

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions

Exercise A, Question 1

Question:

Draw diagrams, as in Examples 1 and 2, to show the following angles. Mark in the acute angle that OP makes with the x -axis.

(a) -80°

(b) 100°

(c) 200°

(d) 165°

(e) -145°

(f) 225°

(g) 280°

(h) 330°

(i) -160°

(j) -280°

(k) $\frac{3\pi}{4}$

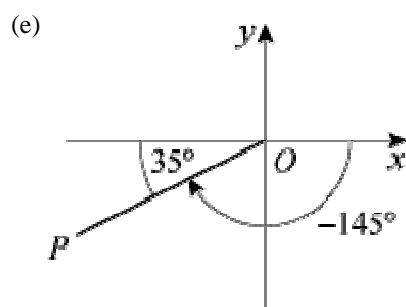
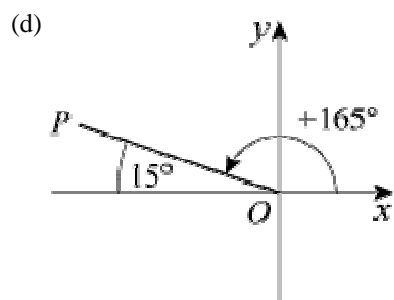
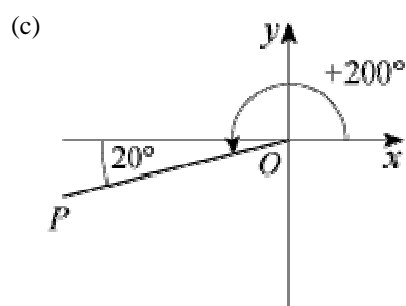
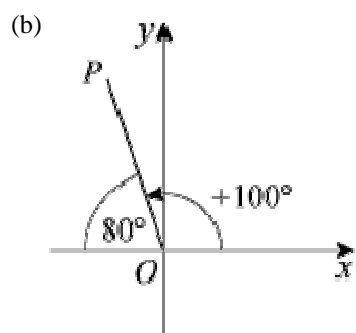
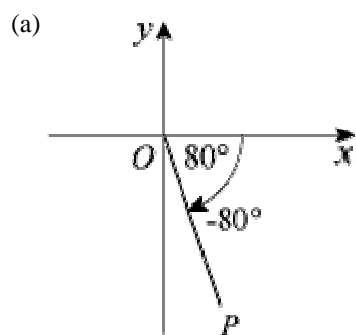
(l) $\frac{7\pi}{6}$

(m) $-\frac{5\pi}{3}$

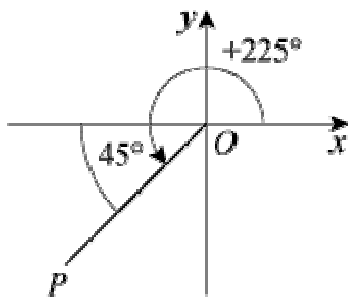
(n) $-\frac{5\pi}{8}$

(o) $\frac{19\pi}{9}$

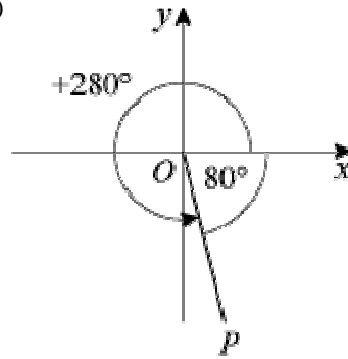
Solution:



