K^* k}{4\sigma^* T'^3_\infty} (\text{Radiation parameter})$$

$$N = \frac{K^* k}{4\sigma^* T'^3_\infty} (\text{Radiation parameter})$$

$$h = \frac{L V_0}{u} (\text{velocity slip parameter})$$

$$u^n + u + GT - \frac{1}{K} u = 0 \tag{10}$$

$$(3N + 4)T^n + 3PNT' + 3PNEu^2 = 0 \tag{11}$$

Where a prime denotes differentiation with respect to y.

The corresponding boundary conditions become

$$u = 0, T = 1, \text{ at } y = 0$$

$$u \rightarrow 0, T \rightarrow 0, \text{ as } y \rightarrow \infty \tag{12}$$

In order to obtain a solution of the above coupled non linear system of equations (10) and (11) we expand u and T in powers of Eckert number E , assuming that it is very small.

Hence we can write

$$u = u_0 + E u_1 + O(E^2) \tag{13}$$

$$T = T_0 + E T_1 + O(E^2)$$

Substituting equations (13) into equations (10) and (11) and equating the coefficients of the powers of E and neglecting terms in E^2 and higher order, we get

$$u_0^n + u_0' + G T_0 - \frac{1}{K} u_0 = 0 \tag{14}$$

$$u_1^n + u_1' + G T_1 - \frac{1}{K} u_1 = 0 \tag{15}$$

$$(3N + 4)T_1^n + 3PNT_0' + 3PNT_1' = 0 \tag{16}$$

$$(3N + 4)T_1^n + 3PNT_1' + 3PNu_1'^2 = 0 \tag{17}$$

With the corresponding boundary conditions:

$$U = L \frac{\partial U}{\partial y} \tag{18}$$

Solving equations (14) - (17) under the boundary conditions (18) and substituting the solutions

$$u' \rightarrow 0, T' \rightarrow T'_\infty \Rightarrow y^* \rightarrow \infty$$

$$y' \rightarrow 0, u^1 = \frac{L_1 \partial u}{\partial u} T' \rightarrow T'_w$$

into equations (13), we take

$$u(y) = C_2 e^{-R_2 y} + A_1 e^{-m y} + E \left[(4e^{-m_2 y} + A_5 e^{-m_2 y} + A_6 e^{-2m y} + A_7 e^{-R_2 y} + A_8 e^{-(R_2+m_2)y}) \right]$$

$$T(y) = e^{-m y} + E \left[C_2 e^{-m_2 y} + A_2 e^{-R_2 y} + A_3 e^{-m_7 y} + A_4 e^{-(R_2+m_2)y} \right]$$

$$R_{1,2} = \frac{-1 \pm \sqrt{1 - 4/K}}{2}$$

$$m_2 = \frac{-3PN}{3N + 4}$$

$$A_1 = \frac{G_r}{(m_2 + R_1)(m_2 - R_2)} A_2 = \frac{R_2 C_2^2 (3N + 4)^{22}}{2(2R_2 - m_2)}$$

$$A_3 = \frac{-m_2 A_1^2 (3N + 4)}{2} A_4 = \frac{2C_2 m_2^2 (3N + 4)}{(R_2 + m_2)}$$

$$A_5 = \frac{-2m_1 A_2 C_7}{(2m_2 + m_1)(2m_2 - m_1)}, A_6 = \frac{-2m_1 A_2 C_7}{(2m_2 + m_1)(2m_2 - m_1)}$$

$$A_7 = \frac{-2m_1 A_3 C_7}{(2R_2 + m_1)(2R_2 - m_1)}, A_8 = \frac{-2m_1 A_4 C_7}{(R_2 + m_2 + m_1)(R_2 + m_2 - m_1)}$$

Where A_i are constants.

$$i = 1, 2, 3 \dots \dots 8$$

II. DISCUSSION AND CONCLUSION

In order to point out the influence of the radiation parameter into the problem on the velocity field the following considerations are made:

- (1) The dimensionless parameter G takes positive value. This case corresponds to an externally cooled plate by the free convection currents.
 - (2) The prandtl number P is taken equal to 0.71 and this value corresponds to the air.
- Under these assumptions in figure shows the variation of the velocity field when $E=0.001, G=6$ and $K=3$. From this figure we conclude that the velocity decreases when the radiation parameter increases.

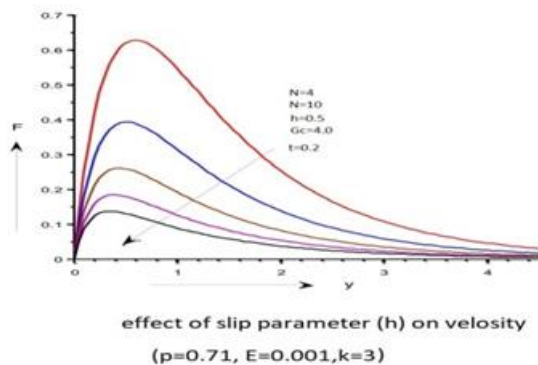


Figure.1

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