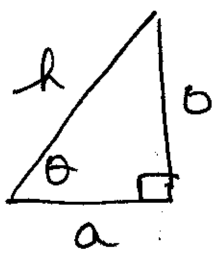
Math 1496 - Calc I

Review – Trig



o – opposite
a – adjacent
h – hypotenuse

Six Trig Functions

$\sin\theta = \frac{o}{h}$, $\cos\theta = \frac{a}{h}$, $\tan\theta = \frac{o}{a}$

$\csc\theta = \frac{h}{o}$ $\sec\theta = \frac{h}{a}$ $\cot\theta = \frac{a}{o}$

In the 1st Quadrant

θ		$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\pi/3$	$\pi/2$
$\sin\theta$	$\sqrt{\frac{0}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{4}{4}}$
	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞ (undefined)

Trig Identities

$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$

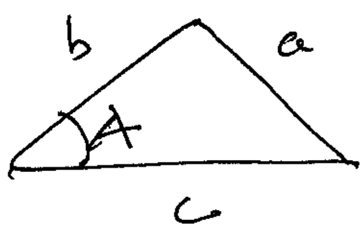
Double Angle Formula's

$\sin 2\theta = 2\sin\theta\cos\theta$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Law of Cosines



$a^2 = b^2 + c^2 - 2bc\cos A$

Even / Odd Identities

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$

$\sec(-\theta) = \sec\theta$

$\cot(-\theta) = -\cot\theta$

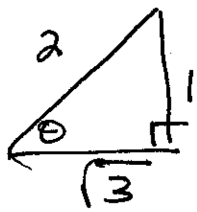
Sum Difference

$\sin(a \pm b) = \sin a\cos b \pm \sin b\cos a$

$\cos(a \pm b) = \cos a\cos b \mp \sin a\sin b$

Ex If $\sin\theta = \frac{1}{2}$ for $0 \le \theta \le \pi/2$

Find all trig function



$\sin\theta = \frac{o}{h}$ by Pythagorean Thm $a^2 + b^2 = c^2$

$\cos\theta = \frac{a}{h} = \frac{\sqrt{3}}{2}$ $\tan\theta = \frac{o}{a} = \frac{1}{\sqrt{3}}$

$\sec\theta = \frac{2}{\sqrt{3}}$ $\cot\theta = \sqrt{3}$ $\csc\theta = 2$

Note $\cot\theta = \frac{1}{\tan\theta}$, $\sec\theta = \frac{1}{\cancel{\sin\theta}\ \cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

Ex Find all values of θ where

$\cos^2\theta + \sin\theta = 1$ (pg 39 # 41)

1st $\cos^2\theta = 1 - \sin^2\theta$ so $\sin\theta = 0$

so $\cancel{1} - \sin^2\theta + \sin\theta = \cancel{1}$ or $\sin\theta = 1$

$-\sin\theta(\sin\theta - 1) = 0$ right away

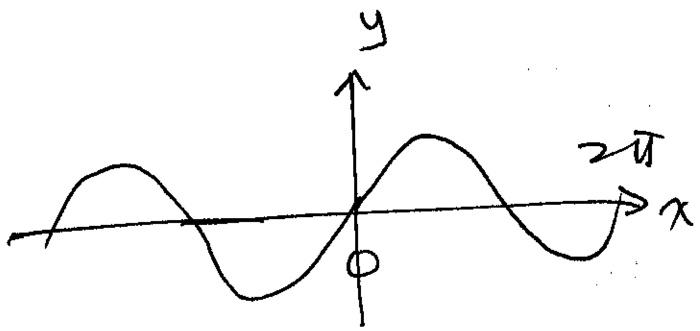
we know $\theta = 0, \pi/2$

but are there others?

