International Monetary Equilibrium with Default*

M. Udara Peiris
ICEF, NRU Higher School of Economics, Moscow.

Dimitrios P. Tsomocos
Saïd Business School and St. Edmund Hall, Oxford.

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Abstract

We present an integrated framework for the study of the international financial economy with trade, fiat money, monetary and fiscal policy, endogenous default and regulation. Money is introduced via a cash-in-advance requirement and real trade is endogenous. The standard international finance pricing results obtain. Market incompleteness and positive default in equilibrium allow for the study of the transmission of default through the international financial markets and imply a positive role for policy. Finally, we present an example where, due to the trade-off between the non-pecuniary cost of default and the resulting allocation, a Pareto improvement occurs following an increase in interest rates.

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1 Introduction

The study of the international financial economy has traditionally followed two distinct strands[1]. In the finance literature, the focus has been on international risk sharing, such as International CAPM. However the asset structure has predominantly either been complete, in the Arrow-Debreu sense, or incomplete by considering only a single bond. The open-economy macroeconomics literature, on the other hand, has focused on real quantities and terms of trade effects and has largely ignored the portfolio choice problem[2]. In both strands the role that monetary and fiscal policy could play within a fully integrated monetary, financial and real sector has not been studied. Here, we attempt to present such a framework within the general equilibrium paradigm: International Monetary Equilibrium with Default (IMED).

Our model extends Geanakoplos and Tsomocos (2002) and Tsomocos (2008) to incomplete markets and default. The interaction between market incompleteness and default allows monetary policy to be non-neutral and non-trivial (the optimal interest rate is not zero, for example). It is consistent with the asset pricing flavour of Lucas (1982), however has several advantages over it. The requirement there that agents sell all of their endowment has several major shortcomings including specifying global transactions beforehand. We focus on the interaction between nominal and real variables by removing the requirement that agents sell all of their endowment. As a result, the financing constraint (ability to borrow money from a national monetary-fiscal authority at a positive interest rate) interacts with the cash-in-advance constraint, allowing monetary policy to have non-neutral effects.

Money is the stipulated medium of exchange in both goods and assets in IMED. Trade is facilitated by the monetary-fiscal authority offering loans before the commodity markets open and are repaid afterwards. Trade within

[1]See Pavlova and Rigobon (2010) for an excellent overview of recent work on international macro-finance.

[2]See Geanakoplos and Tsomocos (2002) for an extended discussion of this.

countries must occur in the fiat currency of that country. Foreign currency can be obtained via a foreign exchange market at market prices. Repayments are made by selling a fraction of their commodity endowments or by rolling over obligations to the future. Thus, the demand for money in our model stems from the immediate transactions need as well as intertemporal and speculative motives.

The ability to roll over monetary obligations implies an endogenous term structure that will be determined in equilibrium. Although the profit of the monetary-fiscal authority will always be the seigniorage revenue (since all fiat money exits the system in the end) different patterns of the term structure will have different consequences on trade and consumption and vice versa. Moreover, this results in the financing cost being an addition to the correlation between aggregate consumption and real asset payoffs in determining the risk-premia in asset prices as in Espinoza et al. (2009). This risk premia exist whenever the volume of trade is positive and is independent of aggregate uncertainty, unlike in representative agent models. Financing costs are generated within the framework of a monetary general equilibrium model, with cash-in-advance constraints built along the lines of (Dubey and Geanakoplos, 1992, 2003*b*,*a*, 2006), Espinoza and Tsomocos (2014), Geanakoplos and Tsomocos (2002), Goodhart et al. (2006) and Tsomocos (2008). Demand for money is endogenous in our model and depends on commodity prices, exchange rates, yield curve and asset prices in the world economy. Given the existence of private monetary endowments in each state and positive default, nominal determinacy is obtained[3]. In an international context, Tsomocos (2008) shows nominal determinacy under the presence of private liquid wealth contrary to the result of Kareken and Wallace (1981)[4].

In the IMED, agents sell contracts (both real and nominal), but need not honour their contractual obligation. As markets are incomplete, by changing the span and asset prices, default may be desirable in equilibrium and, furthermore, may be affected by monetary policy. In our model the decision to default is a decision variable as agents weigh the costs of a non-pecuniary (utility) punishment against the benefits of defaulting. In this we follow Shubik and Wilson (1977) and Dubey et al. (2005). We could have mod-

[3] Lin et al. (2014) show how default alone, without initial private monetary endowments, can support a determinate equilibrium in a monetary economy.

[4] See Woodford (1994) for the fiscal theory of price level. Bloise and Polemarchakis (2006) gives an overview on the non-neutrality of monetary policy and price level determinacy with non-Ricardian fiscal policy.

elled default using collateral such as in Geanakoplos (1997), Modica et al. (1998), Araujo and Páscoa (2002), Sabarwal (2003) and Geanakoplos and Zame (2007) [5]. We chose the former modelling method for two reasons. First, almost always international default is partial, see for example the current Eurozone sovereign debt crisis, and is associated with reputation costs that are reflected in higher sovereign bond yields and Credit Default Swap (CDS) spreads. Second, we endeavour to introduce welfare improvements by designing the appropriate monetary policy (as in the numerical example in Section 3), in the spirit of Dubey et al. (2005) which considers variations in default penalties.

New Open Economy Models (NOEM), inspired by Obstfeld and Rogoff (1995), study the transmission of domestic shocks via purchasing power parity and uncovered interest rate parity through the interaction of nominal rigidities and imperfect competition. In our model, markets are perfectly competitive and prices are fully flexible yet exchange rate movements play a similar role, though via a different channel. This is because monetary policy not only affects the price level, but also the interest rate and hence the liquidity available to facilitate the trading process. Expansionary monetary policy in IMED results in an increase in real exports by making export prices cheaper through depreciating the exchange rate, and also increases the real quantity of goods available for export by making them more liquid. In the numerical example presented in Section 3, we show that an increase in the nominal interest rate can Pareto improve because of the trade-off agents face between substituting between consumption and the non-pecuniary cost of default.[6].

Section 2 describes and defines IMED and the notion of a refined equilibrium whose existence we prove. In Section 3 we present a numerical example and describe the properties of IMED.

[5]Moreover, since we focus on competitive, mass, anonymous markets with perfect information we refrain from addressing issues of restricted participation as they relate with more realistic default penalties.

[6]Another numerical example of IMED in Peiris (2010) shows that expansionary monetary policy (lowering interest rates) can result in terms of trade moving away from the home country, lower long term yields domestically, and can transmit abroad resulting in higher leverage, and ultimately default, globally when interest rates eventually rise.

2 The Model

The monetary economy within each country of our model is as follows. There is a monetary-fiscal authority in each period who lends money to agents at an endogenously determined interest rate. All agents in our model can only borrow from the monetary-fiscal authority of their country of origin. Households (endowed with goods and fiat money) will, given reasonable money market interest rates, borrow from the monetary-fiscal authority in each period. All goods transactions occur in the currency of the country of their origin.

2.1 The International Monetary Economy

We consider an exchange economy which extends over two dates, $t \in T = \{0, 1\}$, with the second period having S possible states of nature which we index with $s \in S = \{1, .., S\}$. Including date 0, there are $S + 1$ date-events lying in the set $S^* := \{0, 1, .., S\}$. There are C countries indexed by $c \in C = \{1, 2, ..., C\}$ where trade occurs at prices denominated in the local currency[7]. At every $s \in S^*$ there are L perishable consumption goods in the world economy, indexed with $l \in L = \{1, ..., L\}$ and traded at domestic nominal spot prices p_{sl}. The price vector at state $s \in S^*$ is $p_s = (p_{s1}, ..., p_{sL}) \in \mathbb{R}^L_+$ and an overall price vector $p = (p_0, ..., p_s, ...p_S) \in \mathbb{R}^{(S+1)L}_{++}$. We also associate each commodity with a single country, and we write for example $l \in L^\alpha$[8]. That is, $l \in L = \bigcup_{\alpha \in C} L^\alpha$.

The nominal exchange rate is the value of a unit of currency $\forall \alpha, \beta \in C$ in terms of currency 1 and, for $s \in S^*$, is $\pi_s = (1, ..., \pi_{sC})$. The overall nominal exchange rate vector is $\pi = (\pi_0, ..., \pi_S) \in \mathbb{R}^{(S+1)C}_{++}$. We find it convenient to use the notation $\pi_{s\alpha\beta}$ to denote the α-currency value of a unit of β-currency and trivially $\pi_{s\alpha\beta} = \frac{\pi_{s\beta}}{\pi_{s\alpha}}$.

At $t = 0$ there are asset markets for $J \leq S - 1$ financial contracts indexed with $j \in J = \{1, .., J\}$. We associate each asset with a country and write $j \in J^\alpha$ with the entire set being $j \in J = \bigcup_{\alpha \in C} J^\alpha$. Each asset is a promise to deliver A_{sj} ($A_{sj} \geq 0$ and $\sum_s A_{sj} > 0$) units of domestic currency at a

[7]Where we generically denote a country $\alpha \in C$ we denote another country as $\beta \in C \backslash \alpha$. Note that countries will be synonymous with the location of a market.