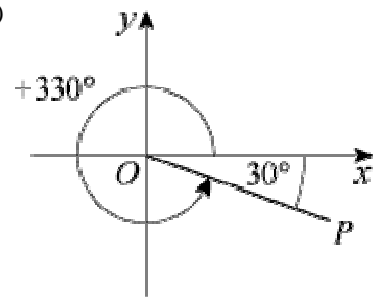
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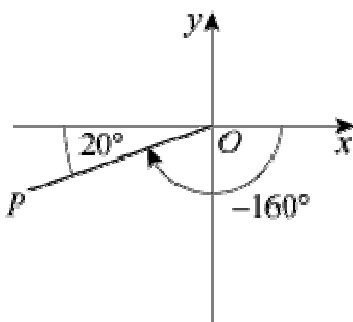
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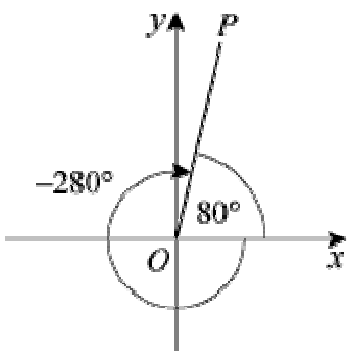
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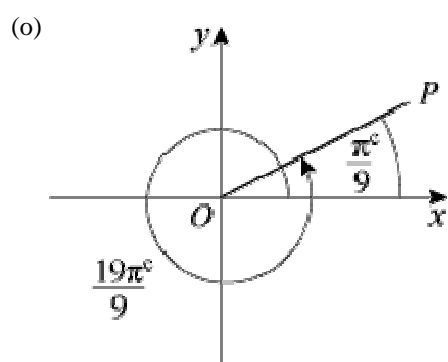
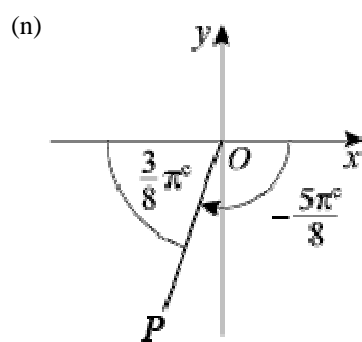
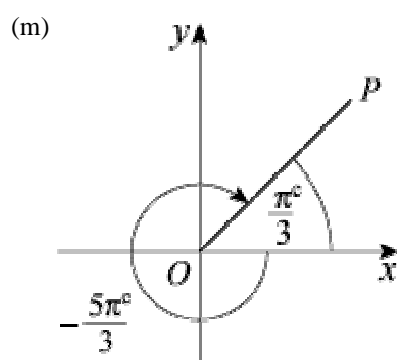
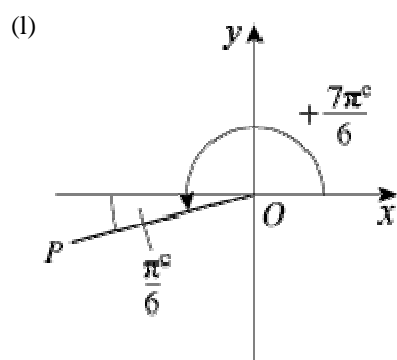
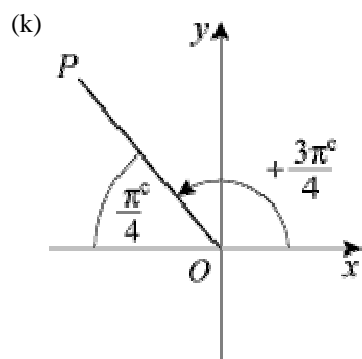


(i)



(j)





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Exercise A, Question 2

Question:

State the quadrant that OP lies in when the angle that OP makes with the positive x -axis is:

(a) 400°

(b) 115°

(c) -210°

(d) 255°

(e) -100°

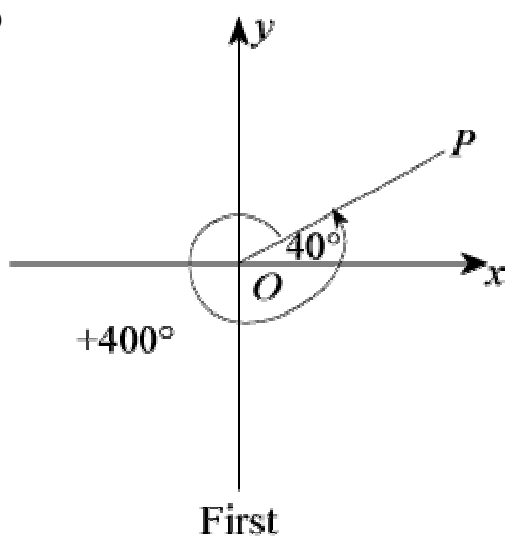
(f) $\frac{7\pi}{8}$

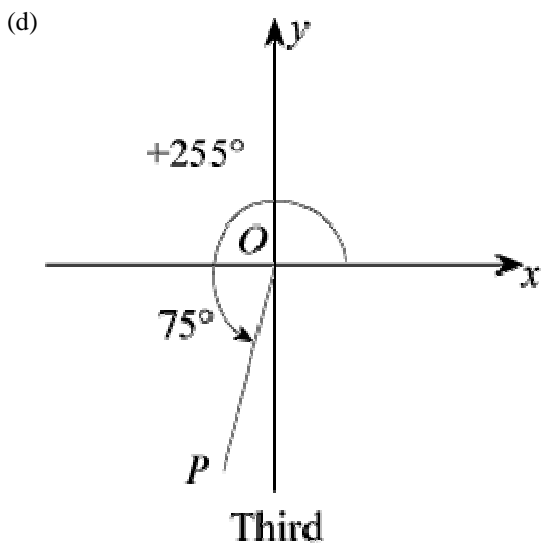
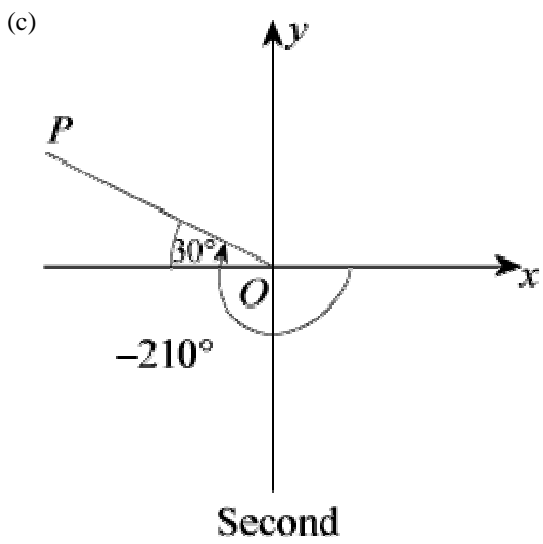
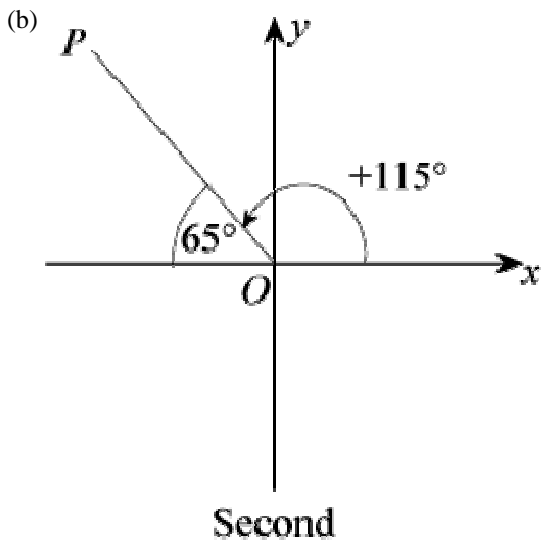
(g) $-\frac{11\pi}{6}$

(h) $\frac{13\pi}{7}$

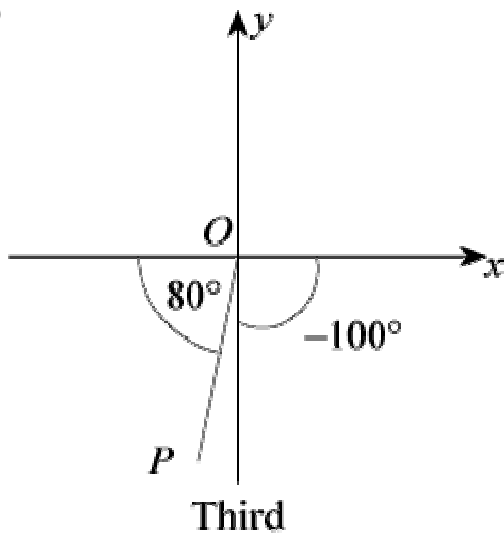
Solution:

(a)