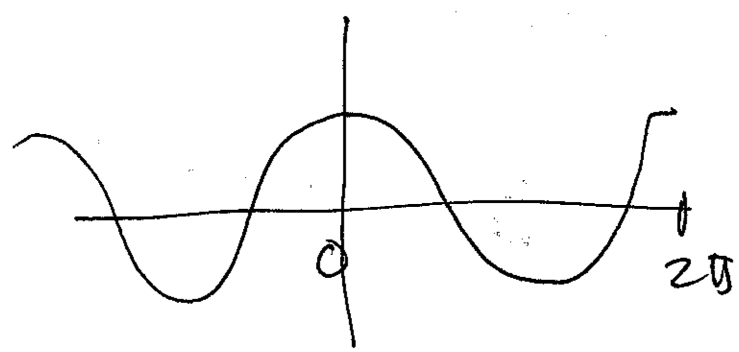
Trig Functions & Their Graphs

So we consider

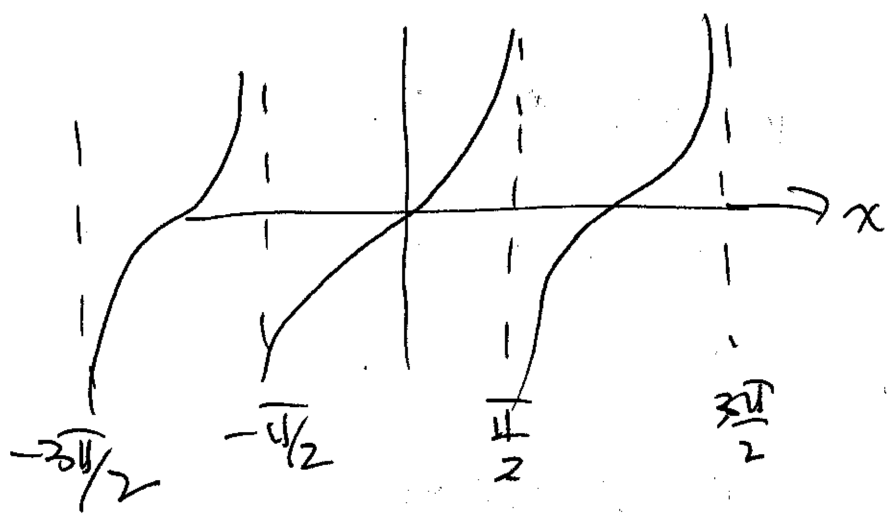
$f(x) = \sin x \qquad f(x) = \cos x \qquad f(x) = \tan x$