[8]For the sake of simplicity, we claim there is a single type of good in the international economy but that is endowed in both countries and hence is characterised by the country of origin. For example the good may be cars but the cars in the UK would be British Cars and would be distinct from American Cars

price of ψ_j in the corresponding currency. The set of asset prices is $\psi = (\psi_1, ..., \psi_J) \in \mathbb{R}^J_{++}$. Sellers of the assets may choose to default and will incur a private marginal cost of default of $\lambda \in \mathbb{R}^J_{++}$. Finally as contracts are nominal, we require an index through which to obtain the real value of default. For $j \in J^\alpha$, we denote this by v_j which is an exogenously specified L dimensional vector with L^α dimensional vector of positive numbers and the remainder zeros, which, when multiplied by the price of goods at a date-event, gives the price index. Note that $v = \{v_1, ..., v_j\} \in \mathbb{R}^{JL}_+$. Consequently assets are defined by the vector (A, λ, v). The asset market is an anonymous market with promises between different sellers not allowed to be distinguished even though they may deliver differently. The possibility of default on assets means that the expected delivery rates, K, are macro variables taken as given by agents. All deliveries are pooled and buyers of the pool for each asset receive a pro rata share of the net deliveries. Each ownership share of the pool receives a fraction K_{sj} of the promised delivery A_{sj} for all $j \in J$. Formally $K_j = (K_{1j}, ..., K_{SJ}) \in [0, 1]^S$ is the vector of delivery rates for asset j and $K = (K_1, ..., K_J) \in [0, 1]^{SJ}$ is the overall set of delivery rates of financial contracts. The perfectly competitive and anonymous nature of asset markets means that creditors face both moral hazard and adverse selection as they are unable to induce debtors to honour their obligations fully nor can they identify relatively worse credit risks. The existence of sufficiently large non-pecuniary punishments for default means that the market does not fully unravel.

In addition to the financial contracts, there are long term (inter-period) bonds available to trade in each currency $\alpha \in C$ at $t = 0$ on which default is precluded[9] at a price $\frac{1}{1+\bar{r}_\alpha}$. The set of long-term interest rates is $\bar{r} = (\bar{r}_1, .., \bar{r}_C) \in \mathbb{R}^C_+$. Finally at each $s \in S^*$ and $c \in C$ there are short term (intraperiod) default-free bonds[10]. The set of short-term interest rates is $r_\alpha = (r_{0\alpha}, ..., r_{S\alpha}) \in \mathbb{R}^{S+1}_+$ with the overall set of short-term interest rates being $r = (r_1, ..., r_C) \in \mathbb{R}^{C(S+1)}_+$. Given this, we denote the macro variables that are determined in equilibrium, and that every agent regards as given, by $\eta = (p, \pi, \psi, r, \bar{r}, K)$.

[9] They can be considered long-term government securities.

[10] These can be considered money-market loans.

2.2 Households

The world economy has H agents, with each agent belonging to a subset of agents resident in each country. That is each agent $h \in H^{\alpha}$ belongs to country $\alpha \in C$, with $h \in H = U_{\alpha \in C} H^{\alpha}$ being set of agents in the international monetary economy.

Each country has its own currency that needs to be used to purchase goods from that country. Exchange of currency occurs only at the foreign exchange market. Agents are endowed with goods and/or money.

Each date event $s \in S^*$ is composed of four transaction moments. This is a consequence of cash-in-advance constraints being in place for the money, foreign exchange, goods, assets and bond markets. The money market opens in the first transaction moment and closes in the last transaction moment. In the first moment, the money market opens however agents may only lend in the money market. In the next moment money from any loans incurred in the money market is received, and the foreign exchange market opens though money may only be sold to the foreign exchange market. In the third moment income transmitted to foreign currencies is received and the goods, asset and intertemporal bond markets open. However only purchases of goods, assets and bonds can occur. In the fourth moment money is received from sales of goods, assets and bonds plus any income from deposits in the money market. This money can then be used to repay any loans in the money market. In the second period the timing is similar though repayments of assets and intertemporal bonds occur before revenue from purchases of assets and bonds are received.

Figure 1 indicates our time line, including the moments at that the various loans come due. We make the sequence precise when we formally describe the budget set.

For every household $h \in H$, define the (consumption good) endowment vector $e^h_s = (e^h_{s1}, ..., e^h_{sL}) \in \mathbb{R}^L_+$, and the overall (consumption good) endowment vector to be $e^h := (e^h_s)_{s \in S^*}$; the monetary endowments to be $m^h_s = (m^h_{s1}, ..., m^h_{sC}) \in \mathbb{R}^C_+$ and the overall monetary endowment vector to be $m^h := (m^h_s)_{s \in S^*}$. Similarly the vector of consumption is $x^h \in \mathbb{R}^{L \times S^*}_+$; vector of sales of goods is $q^h \in \mathbb{R}^{L \times S^*}_+$; vector of money offered to purchase goods and foreign exchange is $b^h \in \mathbb{R}^{L \times S^* + C(C-1) \times S^*}_+$, where b^h_{sl} denotes the money offered by h to purchase goods $l \in L$ while, for $(\alpha, \beta) \in C$, $b^h_{s\alpha\beta}$ is the α money offered by h to purchase β currency in state $s \in S^*$; the vector of deposits in the money market is $d^h \in \mathbb{R}^{S^* \times C}_+$; the vector of money owed

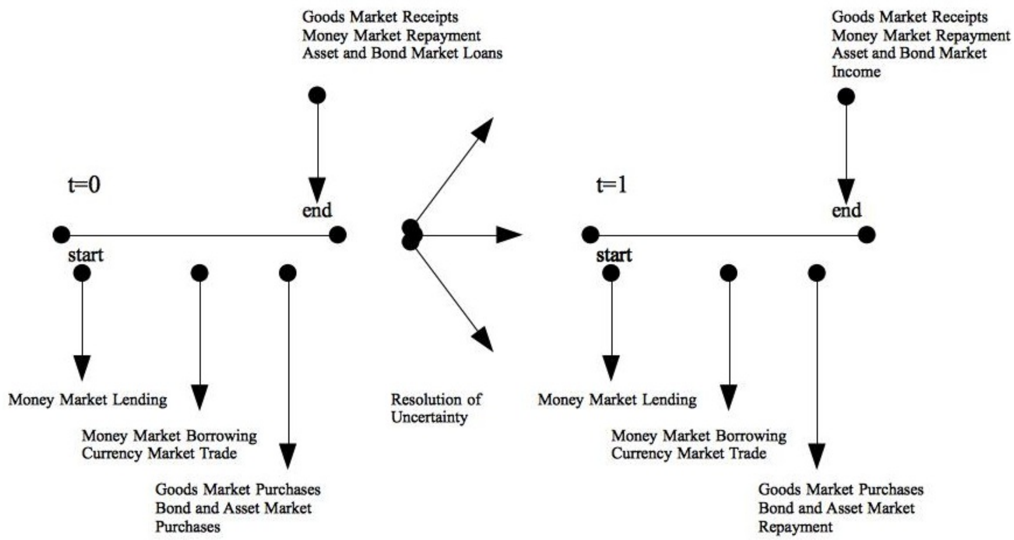

Figure 1: Time Structure

in the money market is $\mu^h \in \mathbb{R}_+^{S^* \times C}$; the vector of money spent purchasing bonds is $\bar{d}^h \in \mathbb{R}_+^C$; the vector of money owed in the bond market is $\bar{\mu}^h \in \mathbb{R}_+^C$; the vector of the monetary value of assets bought is $\theta^h \in \mathbb{R}_+^J$; the vector of the monetary value of assets sold is $\phi^h \in \mathbb{R}_+^J$; and the vector of deliveries (repayment) on assets sold is $D^h \in \mathbb{R}_+^{S \times J}$.

In the following, the Δ refers to any unused wealth from the corresponding budget constraint. For period 0 in country $\alpha \in C$:

$$(0\alpha 1) : d^h_{0\alpha} \leq m^h_{0\alpha}$$

$$(0\alpha 2) : \sum_{\beta \in C} b^h_{0\alpha\beta} \leq \Delta(0\alpha 1) + \frac{\mu^h_{0\alpha}}{1 + r_{0\alpha}}$$

$$(0\alpha 3) : \sum_{l \in L^\alpha} b^h_{0l} + \bar{d}^h_\alpha + \sum_{j \in J^\alpha} \theta^h_j \leq \Delta(0\alpha 2) + \sum_{\beta \in C} b^h{}_{0\beta\alpha}\pi_{0\beta\alpha}$$

$$(0\alpha 4) : \mu^h_{0\alpha} \leq \Delta(0\alpha 3) + \sum_{l \in L^\alpha} p_{0l} q^h_{0l} + (1 + r_{0\alpha}) d^h_{0\alpha} + \frac{\bar{\mu}^h_\alpha}{1 + \bar{r}_\alpha} + \sum_{j \in J^\alpha} \psi_j \phi^h_j$$

$$(0\alpha 5) : q^h_{0l} \leq e^h_{0l}$$

$$(0\alpha 6) : x^h_{0l} \leq \Delta(0\alpha 5) + \frac{b^h_{0l}}{p_{0l}}$$

Budget constraint $(0\alpha 1)$ says that deposits in money market ($d^h_{0\alpha}$) is at most equal to initial money endowments ($m^h_{0\alpha}$). $(0\alpha 2)$ says that money offered to currency markets ($b^h_{0\alpha\beta}$) is at most equal to unused wealth from $(0\alpha 1)$ + loans from money market and bond market (face-value of money-market debt is $\mu^h_{0\alpha}$. $(0\alpha 3)$ says that the money offered to goods market (b^h_{0l}) plus bond (purchases in long-term bond market is $\bar{d}^h_\alpha$) and asset market (money offered to purchase assets is θ^h_j) is at most equal to unused wealth from $(0\alpha 2)$ + money from currency markets ($b^h_{0\beta\alpha}$). $(0\alpha 4)$ says that repayment to money market + money carried over to the next period ($\hat{m}^h_{0\alpha}$) is at most equal to any remaining income from $(0\alpha 3)$ + income from goods sales (quantity of good l sold is q^h_{0l}+ income from money market deposits + sales in the bond market (face value of long-term debt is $\bar{\mu}^h_\alpha$)+ asset sales (quantity of assets sold is ϕ_j^h). In the second period for states $s \in S$ we have