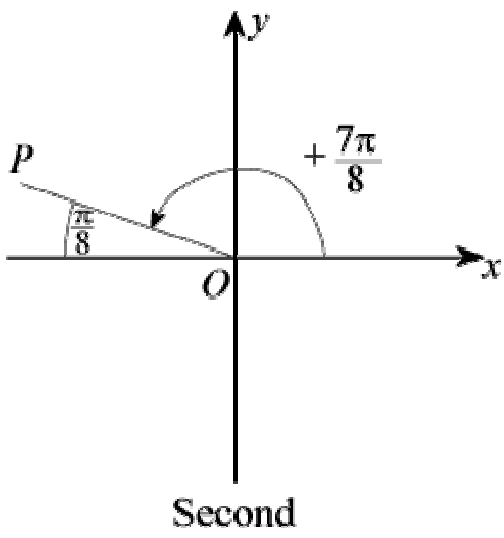




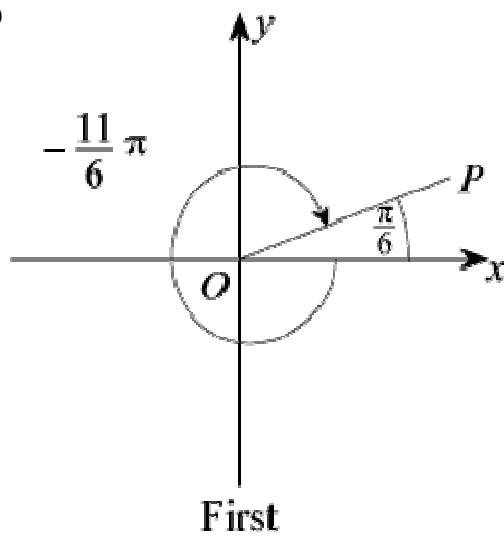
(e)



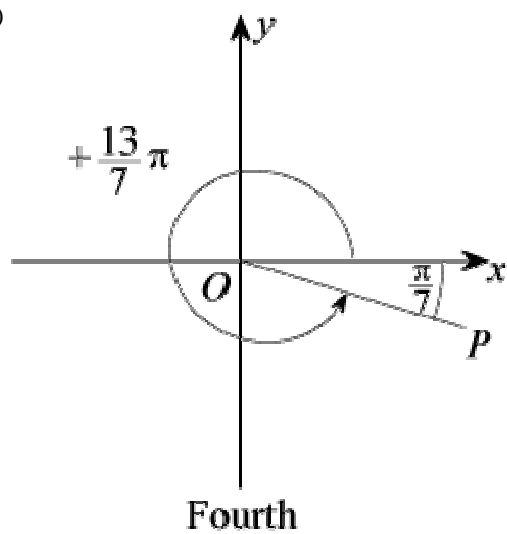
(f)



(g)



(h)



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Exercise B, Question 1

Question:

(Note: do not use a calculator.)

Write down the values of:

(a) $\sin (-90)^\circ$

(b) $\sin 450^\circ$

(c) $\sin 540^\circ$

(d) $\sin (-450)^\circ$

(e) $\cos (-180)^\circ$

(f) $\cos (-270)^\circ$

(g) $\cos 270^\circ$

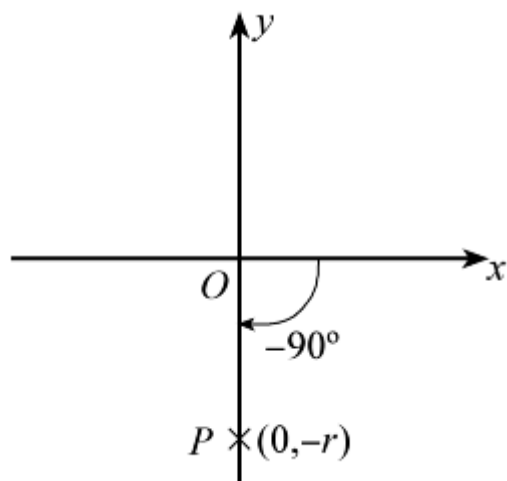
(h) $\cos 810^\circ$

(i) $\tan 360^\circ$

(j) $\tan (-180)^\circ$

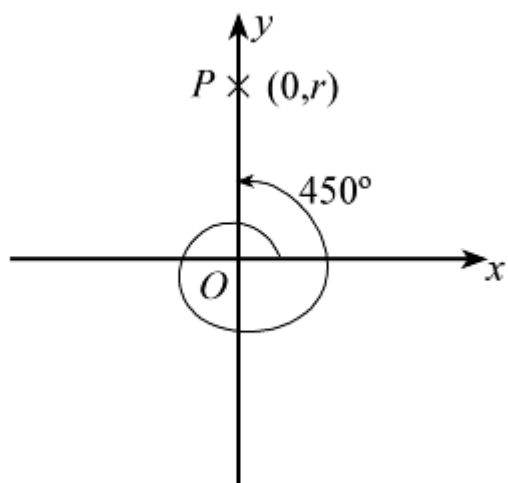
Solution:

(a)