$y = \sin x$



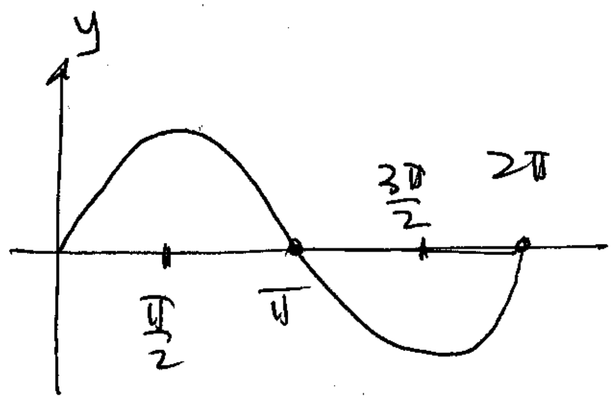
$y = \cos x$



$y = \tan x$

so to finish answer question

consider $y = \sin x$ on $[0, 2\pi]$

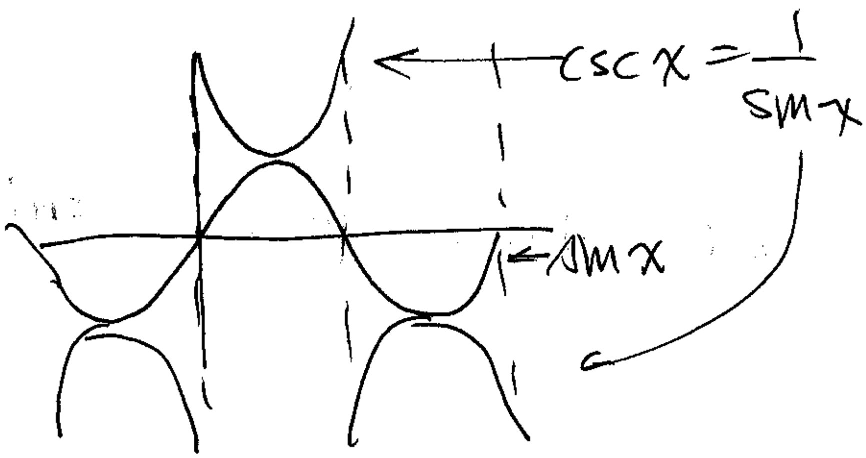


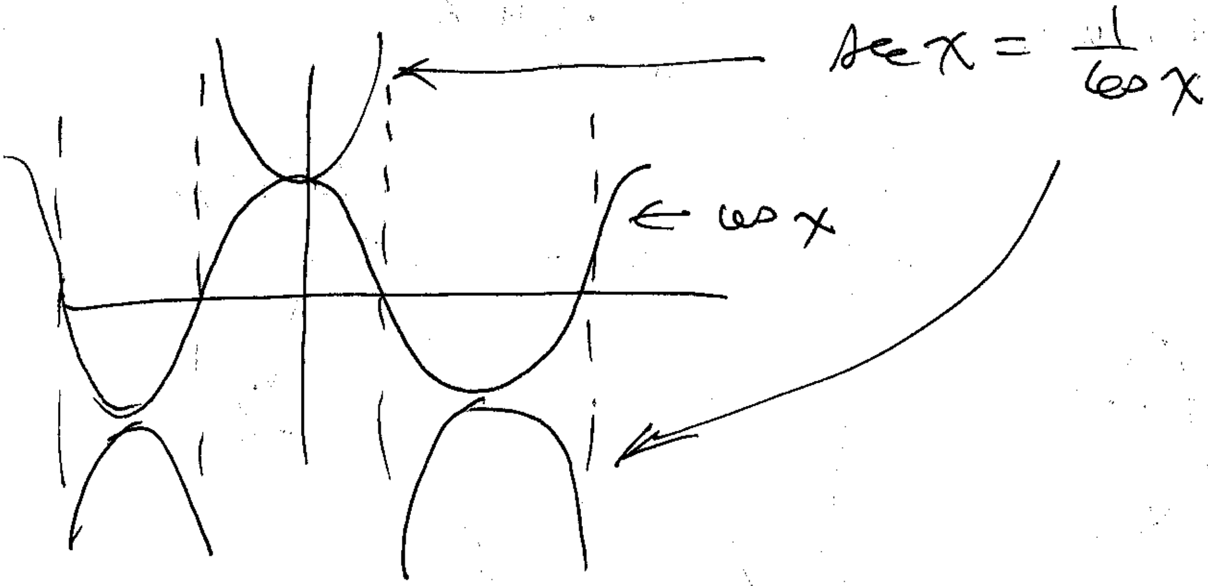
$\sin\theta = 0$

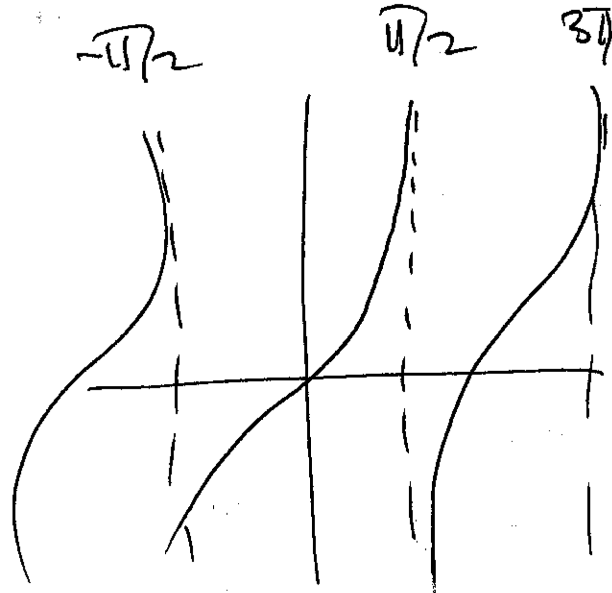
$\theta = 0, \pi, 2\pi$

$\sin\theta = 1 \quad \theta = \pi/2$ only