$$(s\alpha 1): d_{s\alpha}^h \le \Delta(0\alpha 4) + m_{s\alpha}^h$$

$$(s\alpha 2): \sum_{\beta \in C} b_{s\alpha\beta}^h \le \Delta(s\alpha 1) + \frac{\mu_{s\alpha}^h}{1 + r_{s\alpha}}$$

$$(s\alpha 3): \sum_{l \in L^\alpha} b_{sl}^h + \bar{\mu}_\alpha^h + \sum_{j \in J^\alpha} D_{sj}^h \le \Delta(s\alpha 2) + \sum_{\beta \in C} {b^h}_{s\beta\alpha}\pi_{s\beta\alpha}$$

$$(s\alpha 4): \mu_{s\alpha}^h \le \Delta(s\alpha 3) + \sum_{l \in L^\alpha} p_{sl} q_{sl}^h + (1 + r_{s\alpha}) d_{s\alpha}^h + \bar{d}_\alpha^h (1 + \bar{r}_\alpha) + \sum_{j \in J^\alpha} K_{sj} A_{sj} \frac{\theta_j^h}{\psi_j}$$

$$(s\alpha 5): q_{sl}^h \le e_{sl}^h$$

$$(s\alpha 6): x_{sl}^h \le \Delta(s\alpha 5) + \frac{b_{sl}^h}{p_{sl}}$$

$(s\alpha 1)$ says that money carried over from date 0 $(\Delta(0\alpha 4))$+deposits in money market $(d_{s\alpha}^h)$ is at most equal to initial money endowments $(m_{s\alpha}^h)$. $(s\alpha 2)$ says that money offered to currency markets $(b_{s\alpha\beta}^h)$ is at most equal to unused wealth from $(s\alpha 1)$ + loans from money market and bond market (face-value of money-market debt is $\mu_{s\alpha}^h$). $(s\alpha 3)$ says that money offered to goods market (b_{sl}^h) plus repayment of long-term bonds $(\bar{\mu}_\alpha^h)$ and assets (D_{sj}^h) is at most equal to unused wealth from $(s\alpha 2)$ + money from currency markets $(b_{s\beta\alpha}^h)$. $(s\alpha 4)$ says that repayment to money market is at most equal to any remaining income from $(s\alpha 3)$ + income from goods sales (quantity of good l sold is q_{sl}^h+ income from money market deposits + income from long-term bonds maturing $(\bar{d}_\alpha^h(1 + \bar{r}_\alpha))$+ income from assets purchased $(K_{sj}A_{sj}\frac{\theta_j^h}{\psi_j})$. Finally $(0\alpha 5)$ and $(s\alpha 5)$ state that the quantity of goods sold must be less than the endowment, while $(0\alpha 6)$ and $(s\alpha 6)$ state that the quantity of goods consumed is less than any unsold goods plus new purchases of that good.

Denote the choices of agent h by $\sigma^h \in \Sigma^h(\eta)$, where

$$\sigma^h = (x^h; q^h; b^h; \tilde{\theta}^h; \tilde{\phi}^h; D^h)$$

and $\tilde{\theta}^h \equiv \{d^h, \bar{d}^h, \theta^h\}$ and $\tilde{\phi}^h \equiv \{\mu^h, \bar{\mu}^h, \phi^h\}$. Given this, the budget set $B^h(\eta)$ consists of choices $\sigma^h \in \Sigma^h$ satisfying (1-6) where (1-6) are the budget constraints presented earlier.

The payoff to h of the outcome is given by:

$$\Pi^h(x^h, \phi^h, D^h) = u^h(x^h) - \sum_{j \in J} \sum_{s \in S} \lambda_j \frac{max\{0, [A_{sj}\phi_j^h - D_{sj}^h]\}}{p_s \cdot v_j}.$$

where $u^h(x^h)$ is the utility of consumption and where the nominal disutility from defaulting on asset j obligations in state s is $\lambda_j max\{0, [A_{sj}\phi_j^h - D_{sj}^h]\}$ with $p_s \cdot v_j$ is the price deflator. $\Pi^h : \mathbb{R}_+^{S^* \times L} \times \mathbb{R}_+^J \times \mathbb{R}_+^{S \times J} \Rightarrow \mathbb{R}$.

2.3 Monetary-Fiscal authority

There is a combined monetary-fiscal authority in each country, $\forall \gamma \in C$, that has the authority to supply money in the money market and in exchange for bonds, and whose actions will be taken as exogenous. The (multi-currency) monetary endowments of agents (m^h) can be considered as either money balances carried over from an unmodeled previous period, or as fixed seigniorage transfers which ultimately constitute profits of the monetary-fiscal authority and correspond to the non-Ricardian fiscal regime present in each country. The operations of the monetary-fiscal authority are now specified.

- **Discount Window (intratemporal money market):** The monetary-fiscal authority supplies money to agents in each state by operating a discount window at the beginning of each state in which an amount of money is lent to agents (M_s^γ in state $s \in S^*$, and the vector of such monies is $M^\gamma = \{M_0^\gamma, ..., M_S^\gamma\}$) at an interest rate ($r_{s\gamma}$) and whose loan is due at the end of the state.

- **Open Market Operations (intertemporal money market):** At date 0, the monetary-fiscal authority is able to purchase domestic bonds by spending an amount of money $\bar{M}^\gamma$ in exchange for quantities of bonds at an endogenously determined intertemporal interest rate $\bar{r}_\gamma$[11].

The monetary-fiscal authority actions of country $\gamma \in C$ are given by the vector $(M^\gamma, \bar{M}^\gamma)$. That is, money lent in the money market and money spent in exchange for bonds.

[11] For the sake of minimising notation, we specify that central banks only purchase bonds, though nothing is lost in allowing them to sell bonds instead. In other applications it may be useful to also consider central bank purchases of foreign bonds as reserves, a feature which we also drop for simplicity.

2.4 Assumptions

2.4.1 Endowments

We make the following assumptions regarding endowments.

- **(A1).** $\forall s \in S^*$, $\forall \alpha \in C$ and $\forall l \in L$, $\sum_{h \in H} e^h_{sl} > 0$, $M^\alpha_s > 0$ and $\sum_{h \in H} m^h_{s\alpha} > 0$. That is, every commodity is present in the economy and there is a positive amount of both public (inside) and private (outside) money in each economy.

- **(A2).** $\forall h \in H$, $e^h_{sl} > 0$ and/or $m^h_{s\alpha} > 0$ for some $l \in L$, $s \in S^*$ and $\alpha \in C$. That is, no agent has the null endowment of both goods and private money in all states of nature.

- **(A3).** Let $\mathbb{A}$ be the maximum amount of any commodity sl that exists and let 1 denote the unit vector in $\mathbb{A}^{S^*L}$. Then

  $$\exists Q > 0 \text{ such that } u^h(0, ..., Q, ...0) > u^h(\mathbb{A}1)$$

  for Q in an ordinary component (i.e., concavity, continuity and strict monotonicity in every component). Also, continuity and concavity are assumed.

- **(A4).** $\forall j \in J$, $\lambda_j > 0$. That is, for each asset there is a positive cost of defaulting.

2.4.2 Gains-to-Trade Hypothesis

Since money is fiat it can only have value if it is actually used in trade and so we make the assumption that there exist sufficient gains from trade available to agents in the presence of interest payment (liquidity) costs. Specifically, we assume any allocation achievable without money must be far from Pareto efficient. Debreu (1951) introduced the coefficient of resource utilization to measure how far a given allocation is from Pareto-optimal. His measure identifies the fraction of the aggregate resources that can be given up while leaving behind enough to distribute so as to maintain the same utility levels as before. In contrast, in (Dubey and Geanakoplos, 1992, 2003*b*,*a*), Geanakoplos and Tsomocos (2002), Tsomocos (2003) and Tsomocos (2008) the maximum tax on traded resources that would still leave room for Pareto improvement is considered, rather than tax the aggregate resources, as in Debreu. We follow these papers in stating our Gains-to-Trade hypothesis.

Definition: Let $x^h \in \mathbb{R}_+^{S^* \times L}$ $\forall h \in H^\alpha, s \in S, \alpha \in C$. $\forall \delta_{s\alpha} > 0$, we will say that $(x^1, ..., x^H) \in \mathbb{R}_+^{S^* \times L \times H}$ permits at least $\delta_{s\alpha}$-gains-to-trade in state $s \in S$ and $\alpha \in C$ if there exist $\tau_s^1, ..., \tau_s^H$ in $\mathbb{R}^L$ such that:

$$\sum_{h \in H} \tau_s^h = 0$$

and

$$x_s^h + \tau_s^h \in \mathbb{R}_+^L, \quad \forall h \in H$$
$$u^h(\bar{x}^h(\delta_{s\alpha}, \tau_s^h)) > u^h(x^h), \quad \forall h \in H^\alpha, \alpha \in C$$

where for $h \in H^\alpha$

$$\bar{x}^h(\delta_{s\alpha}, \tau^h)_{tl} = \left\{ \begin{array}{ll} x_{tl}^h & t \in S^* \backslash \{s\} \\ x_{tl}^h + min\{\tau_{tl}^h, \tau_{tl}^h/(1+\delta_{s\alpha}), & \text{for } l \in L \text{ and } t = s \end{array} \right\}$$

Note that when $\delta_{s\alpha} > 0$,

$$\bar{x}^h(\delta_{s\alpha}, \tau_s^h)_{sl} < x_{sl}^h + \tau_{sl}^h,$$

if $\tau_{sl}^h > 0$ and

$$\bar{x}^h(\delta_{s\alpha}, \tau_s^h)_{sl} = x_{sl}^h + \tau_{sl}^h$$

if $\tau_{sl}^h \leq 0$.