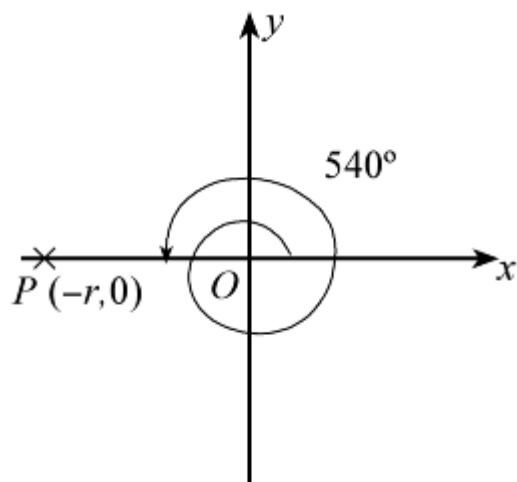
$$\sin \left(-90 \right)^\circ = \frac{-r}{r} = -1$$

(b)



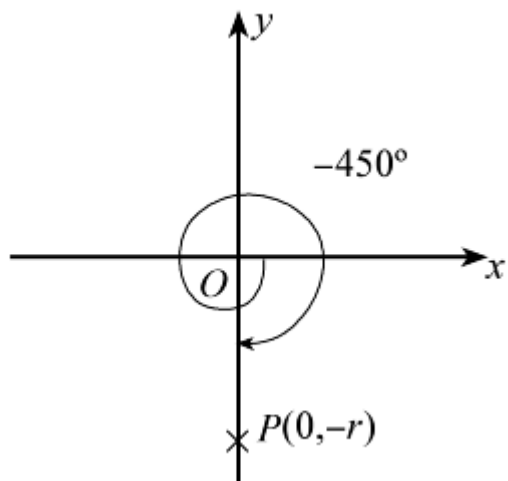
$$\sin 450^\circ = \frac{r}{r} = 1$$

(c)



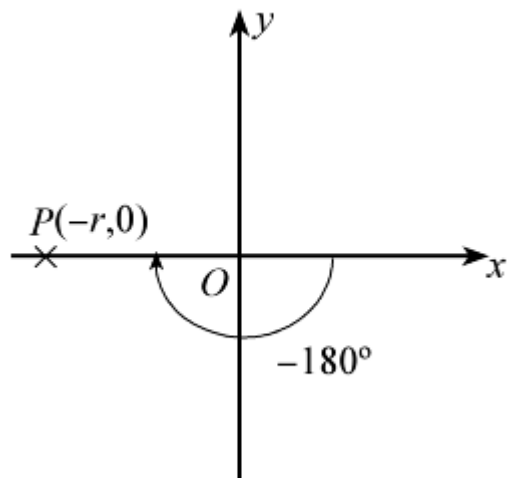
$$\sin 540^\circ = \frac{0}{r} = 0$$

(d)



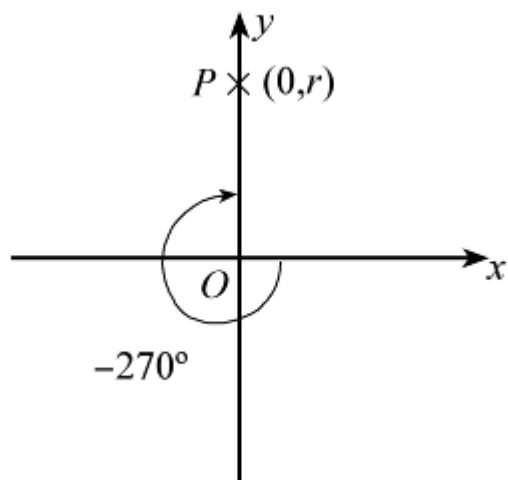
$$\sin \left(-450 \right)^\circ = \frac{-r}{r} = -1$$

(e)



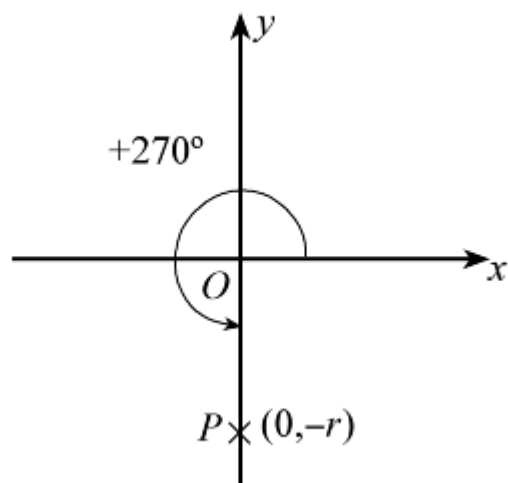
$$\cos \left(-180 \right)^\circ = \frac{-r}{r} = -1$$

(f)