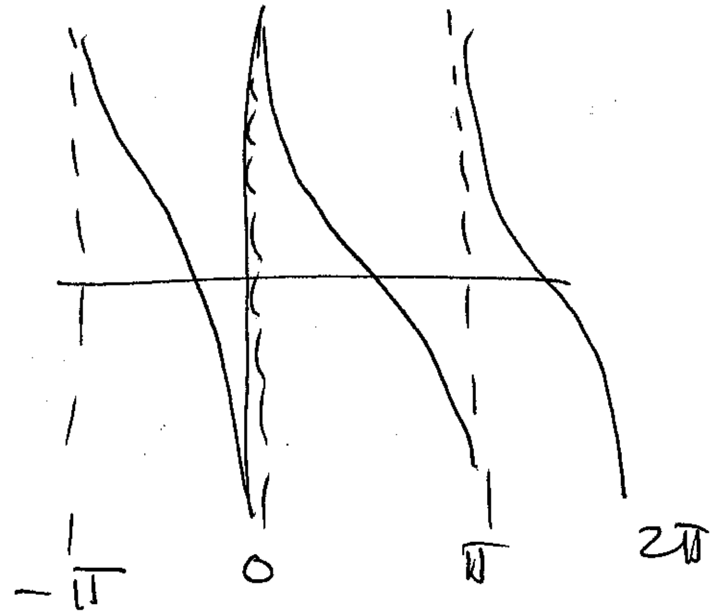
More Graphs







$$y = \tan x = \frac{\sin x}{\cos x}$$

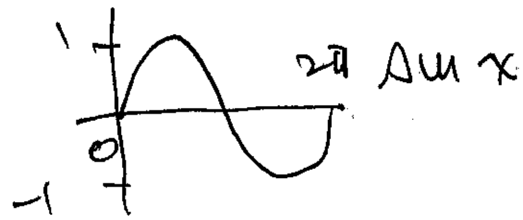


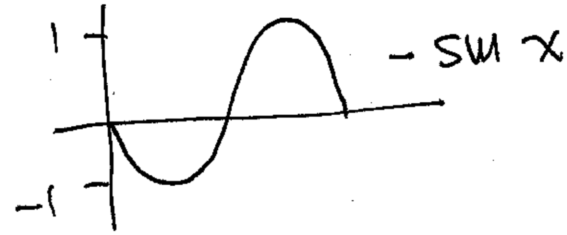
$$y = \cot x = \frac{\cos x}{\sin x}$$

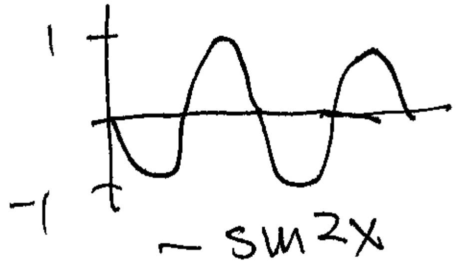
Consider

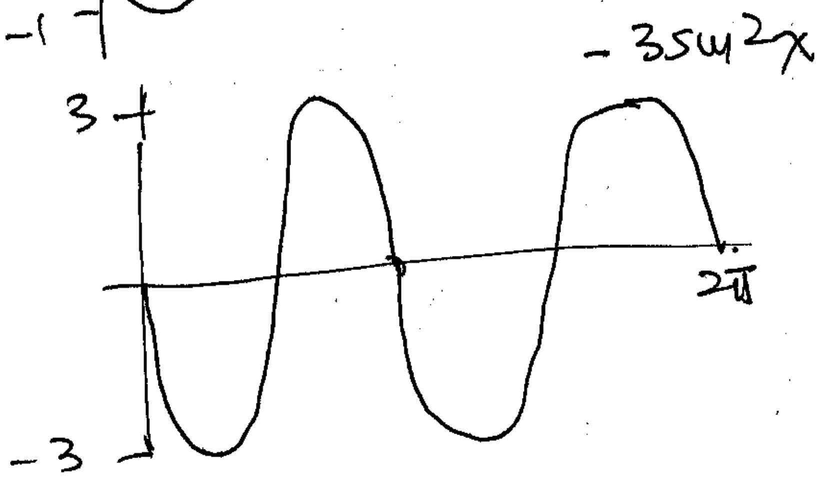
$$f(x) = -3\sin 2x$$

we start with the basic $\sin x$ picture and then modify it!