Formally, the condition we impose on the world economy for sufficient gains from trade **(G to T)** is:

(G to T): For all $s \in S$ and $\alpha \in C$ the initial endowment $(e^h)_{h \in H}$ permits at least $\delta_{s\alpha}$ gains from trade where $\delta_{s\alpha} = \frac{\sum_{h \in H} m_{0\alpha}^h + \sum_{h \in H} m_{s\alpha}^h}{M_s^\alpha}$.

This corresponds to the highest possible interest rate in that state as M_s^α is lent by the government, and the total possible seigniorage profits is the sum of all private monetary endowments of that state and date 0. Note that the condition **(G to T)** needs to be valid $\forall s \in S$ but not for $s = 0$.

2.5 Equilibrium

We say that $(\eta, (\sigma^h)_{h \in H})$ is an **International Monetary Equilibrium with Default** and denote it IMED for the world economy $E = \left\{(u^h, e^h, m^h)_{h \in H}, \{A_j, \lambda_j, v_j\}_{j \in J}, (M^\gamma, \bar{M}^\gamma)_{\gamma \in C}\right\}$ if and only if $\forall s \in S$, $\forall s^* \in S^*$, $\forall \alpha, \beta \in C$, $\forall j \in J$ and $\forall h \in H$:

1. $(\sigma^h) \in \text{argmax}_{\sigma^h \in B(\eta)} \Pi(x^h, \phi^h, D^h)$
2. $p_{s^*l} \sum_{h \in H} q^h_{s^*l} = \sum_{h \in H} b^h_{s^*l}$
3. $\pi_{s^*\alpha\beta}(\sum_{h \in H} b^h_{s^*\alpha\beta}) = \sum_{h \in H} b^h_{s^*\beta\alpha}$
4. $\sum_{h \in H} \mu^h_{s^*\alpha} = (1 + r_{s^*\alpha}) \left[\sum_{h \in H} d^h_\alpha + M^\alpha_{s^*} \right]$
5. $\sum_{h \in H} \bar{\mu}^h = (1 + \bar{r}_\alpha) \left[\sum_{h \in H} \bar{d}^h_\alpha + \bar{M}^\alpha \right]$
6. $\psi_j \sum_{h \in H} \phi^h_j = \sum_{h \in H} \theta^h_j$
7. $K_{sj} = \left\{ \begin{array}{ll} \frac{\sum_{h \in H} D^h_{sj}}{\sum_{h \in H} A_{sj}\phi^h_j} & \text{if } \sum_{h \in H} A_{sj}\phi^h_j > 0 \\ \text{arbitrary} & \text{if } \sum_{h \in H} A_{sj}\phi^h_j = 0 \end{array} \right\}$

Condition 1 says that all agents optimise; 2 says that all commodity markets clear, or equivalently that price expectations are correct; 3 says that all currency markets clear, or equivalently, that currency forecasts are correct; 4 says that all money markets clear, or equivalently, that predictions of money market interest rates are correct; 5 says that all bond markets clear, or equivalently, that predictions of bond interest rates are correct; 6 states that asset markets clear while 7, together with the budget set, says that each potential buyer of an asset is correct in his expectation about the fraction of promises that get delivered.

Note that all transactions must be denominated in the currency of the country in which they occur. Conditions 2 - 7 come from the strategic market games literature and, in this context, can be seen as the price formation mechanism in this economy. They hold true for positive levels of trade and $0 < p_{s^*l} < \infty$, $0 < \pi_{s^*\alpha\beta} < \infty$, $0 \leq r_{s^*\alpha} < \infty$, $0 \leq \bar{r}_\alpha < \infty$ and $0 < \phi^h_j < \infty$ respectively.

2.6 Equilibrium Refinement

Condition 7 in Section 2.5 ensures that expected deliveries of assets, loans and deposits are equal to realized deliveries in equilibrium. A difficulty associated with allowing default in a rational expectations economy arises for untraded assets. Agents decide whether to buy or sell an asset depending on the price and delivery rates associated with the asset. Note that in section 2.5 in part 7 of the definition of equilibrium, the specification of expectations for inactive markets is arbitrary. Thus, the model does not rule out trivial equilibria in which there is no trade. Dubey and Shubik (1978) and Dubey et al. (2005) excluded trivial equilibria by adding an external agent (who can be considered the government) that sells and buys $\varkappa = (\varkappa_j)_{j \in J} \gg 0$ of every asset and never abrogates his contractual obligation. We follow this approach and then let $\varkappa \to 0$. Note that as all deliveries are made in units of currency[12], the external agent injects a monetary amount of $A_{sj}\varkappa$ and receives $K_{sj}A_{sj}\varkappa$, resulting in a net injection of money of $(1 - K_{sj})A_{sj}\varkappa$ for each asset in each state[13].

2.7 Refined Equilibrium

An equilibrium $(\eta, (\sigma^h)_{h \in H})$ obtained with the $\varkappa$-agent is called an *$\varkappa$-boosted equilibrium.* Thus any such

$$\eta = (p, \pi, \psi, r, \bar{r}, K)$$

and

$$\sigma^h = (x^h; q^h; b^h; \tilde{\theta}^h; \tilde{\phi}^h; D^h).$$

We say that $(\eta, (\sigma^h)_{h \in H})$ is an **$\varkappa$-boosted International Monetary Equilibrium with Default** and denote it IMED($\varkappa$) for the world economy $E = \{(u^h, e^h, m^h)_{h \in H}, \{A_j, \lambda_j, v_j\}_{j \in J}, (M^\gamma, \bar{M}^\gamma)_{\gamma \in C}\}$ $\forall s \in S$, $\forall s^* \in S^*$, $\forall \alpha, \beta \in C$, $\forall j \in J$ and $\forall h \in H$ if and only if:

1. $(\sigma^h) \in \mathrm{argmax}_{\sigma^h \in B(\eta)} \quad \Pi(x^h, \phi^h, D^h)$

2. $p_{s^*l} \sum_{h \in H} q^h_{s^*l} = \sum_{h \in H} b^h_{s^*l},$

3. $\pi_{s^*\alpha\beta} \sum_{h \in H} b^h_{s^*\alpha\beta} = \sum_{h \in H} b^h_{s^*\beta\alpha}(\varkappa)$

[12] Currency in which the assets are denominated in.

[13] For an extensive discussion of this equilibrium refinement and how it is related to Selten (1975), see Dubey et al. (2005).

4. $\frac{\sum\limits_{h\in H}\mu^h_{s^*\alpha}}{(1+r_{s^*\alpha})} = \left\{ \begin{array}{ll} \left[\sum\limits_{h\in H} d^h_{s^*\alpha} + M^\alpha_{s^*}\right] & \text{for } s^* = 0 \\ \left[\sum\limits_{h\in H} d^h_{s^*\alpha} + M^\alpha_{s^*} + \sum\limits_{j\in J^\alpha}(1-K_{s^*j}(\varkappa))A_{s^*j}\varkappa\right] & \text{for } s^* = \{1,...,S\} \end{array} \right\}$

5. $\sum\limits_{h\in H}\bar{\mu}^h = (1+\bar{r}_\alpha)\left[\sum\limits_{h\in H}\bar{d}^h_\alpha + \bar{M}^\alpha\right]$

6. $\psi_j \sum\limits_{h\in H}\phi^h_j = \sum\limits_{h\in H}\theta^h_j$

7. $K_{sj} = \left\{ \begin{array}{ll} \frac{A_{sj}\varkappa + \sum\limits_{h\in H} D^h_{sj}}{A_{sj}\varkappa + \sum\limits_{h\in H} A_{sj}\phi^h_j} & \text{if } A_{sj} > 0 \\ 1 & \text{if } A_{sj} = 0 \end{array} \right\}$

We now state the existence theorem:

Theorem 1

There always exists an IMED($\varkappa$) of $E = \left\{(u^h, e^h, m^h)_{h\in H}, \{A_j, \lambda_j, \upsilon_j\}_{j\in J}, (M^\gamma, \bar{M}^\gamma)_{\gamma\in C}\right\}$ provided

1. Gains-to-trade hypothesis holds
2. Assumptions **(A1)** - **(A4)** hold.

The proof of this theorem extends Geanakoplos and Tsomocos (2002) to the case of incomplete markets. Besides addressing the Hahn problem of the value of money in finite horizon economies, it also resolves the non-existence problem of Hahn (1965). Positive interest rates and bankruptcy penalties compactify the choice space by binding the potential transactions in the asset markets. Fiat money is an institutionalized symbol of trust and the monetary-fiscal authority compels its acceptance as a means of payment. Agents use fiat money because the benefit it gives them exceeds the interest rate loss. The existence argument relies not only upon the cash-in-advance constraint but also on the presence of the government and finally, the potential gains from trade.

