

Applications

Falling Bodies



$$F = ma = mg$$

$$m \frac{dv}{dt} = mg$$

$$\text{so } \frac{dv}{dt} = g \quad v = gt + v_0$$

$$\text{if at } t=0 \quad v(0) = 0 \quad g = 32 \text{ ft/sec}^2$$

$$v = 32t$$

this predicts that the velocity could be very large if t is large.

This model doesn't include air resistance

New Model

$$m \frac{dv}{dt} = mg - kv$$

air resistance
proportional to v

(2)

$$\text{so } \frac{m dv}{mg - kv} = dt$$

$$\text{if } W = mg - kv$$

$$dW = -k dv$$

$$\text{so } -\frac{m}{k} \frac{dW}{W} = dt$$

$$-\frac{m}{k} \ln|W| = t - \frac{m}{k} \ln C$$

$$\text{so } \ln|W| = -\frac{k}{m} t + \ln C$$

$$W = C e^{-\frac{k}{m} t}$$

$$mg - kv = C e^{-\frac{k}{m} t}$$

$$v = \frac{mg}{k} + \frac{C}{k} e^{-\frac{k}{m} t}$$

$$t \rightarrow \infty \quad v \rightarrow \frac{mg}{k}$$

Interesting Project

Free fall

Then open
parachute

$$(1) \quad m \frac{dv}{dt} = mg$$

(2) at same t

$$m \frac{dv}{dt} = mg - kv$$

(2) Newton's Law of Cooling

(3)

If T is temp of body & T_{∞} is temp of surrounding then

$$\frac{dT}{dt} \propto (T - T_{\infty})$$

$$\text{a } \frac{dT}{dt} = k(T - T_{\infty}) \quad T_{\infty} \neq$$

$$\text{Sep } \frac{dT}{T - T_{\infty}} = k dt$$

$$\ln |T - T_{\infty}| = kt + \ln C$$

$$T - T_{\infty} = C e^{kt}$$

$$\boxed{T = T_{\infty} + C e^{kt}}$$

Murder mystery

4

You show up at a murder scene at 12 midnight
Trump is dead. An eye witness saw Hilary
do it at 11:30 pm. She has an alibi until
11:15 pm

You take body temp.

$$t=0 \quad T=88^{\circ}\text{F}$$

$$t=20 \quad T=85^{\circ}\text{F}$$

Ambient temp is $60^{\circ}\text{F} = T_{\infty}$

$$\text{so } T = T_{\infty} + C e^{-kt}$$

$$T = 60 + C e^{-kt}$$

$$T(0) = 60 + C e^0 = 88 \Rightarrow C = 28$$

$$T(t) = 60 + 28 e^{-kt}$$

$$T(20) = 60 + 28 e^{-20k} = 85$$

$$28 e^{-20k} = 85 - 60 = 25$$

$$e^{\frac{20}{28}} = \frac{25}{28} \approx$$

$$k = \frac{1}{20} \ln \frac{25}{28} = -0.00566643$$

$$-0.00566643t$$

$$T = 60 + 28 e$$

Now when was $T = 98.6$

$$-0.00566643t$$

$$60 + 28 e = 98.6$$

$$-0.00566643t = \frac{98.6 - 60}{28}$$

Solving for $t = -56.66$ minutes ago

or at 11:03 pm not 11:30 pm