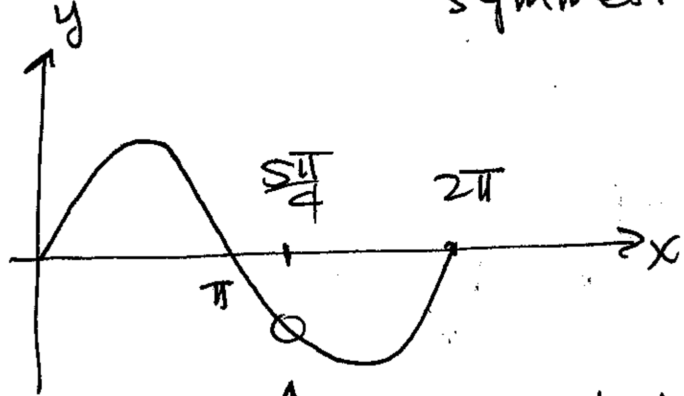






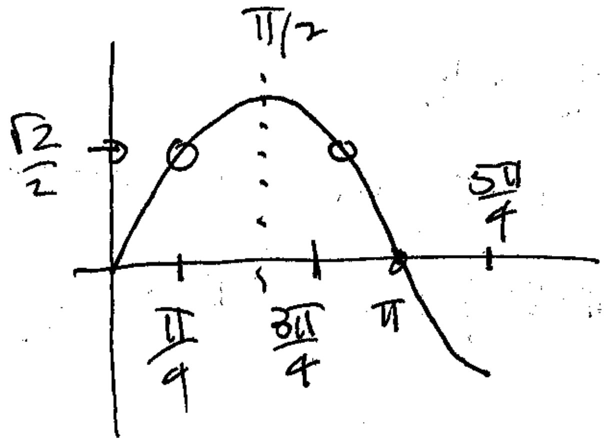
Finally, find $\sin \frac{5\pi}{4}$

Let's consider the sin curve. There's lots of symmetry in this graph



there's what we're looking for

So let's explore the symmetry here



since $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

from the graph

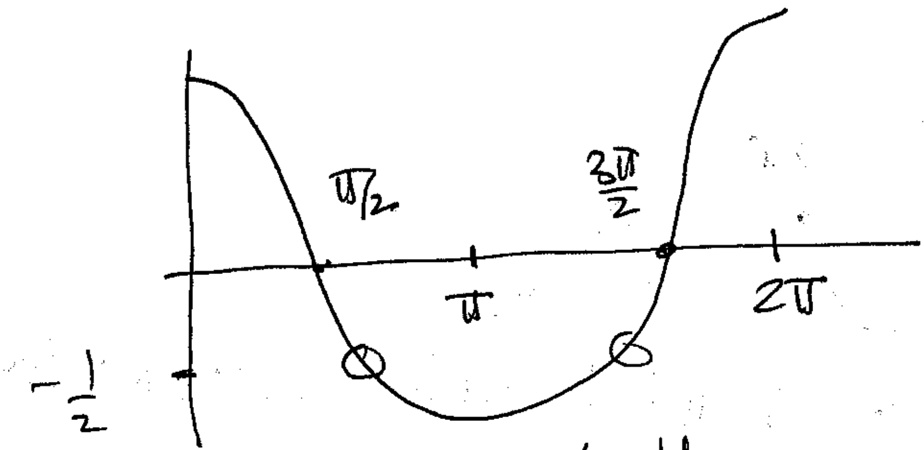
$\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$ also

and the symmetry in $x = \pi$

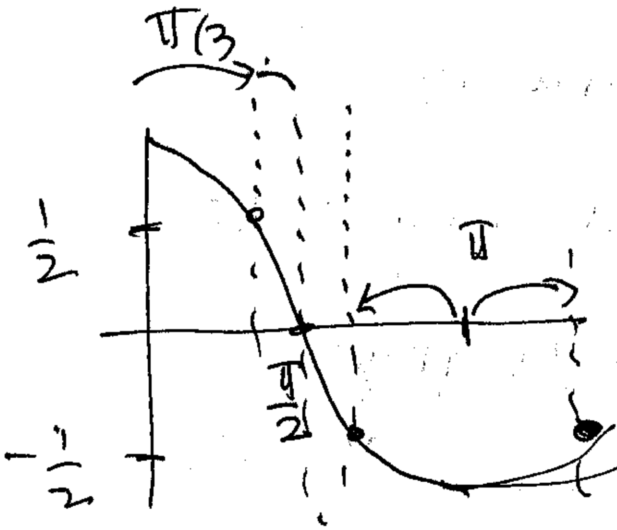
gives $\sin \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$

Ex Find $\cos^{-1}\left(-\frac{1}{2}\right)$ or $\cos\theta = -\frac{1}{2}$

so cosine graph



we want these



we know

$\cos \pi/3 = \frac{1}{2}$

a symmetry in π/2 gives

$\cos(\pi - \pi/3) = \cos \frac{2\pi}{3} = -\frac{1}{2}$

Also $\cos(\pi + \pi/3) = -\frac{1}{2}$

so $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$