3 Properties of IMED

Our IMED integrates monetary, financial and real sectors allowing for the study of the international transmission of both real and nominal domestic shocks. The total quantity of money is related to the nominal value of transactions (the quantity theory of money holds with endogenous velocity[14]), changes in nominal quantities affect real international trade (money is non-neutral) and there is a fully integrated term structure of interest rates (nominal quantities determine both date 0 and date 1 interest rates simultaneously). Standard results from finance such as martingale pricing (for bonds, assets and exchange rates[15]) hold, with endogenous nominal and real risk premia. Rather than presenting these features formally we present a numerical example of IMED in which these properties can be examined[16]. We first examine the properties of equilibrium, and then show that, once default is allowed, raising nominal interest rates can Pareto improve.

In Table 1 we present the data of a fictitious IMED economy comprised of two countries each inhabited by a continuum of agents endowed with distinct commodities in each country. Without loss of generality we remove the cash-in-advance restrictions on currencies, asset and bond markets. Specifically budget constraints are $\forall(\alpha, \beta) \in (1, 2)$ and let the good and asset in country

[14] Less than 1.

[15] i.e. Uncovered Interest Parity.

[16] The properties of such equilibria can be found in several related papers such as (Dubey and Geanakoplos, 2003*b*,*a*), Geanakoplos and Tsomocos (2002) and Tsomocos (2003).

α be denoted with α. The budget constraints for agent α is

$$
\begin{aligned}
&(0\alpha 1) : d^h_{0\alpha} + b^h_{0\alpha\beta} + b^h_{0\alpha} + \bar{d}^h_\alpha + \theta^h_\alpha \\
&\qquad\qquad \leq m^h_{0\alpha} + \frac{\mu^h_{0\alpha}}{1+r_{0\alpha}} + \frac{\bar{\mu}^h_\alpha}{1+\bar{r}_\alpha} + b^h{}_{0\beta\alpha}\pi_{0\beta\alpha} + \psi_\alpha \phi^h_\alpha \\
&(0\alpha 2) : \mu^h_{0\alpha} \leq \Delta(0\alpha 1) + p_{0\alpha} q^h_{0\alpha} + (1+r_{0\alpha}) d^h_{0\alpha} \\
&(0\alpha 3) : q^h_{0\alpha} \leq e^h_{0\alpha} \\
&(0\alpha 4) : x^h_{0\alpha} \leq \Delta(0\alpha 3) + \frac{b^h_{0\alpha}}{p_{0\alpha}} \\
&(s\alpha 1) : d^h_{s\alpha} + b^h_{s\alpha\beta} + b^h_{s\alpha} + \bar{\mu}^h_\alpha + D^h_{s\alpha} \\
&\qquad\qquad \leq \Delta(0\alpha 2) + m^h_{s\alpha} + \frac{\mu^h_{s\alpha}}{1+r_{s\alpha}} + b^h{}_{s\beta\alpha}\pi_{s\beta\alpha} + \bar{d}^h_\alpha(1+\bar{r}_\alpha) + K_{s\alpha} A_{s\alpha} \theta^h_\alpha / \psi_\alpha \\
&(s\alpha 2) : \mu^h_{s\alpha} \leq \Delta(s\alpha 1) + p_{s\alpha} q^h_{s\alpha} + (1+r_{s\alpha}) d^h_{s\alpha} \\
&(s\alpha 3) : q^h_{s\alpha} \leq e^h_{s\alpha} \\
&(s\alpha 4) : x^h_{s\alpha} \leq \Delta(s\alpha 3) + \frac{b^h_{s\alpha}}{p_{s\alpha}}.
\end{aligned}
$$

Households have von Neumann-Morgenstern expected utilities with constant relative risk aversion instantaneous utility functions and (expected) utility punishments for nominal default deflated by the price level in the country and date-event of the asset at which default occurs. For convenience, we fix the short term nominal interest rates in each country and date-event (as opposed to money supply), and we make them identical across countries. The monetary endowments (m) of the representative agents in each country are similar at date 0 but negatively correlated in the second period. Similarly, the real endowments (e) are negatively correlated in the second period though higher in the second country at date 0. Households in Country 1 believe that states 1-4 are increasingly likely to occur (π) while households in Country 2 believe states are equally likely to occur. In other respects the countries are identical, having the identical time discount rates (β), coefficient of relative risk aversion (ρ) and preference for goods (they are equally preferred). In each country there is an intertemporal (government) bond (nominally riskless and default-free) which agents and governments (of that country) can trade (the quantity of bonds governments purchase is $\bar{M}$). Additionally, there is a nominally riskless bond which (private) agents can trade in Country 1 but incurs a (marginal) non-pecuniary penalty (λ) if agents choose to default upon it[17].

We consider two equilibria, one where the marginal non-pecuniary cost of default is infinite for both agents (i.e. the equilibrium is default free), and one where the cost is what is reported in Table 1. The parameters with an asterix are 1000 times larger than what was used in the numerical simulation. π, m^*, e^*, ρ and β correspond to the values for beliefs of states, monetary endowment, goods endowment, coefficient of relative risk aversion and discount factor respectively, of the domestic agent in that country[18] while $\bar{M}$ and λ correspond to the trade of the domestic government in the domestic default-free bond and marginal punishment for the domestic private (defaultable) asset respectively.

	Country 1					Country 2				
s	0	1	2	3	4	0	1	2	3	4
π		0.10	0.20	0.30	0.40		0.25	0.25	0.25	0.25
m^*	5.75	5.6	7.2	8.8	10.4	5.25	12.0	9.6	4.0	2.4
e^*	60	70	90	110	130	120	150	120	50	30
r	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05
ρ	0.6					0.6				
β	0.95					0.95				
$\bar{M}$	0					0				
λ	10.0					∞				

Table 1: Parameters of Initial Equilibrium

17 The marginal cost for each agent is reported below.

18 They are only endowed with money and goods in their native country but have preference for goods in both countries. The goods are not perfect substitutes.

3.1 Equilibrium

The marginal punishment for default for the private asset in Country 1 (λ) is either 10 or infinite. When the punishment is infinite, default is ruled out and the private asset is collinear with the risk-free government bond in which case we assume there is no trade in the private asset. When the punishment for default is finite, then debtors would rather sell the private asset than short-sell the risk-free government security. As the monetary-fiscal authority of either country has no net position of the bond in that country, net trade in the risk-free government debt goes to zero (for that asset/country). In Table 2 below we present the equilibrium values for an economy with default on a privately traded asset and the numbers in brackets are the values in an economy without default.

3.1.1 Equilibrium Analysis

The General Equilibrium formulation of our model means that we can describe the standard national accounting results as well as standard pricing relationships. We will go through each of these now (values in brackets are for the equilibrium without default):

Real Exchange Rate and Purchasing Power Parity
As all goods are tradeable, and there are no impediments to trade, Absolute Purchasing Power Parity holds and the Real Exchange Rate is 1. However, the cost of liquidity (the positive short term interest rate) means that the effective cost of purchasing (importing) a unit of foreign goods is higher than the effective income from selling (exporting) a unit of domestic goods (or equivalently the effective opportunity cost of consuming a unit of domestic goods).

Fisher Equation
As markets are incomplete the Fisher equation holds with an additional endogenous risk premium term. The different beliefs that the two agents have about the expected rates of inflation motivate their risk-sharing incentives and asset trade: In Country 1, for domestic households the expected rate of inflation in Country 1 is .8% (2.4%) while Country 2 households expect -11.3% (-12.4%). In Country 2, Country 1 household expect the rate of inflation to be 80.2% (96.2%) while Country 2 households expect 107.7% (143.4%). Country 1 households borrow domestically (either through the private or government bond) and invest abroad (through the government

s	0	1	2	3	4
	Consumption/Trade in Goods				
$x_{s1}^2(q_{s1}^1)$	0.030 (0.029)	0.047 (0.053)	0.056 (0.058)	0.049 (0.049)	0.040 (0.037)
$x_{s2}^1(q_{s2}^2)$	0.056 (0.057)	0.045 (0.032)	0.040 (0.038)	0.026 (0.026)	0.020 (0.020)
	Price Levels				
p_{s1}	4.057 (4.141)	2.522 (2.213)	2.678 (2.609)	3.756 (3.774)	5.432 (5.909)
p_{s2}	1.983 (1.941)	5.624 (7.908)	5.006 (5.254)	3.280 (3.266)	2.565 (2.465)
	Exchange Rate				
π_{s21}	1.352 (1.409)	0.289 (0.182)	0.457 (0.425)	1.832 (1.848)	5.019 (5.666)

	Country 1	Country 2
	International Bond Trade	
$\left\{\bar{\mu}_1^1, \bar{d}_2^1\right\}$, $\left\{\bar{d}_1^2, \bar{\mu}_2^2\right\}$	0 (0.076), 0.024 (0.020)	0 (0.068), 0.021 (0.025)
	International Bond Interest Rate	
$\bar{r}_1$, $\bar{r}_2$	14.3% (11.7%)	5.0% (5.6%)
	International Asset Trade	
$\{\phi_2^1, \phi_1^1\}$, $\left\{\frac{\theta_1^2}{\psi_1}, \frac{\theta_2^2}{\psi_2}\right\}$	0.069 (0),0 (0)	0.069 (0) ,0 (0)
	International Asset Price	
ψ_1, ψ_2	0.799 (0)	0 (0)
	Delivery Rates on International Asset Trade for $s = \{1, 2, 3, 4\}$	
K_{s1}, K_{s2}	0.740 (1),1(1),1(1),1(1),1(1)	1(1),1(1),1(1),1(1),1(1)

Country 1 Agents	**Country 2 Agents**
Total Payoff	
2.729 (2.736)	2.839 (2.843)
Dead-weight cost of Default	
-0.007 (0)	0 (0)

Table 2: Variables of Equilibrium

bond there).

Uncovered Interest Parity

The nominal long-term (intertemporal) interest rate in Country 1 is higher than in Country 2. From Uncovered Interest Parity the nominal exchange rate in Country in Country 1 is expected (in a risk-neutral sense) to depreciate. The nominal exchange rate is 1.352 (1.409) at date 0 and ranges between 0.289 (0.182) and 5.019 (5.666). The risk neutral expectation of a depreciation (a higher number), reflects a positive correlation between the nominal exchange rate and a valid (though not unique) martingale measure.

National Accounts

Nominal National Income at each date-event in our example simplifies to the total money stock due to the assumption of a unitary velocity of money[19]. The endowments of nominal wealth/money balances agents have at the beginning of date-events will, in equilibrium, be the seigniorage revenue of the monetary-fiscal authorities at that date-event and determine the new money printed at the chosen nominal interest rate. As nominal short term interest rates are positive and the monetary-fiscal authorities have no net position in the long-term bond markets, there is no flow of money between periods.

The trade-balance at each date event is the difference between the domestic value of exports less the domestic value of imports. As all traded goods are exported in this example, we find that the trade balance for Country 1 at date 0 is -0.028 (-0.035) implying a net capital inflow into Country 1. As the distribution of risk across states in the second period is similar across countries (negatively correlated), the trade deficit in the example is driven by the relatively smaller real endowment of Country 1 at date 0.

The trade deficit at date 0 for Country 1 is partly financed by borrowing in the private bond market when the punishment for default is finite (allowed). Default occurs in State 1 in the second period, with a delivery rate of 74%. This results in not only a change in the span, but also a dead-weight cost borne by the residents of Country 1.

Comparing the welfare of the economies with and without default, we can see that the total payoff to the representative agents in both countries falls when default is allowed. This is because although the real terms of trade generally favours Country 1 in State 1 when default is allowed, it is offset by the private cost of default.

[19] A consequence of there only being a cash-in-advance constraint on goods.

3.2 Comparative Statics of an increase in r_{12}

Typically increasing nominal interest rates in our economy reduces the liquidity of endowments and makes trade, and the allocation more inefficient. However market incompleteness means that setting interest rates to zero (making endowments perfectly liquid) will not result in a Pareto efficient allocation. In Table 3 we consider the effect of *increasing* the nominal short term interest rate in State 1 in Country 2 and show that the trade-off between changes in the real terms of trade (allocation) and the dead-weight cost of default results in a Pareto improvement in the default equilibrium. We do this by increasing the nominal interest rate r_{12} by 1/100th of a percent 4 times then recording the direction of the change in the endogenous variables. We have chosen such a small change to ensure we are perturbing around the same equilibrium, and have done it gradually 4 times to see if the change is monotonic (which we found to be true). The signs reflect changes in variables. The sign in brackets is the change under the no-default equilibrium. Finally a 0 indicates no change.

A higher short term interest rate in State 1 in Country 2 reduces liquidity for domestic households, increasing the effective cost of purchasing imports. As a result domestic consumers switch consumption towards domestic goods, and we see a reduction in the quantity of goods exported. The relative scarcity of goods in Country 2 drives up their Country 1 price and Country 1 residents also reduce their demand for those goods in favour of Country 1 goods. In the equilibrium without default we can see that world trade has fallen at this date event as a consequence of a reduction in liquidity in Country 2. In the equilibrium where default is allowed, the representative agent of Country 1 not only chooses to switch consumption between home and foreign goods, but also between consumption and the dead-weight cost of default. Instead of switching towards domestic goods, they choose to use their income from exports to repay a greater real proportion of their debt. In equilibrium there is no change in the quantity exported, but the additional income is used to repay debt and we see the delivery rate, K_{11} increase.

The higher nominal interest rate also reduces the quantity of money, and the price level in Country 2 at that date-event, increasing the real payoff of the bond. Given the relatively larger endowment in Country 2 in this state, the risk sharing opportunities increase stimulating trade in the bonds. In Country 1, the higher repayment rate (and no-change in the price level), also results in a higher real payoff of the private asset stimulating trade there. In the no-default equilibrium, higher short-sales of the government bond by the Country 1 household is financed by greater trade and corresponding lower prices and real-payoffs of the bonds in all states except State 1.

In terms of the allocation, there is an increase in the date 0 nominal trade deficit in Country 1 reflecting higher consumption smoothing: they attempt to export more in the second period to finance the same level of imports. Overall the allocation favours Country 2 residents following the change in monetary policy, and in the no-default equilibrium so does welfare. However in the default equilibrium, the reduction in the dead-weight cost of default for Country 1 residents means that we find a Pareto improvement. This result is reminiscent of Goodhart et al. (2013) in which introducing a tax on capital flows at date 0 can Pareto improve. There, raising the tax reduces asset trade and default, while here it is lower liquidity through higher nominal interest rates that does so.

s	0	1	2	3	4
	Consumption/Trade in Goods				
x^2_{s1}/q^1_{s1}	- (-)	0 (-)	+ (+)	+ (+)	+ (+)
x^1_{s2}/q^2_{s2}	+ (+)	- (-)	- (-)	- (-)	- (-)
	Price Levels				
p_{s1}	+ (+)	0 (+)	- (-)	- (-)	- (-)
p_{s2}	- (-)	- (-)	+ (+)	+ (+)	+ (+)
	Exchange Rate				
π_{s21}	+ (+)	+ (+)	- (-)	- (-)	- (-)

	Country 1	Country 2
	International Bond Trade	
$\{\bar{\mu}^1_1, \bar{d}^1_2\}$, $\{\bar{d}^2_1, \bar{\mu}^2_2\}$	0 (+), + (+)	0 (+) , + (+)
	International Bond Interest Rate	
$\bar{r}_1$, $\bar{r}_2$	- (-)	- (-)
	International Asset Trade	
$\{\phi^1_1, \phi^1_2\}$, $\left\{\frac{\theta^2_1}{\psi_1}, \frac{\theta^2_2}{\psi_2}\right\}$	+ (0), 0 (0)	+ (0), 0 (0)
	International Asset Price	
ψ_1, ψ_2	+ (0)	0 (0)
	Delivery Rates on International Asset Trade for $s = \{1, 2, 3, 4\}$	
K_{s1}, K_{s2}	+ (0), 0 (0), 0 (0), 0 (0), 0 (0)	0 (0), 0 (0), 0 (0), 0 (0), 0 (0)

Country 1 Agent	**Country 2 Agent**
Total Payoff	
+ (-)	+ (+)
Dead-weight cost of Default	
- (0)	0 (0)

Table 3: Comparative Statics of an increase in r_{12}

4 Appendix

Proof of Theorem 1 It may help to describe the outline of the proof. For every $\varepsilon > 0$ we define an ε-IMED($\varkappa$) and show that it exists. An IMED($\varkappa$) is then obtained as a limit of ε-IMED($\varkappa$), $\varepsilon \to 0$.

An ε-IMED($\varkappa$) may be thought of as a Nash equilibrium of a generalized game. First, we utilise the continuum hypothesis by replacing each $h \in H$ by a continuum $(h-1, h]$ of identical players with every $t \in (h-1, h]$ having the characteristics $e^t \equiv e^h$, $\Pi^t \equiv \Pi^h$. The time structure of the model and the corresponding budget constraints determine how each $h \in (0, H]$ move at each stage. The multiple budget constraints indicate the sequence of the moves. At each stage all moves are simultaneous. All past prices are observed and there is an external agent who behaves as a strategic dummy by offering ε units on each side of every market. Put differently the external agent does not posses an objective function and he just participates in every market with an ε bid and an ε offer. An ε-IMED($\varkappa$) will thus correspond to a type-symmetric strategic equilibrium of the generalized game that we have described where all players $(h-1, h]$ employ the same strategy. Now we begin the proof formally. Let $M^*_\alpha \equiv \sum\limits_{s\in S^*} M^\alpha_s + \bar{M}^\alpha + \sum\limits_{h\in H, s\in S^*} m^h_{s\alpha} + \sum\limits_{s\in S, j\in J^\alpha} A_{sj}\varkappa$ be the total quantity of α money appearing in the economy of each country[20]. For each $h \in H^\alpha$, $j \in J^\alpha$, $\alpha \in C$ and $\varepsilon > 0$ let

$$\begin{aligned}
\Sigma^h_\varepsilon = \Big\{ & (x^h, q^h, b^h, \mu^h, \bar{\mu}^h, d^h, \bar{d}^h, \theta^h, \phi^h, D^h_j) \in \mathbb{R}^{L\times S^*}_+ \times \mathbb{R}^{L\times S^*}_+ \times \mathbb{R}^{L\times S^* + C(C-1)\times S^*}_+ \\
& \times \mathbb{R}^J_+ \times \mathbb{R}^J_+ \times \mathbb{R}^{(S^*\times(L+1))\times J}_+ : 0 \le x^h \le 2\mathbb{A}1, \varepsilon \le q^h_{sl} \le \frac{1}{\varepsilon}, \varepsilon m^h_{s\alpha} \le b^h_{sl} \le \frac{1}{\varepsilon}, \\
& \varepsilon m^h_{s\alpha} \le \mu^h_{s\alpha} \le \frac{1}{\varepsilon}, \varepsilon m^h_{0\alpha} \le \bar{\mu}^h_{s\alpha} \le \frac{1}{\varepsilon}, 0 \le d^h_{s\alpha} \le M^*_\alpha, 0 \le \bar{d}^h_\alpha \le M^*_\alpha, \varepsilon m^h_{s\alpha} \le \theta^h_j \le \frac{1}{\varepsilon}, \\
& \varepsilon \le \phi^h_j \le \frac{1}{\varepsilon}, \varepsilon m^h_{s\alpha} \le D^h_{sj} \le \frac{1}{\varepsilon} \Big\},
\end{aligned}$$

that is both compact and convex.

Let the typical element of Σ^h_ε be $\sigma^h \in \Sigma^h_\varepsilon$. Define $B^h_\varepsilon(\eta) = B^h(\eta) \cap \Sigma^h_\varepsilon$. Also let $\sigma = (\sigma^1, ..., \sigma^H) \in \Sigma_\varepsilon = \mathsf{X}_{h\in H}\Sigma^h_\varepsilon$. Define the map $\Psi_\varepsilon : \Sigma_\varepsilon \to N$, where

[20] $\sum\limits_{j\in J^\alpha} A_{sj}\varkappa$ is the maximum amount injected by the "external agent" who guarantees full delivery on every asset.

$$N = \left\{ \eta = (p, \pi, r, \bar{r}, \psi, K) \in \mathbb{R}^{LS^*}_{++} \times \mathbb{R}^{C(C-1)\times S^*}_{++} \times \mathbb{R}^{C\times S^*}_{++} \times \mathbb{R}^{C}_{++} \times \mathbb{R}^{J}_{++} \times \mathbb{R}^{S^*\times J}_{++} \right\}$$

and Ψ is defined by equations 2-7 of 2.7. In addition define (η, σ) to be an ε-IMED($\varkappa$) if $\eta = \Psi_\varepsilon(\sigma)$ and $\sigma^h \in argmax_{\sigma^h \in B^h_\varepsilon(\eta)} \Pi^h(x^h(\sigma^h))$. Note also that all elements of $\Psi_\varepsilon(\sigma) = \eta$ are continuous functions of σ, since in each market some agents are bidding (offering) strictly positive amounts and repayments are bounded away from 0 by the presence of the $\varkappa$ asset trades by the external agent.

Furthermore, define

$$G : N \rightrightarrows \mathsf{X}_{h\in H} \Sigma^h_\varepsilon = \Sigma_\varepsilon, \text{where}$$
$$G^h = \sigma^h \in \underset{\sigma^h \in B^h_\varepsilon(\eta)}{argmax}\, \Pi^h(x^h(\sigma^h)) \text{ and } G = \mathsf{X}_{h\in H} G^h.$$

Finally, let $F = G \circ \Psi : \Sigma_\varepsilon \rightrightarrows \Sigma_\varepsilon$. G is convex-valued since $\sigma \to u^h(x^h(\sigma^h))$ is concave. Recall, $\sigma^h \to x^h(\sigma^h)$ is linear, and that $B^h_\varepsilon(\eta)$ is convex. Since Ψ is a function, $F = G \circ \Psi$ is also convex valued. Moreover, if ε is sufficiently small, G is non-empty, since $m^h_{s\alpha} > 0, \forall h \in H^\alpha$ and $\alpha \in C$. When $\varepsilon > 0$, $p, \pi, r, \bar{r}, \psi, K > 0$, and since $e^h \neq 0$, $B^h_\varepsilon(\eta)$ for $h \in H$ is a continuous correspondence. Hence, by the Maximum Theorem, G is compact-valued and upper semicontinuous, and therefore so is F. Note that since we have restricted the domain of Ψ to Σ_ε and since for each good and money, some $h \in H$ has a strictly positive endowment, the restriction Ψ to strictly positive prices, and interest rates strictly greater than -1 is legitimate. The same applies for K's since an external agent always guarantees a minimum repayment $\varkappa > 0$.

Step 1: An ε-IMED($\varkappa$) exists for any sufficiently small $\varepsilon > 0$.

Proof. The map F satisfies all the conditions of the Kakutani fixed point theorem, and therefore admits a fixed point $F(\sigma)$ such that σ satisfies 1-6 of 2.5 for an ε-IMED($\varkappa$). □

For every small $\varepsilon > 0$, let $(\eta(\varepsilon), \sigma(\varepsilon))$ denote the corresponding ε-IMED($\varkappa$).

Step 2: At any ε-IMED($\varkappa$), $r_{s\alpha}(\varepsilon), \bar{r}_\alpha(\varepsilon) \geq 0, \forall s \in S^*$ and $\alpha \in C$.

Proof. if $r_{s\alpha} < 0$ or $\bar{r}_\alpha < 0$ for any $s \in S^*$ and $\alpha \in C$, then agents could borrow $\epsilon/(1 + r_{s\alpha}) > \epsilon$ at the beginning of the period and repay ϵ making a profit. The same argument applies for $\bar{r}_\alpha$ as money is durable. □

Step 3: At any ε-IMED($\varkappa$) $\exists I, Z < \infty$ such that $r_{s\alpha}(\varepsilon), \bar{r}_\alpha(\varepsilon) < I$ and $\bar{\mu}^h_\alpha(\varepsilon), \mu^h_{s\alpha}(\varepsilon) \leq Z, s \in S^*, h \in H^\alpha, \alpha \in C$.

Proof. Suppose that $r_{s\alpha}(\varepsilon) \to \infty$. Then $\exists h \in H^\alpha$ such that $\mu^h_{s\alpha}(\varepsilon) \to \infty$ and consequently $\mu^h_{s\alpha} - M^*_\alpha \to \infty$. However the maximum amount of money available to repay $\mu^h_{s\alpha}$ is M^*_α, a contradiction. A similar argument applies for $\bar{r}_\alpha(\varepsilon), \bar{\mu}^h_\alpha(\varepsilon)$. □

Step 4: For any ε-IMED($\varkappa$) $\exists \tilde{c} > 0$ such that $p_{sl}(\varepsilon) > \tilde{c}$, $\pi_{s\alpha\beta}(\varepsilon) > \tilde{c}$, $\forall s \in S^*, l \in L^\alpha, \alpha, \beta \in C$.

Proof. Suppose that $p_{sl}(\varepsilon) \to 0$ for some $s \in S^*$, $l \in L^\alpha$. Then choose $h \in H^\alpha$. He could have borrowed Δ more to buy $\Delta/p_{sl}(\varepsilon) \to \infty$. His net gain in utility would be $\frac{\nabla\Pi^h_{sl}}{p_{sl}} > 0$ and by (A3), $\Pi^h(0, ..., Q, ..., 0) > u^h(\mathbb{A}1)$ with Q in the s^*lth place. Thus, $p_{sl}\ \nabla\Pi^h_{sl} > \tilde{c} > 0$.

Similarly an agent from country $\beta \neq \alpha$ with $m^h_{0\beta} > 0$ ($m^h_{s\beta} > 0$) can purchase $m^h_{0\beta}/(\pi_{0\beta\alpha}p_{sl})$ ($m^h_{s\beta}/(\pi_{s\beta\alpha}p_{sl})$) units of good sl for $s \in S$ and $l \in L^\alpha$ by hoarding his β-money, exchanging it for α-money and then purchasing good l. If $\pi_{s\beta\alpha}p_{sl} \to 0$ then he could purchase an infinite amount of good sl, contradicting the fact that we are at an ε-IMED($\varkappa$). Thus $\pi_{s\beta\alpha}p_{sl} > \tilde{c}$. As $p_{sl} > \tilde{c}$ then $\pi_{s\beta\alpha} > \tilde{c}$. □

Step 5: For any ε-IMED($\varkappa$) $\exists \Gamma$ such that $\phi^h_j(\varepsilon) < \Gamma, \forall l \in L^\alpha, j \in J^\alpha, h \in H, \alpha \in C$.

Proof. Suppose that for some $j \in J^\alpha$, $\phi^h_j(\varepsilon) \to \infty$ and $A_{sj} > 0$. Then h can deliver at most $\phi^h_j A_{sj} \leq M^*_\alpha$, and therefore his disutility from default would be $\lambda_j(\phi^h_j A_{sj} - M^*_\alpha)/(p_s\nu_s) < u^h(\mathbb{A}1)$. Otherwise, $\sigma^h(\varepsilon)$ are not optimal. □

Step 6: For all $h \in H, d^h_s, \bar{d}^h, b^h_{sl}, \theta^h_j, u^h \leq 2M^*_\alpha$ and $0 < K \leq 1$.

Proof. From the cash-in-advance specification for the money market, bonds, assets and goods, all variables are constrained by the total amount of money present in the economy. K is given by

$$K_{sj} = \left\{ \begin{array}{ll} \frac{A_{sj}\varkappa + \sum\limits_{h \in H} D^h_{sj}}{A_{sj}\varkappa + \sum\limits_{h \in H} A_{sj}\phi^h_j} & \text{if } A_{sj} > 0 \\ 1 & \text{if } A_{sj} = 0 \end{array} \right\}$$

□

Step 7: For all $h \in H^\alpha$, $\sigma_\varepsilon^h = \underset{\sigma_\varepsilon^h \in B^h(\eta(\varepsilon))}{\arg\max}\ \Pi^h(x^h(\sigma^h))$, for sufficiently small $\varepsilon > 0$.

Proof. From steps 2-6 and the budget constraints ($0\alpha5$) and ($s\alpha5$) of 2.2, the ε-constraint is not binding thus concavity of payoffs guarantees the optimality of $\sigma^h(\varepsilon)$. □

IMED(η, σ) will be constructed by taking the limit of ε-IMED$(\varkappa)$ $(\eta(\varepsilon), \sigma(\varepsilon))$, as $\varepsilon \to 0$. This is achieved by taking limits of sequences of ε and subsequences of subsequences.

Step 8: If for some $\bar{s}\bar{l}, p_{\bar{s}\bar{l}} \to \infty$ then $p_{sl}(\varepsilon) \to \infty$ $\forall \bar{s}, s \in S, \bar{l}, l \in L^\alpha, j \in J^\alpha, \alpha \in C$. Also if either $\psi_j(\varepsilon) \to \infty$ or $p_{0l}(\varepsilon) \to \infty$ then $p_{sl}(\varepsilon) \to \infty$ $\forall l \in L^\alpha, j \in J^\alpha, \alpha \in C$, $s \in S^*$.

Proof. Some h owns $e^h_{\bar{s}\bar{l}} > 0$. If $p_{sl}(\varepsilon)$ stays bounded on some subsequence, then by borrowing very large $\bar{\mu}^h_\alpha$ or $\mu^h_{0\alpha}$ if $s = 0$, h can use it to buy Q units of sl. Then since $\bar{r}_\alpha(\varepsilon)$, $r_{s\alpha}(\varepsilon) < I$, h can sell Δ of $\bar{s}\bar{l}$ acquire $\Delta p_{\bar{s}\bar{l}}(\varepsilon) \to \infty$ to defray his loan and improve his utility, a contradiction.

If $\theta_j(\varepsilon) \to \infty$ and $p_{sl}(\varepsilon) < \infty$, let h borrow $\Delta\theta_j(\varepsilon)/(1 + r_{0\alpha}(\varepsilon))$ and buy $\Delta\phi^j(\varepsilon)/(1 + r_{0\alpha}(\varepsilon))p_{sl}(\varepsilon)$ of sl and improve his utility. If $p_{0\bar{l}}(\varepsilon) \to \infty$, as previously argued then $p_{0l}(\varepsilon) \to \infty$,$\forall l, \bar{l} \in L^\alpha$. Then, by selling Δ of $o\bar{l}$ h can acquire $\Delta p_{0l}(\varepsilon) \to \infty$. If any of $p_{sl}(\varepsilon) \nrightarrow \infty$, $s \in S$ then by inventorying money he can improve upon his utility. □

Step 9: $\exists \nu > 0$ such that $p_{sl}(\varepsilon)/p_{sk}(\varepsilon) < \nu$, $\forall l, k \in L^\alpha$, $l \neq k$, $s \in S$, $\alpha \in C$.

Proof. Suppose the opposite. Then take h with $e^h_{sl} > 0$. Let him reduce Δ of his sales of sl and lose $\Delta(\Pi^h(\mathbb{A}1) - \Pi^h(0))$ at most. Then he could buy more of sk by borrowing $p_{sl}(\varepsilon)/(1 + r_{s\alpha}(\varepsilon))$ and sell Δ of sl. His net gain in utility would be

$$\Delta(\varepsilon)\left\{\frac{p_{sl}(\varepsilon)}{(1 + r_s(\varepsilon)p_{sk}(\varepsilon)}\left(\nabla\Pi^h_{sk}(x^h)\right) - (\Pi(\mathbb{A}1) - \Pi(0)\right\} > 0$$

since $p_{sl}(\varepsilon)/p_{sk}(\varepsilon) \to \infty$ and by step 3, $r_{s\alpha}(\varepsilon) < I$. □

Step 10: $\exists \nu' > 0$ such that $p_{0l}(\varepsilon)/p_{sk}(\varepsilon) < \nu'$,$\forall s \in S^*, l, k \in L^\alpha, l \neq k, \alpha \in C$.

Proof. If $s = 0$ then step 9 obtains. Otherwise, set $\Delta(0\alpha 4)$ of Section 2.2 equal to $\Delta p_{sl}(\varepsilon)/(1+r_{s\alpha}(\varepsilon))$. $\square$

Step 11: $\psi_j(\varepsilon)/\sum_{l\in L^\alpha} p_{0l}(\varepsilon) \nrightarrow \infty, \forall j \in J^\alpha, l \in L^\alpha, \alpha \in C.$

Proof. Suppose the contrary. Let h sell $\frac{\Delta}{(1+\bar{r}_\alpha(\varepsilon))}$ of j and borrow $\frac{\Delta\psi_j(\varepsilon)}{(1+\bar{r}_\alpha(\varepsilon))}$ more. Let him consume $\frac{\Delta\psi_j}{(1+\bar{r}_\alpha(\varepsilon))p_{0l}(\varepsilon)}$ more of some $l \in L^\alpha$ in $s = 0$. Then h can use the proceeds of the asset sale to defray the loan. His net gain of this action will be $\Delta\left(\frac{\psi_j}{(1+\bar{r}_\alpha(\varepsilon)p_{0l}} - \frac{A_{sj}}{(1+\bar{r}_\alpha(\varepsilon)}\right) > 0$ since $\frac{\psi_j(\varepsilon)}{p_{0l}(\varepsilon)} \to \infty$. $\square$

Step 12: If $\psi_j(\varepsilon)\Big/\sum_{l\in L^\alpha} p_{0l}(\varepsilon) \to 0$ then $A_{sj} = 0$, and $\sum_{l\in L^\alpha} p_{0l}(\varepsilon) \to \infty$, whenever $A^j_{sm} > 0, \forall s \in S, j \in J^\alpha, l \in L^\alpha, \alpha \in C$.

Proof. Suppose $A_{sj} > 0$ for some $s \in S$, $l \in L$. Choose $h \in H^\alpha$ with $e^h_{0l} > 0$ for some $l \in L^\alpha$. Let h sell $\frac{\Delta}{(1+\bar{r}_\alpha(\varepsilon))}$ more of $0l$ and increase his loan by $\left(\frac{\Delta}{(1+\bar{r}_\alpha(\varepsilon))}\right) p_{0l}$. Then he could purchase $\frac{\Delta p_{0l}}{(1+\bar{r}_\alpha(\varepsilon))(1+r_{0\alpha}(\varepsilon))\psi_j(\varepsilon)}$ of j. Then, by borrowing in s and defraying his loan by asset deliveries he can improve his payoff, a contradiction. The same argument applies if $A_{sj} > 0$ and $\sum_{l\in L^\alpha} p_{0l}(\varepsilon) \nrightarrow \infty$. $\square$

Step 13: There exists $\tilde{\imath}^*$ such that $p_{sl}(\varepsilon) < \tilde{\imath}^*$ $\forall s \in S^*, l \in L^\alpha, \alpha \in C$.

Proof. Suppose the contrary and w.l.o.g. suppose that $p_{\bar{s}\bar{l}} \to \infty$ for some $\bar{s}, s \in S^*, \bar{l}, l \in L^\alpha, \alpha \in C$. Since $p_{sl}(\varepsilon) = \frac{\sum_{h\in H} b^h_{sl}(\varepsilon)}{\sum_{h\in H} q^h_{sl}(\varepsilon)} \leq \frac{M^*_\alpha}{\sum_{h\in H} q^h_{sl}(\varepsilon)}$, it must necessarily be that $q^h_{sl}\to 0$ as $\varepsilon \to 0$ for all $s \in S^*$, $l \in L$ by step 8.

At any ε-IMED($\varkappa$), $\bar{r}_\alpha(\varepsilon)r_{s\alpha}(\varepsilon) < \delta_s$ by step 3. Hence, at any ε-IMED($\varkappa$), there are less than δ_s-gains from trade. By continuity, there are less than δ_s-gains from trade at $(e^h)_{h\in H}$. However, **G to T** hypothesis guarantees that there are more than δ_s-gains from trade $\forall s \in S$, a contradiction. $\square$

Step 14: There are $0 < \tilde{c} < \tilde{c}^*$ such that the exchange rates are bounded:

$$\tilde{c} < \pi_{s\alpha\beta}(\varepsilon) < \tilde{c}^* \quad \forall s \in S^*, \alpha, \beta \in C.$$

Proof. We showed in Step 4 that $\pi_{s\beta\alpha}(\varepsilon) > \tilde{c}$ $\forall \alpha, \beta \in C$. But $\pi_{s\beta\alpha}(\varepsilon) = \pi^{-1}_{s\alpha\beta}(\varepsilon)$, hence $\pi_{s\beta\alpha}(\varepsilon)$ is bounded from above. $\square$

Step 15: $\eta = \lim_{\varepsilon \to 0} \eta(\varepsilon)$ and $\lim_{\varepsilon \to 0} (\eta(\varepsilon), \sigma(\varepsilon)) = (\eta, \sigma)$.

Proof. From the previous steps, $\eta(\varepsilon)$ is bounded in all components. The same applies for $\sigma(\varepsilon)$. Thus, a convergent subsequence can be selected that obtains (η, σ) in the limit. By continuity of $\Pi^h(\sigma^h)$, (η, σ) is an IMED($\varkappa$), and the artificial upper and lower bounds on choices are irrelevant since they are not binding and payoff functions are concave in actions. $\square$

Thus we have shown that (η, σ) is an IMED($\varkappa$). Letting $\varkappa \to 0$ and taking limits we obtain a refined equilibrium, and proves the theorem.

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