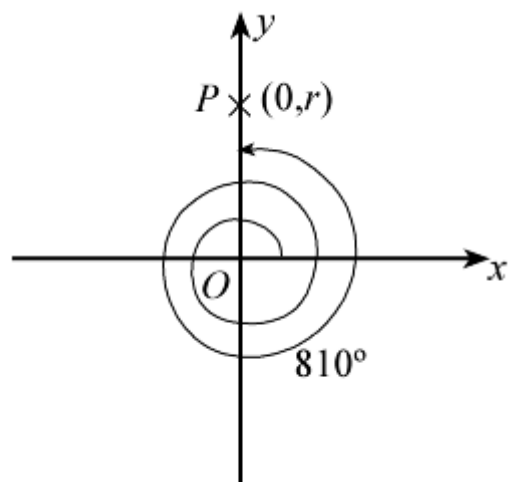
$$\cos \left(-270 \right)^\circ = \frac{0}{r} = 0$$

(g)



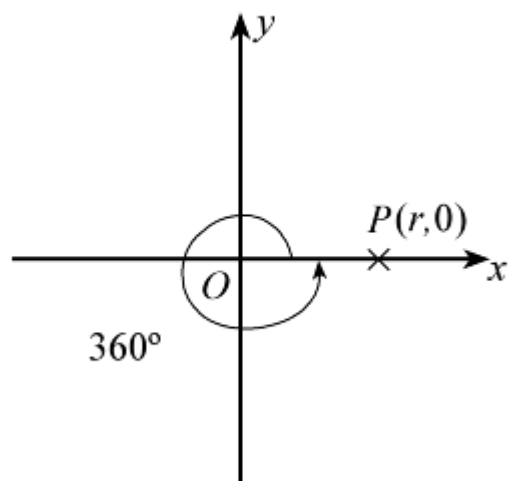
$$\cos 270^\circ = \frac{0}{r} = 0$$

(h)



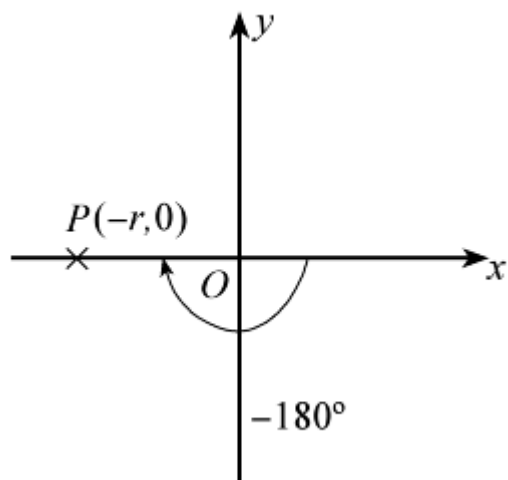
$$\cos 810^\circ = \frac{0}{r} = 0$$

(i)



$$\tan 360^\circ = \frac{0}{r} = 0$$

(j)



$$\tan \left(-180 \right)^\circ = \frac{0}{-r} = 0$$

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Exercise B, Question 2

Question:

(Note: do not use a calculator.)

Write down the values of the following, where the angles are in radians:

(a) $\sin \frac{3\pi}{2}$

(b) $\sin \left(-\frac{\pi}{2} \right)$

(c) $\sin 3\pi$

(d) $\sin \frac{7\pi}{2}$

(e) $\cos 0$

(f) $\cos \pi$

(g) $\cos \frac{3\pi}{2}$

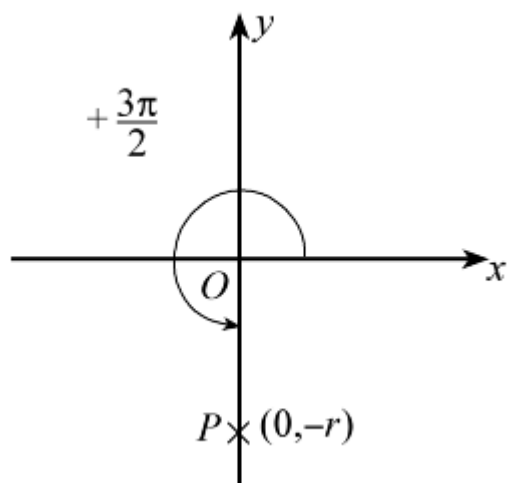
(h) $\cos \left(-\frac{3\pi}{2} \right)$

(i) $\tan \pi$

(j) $\tan (-2\pi)$

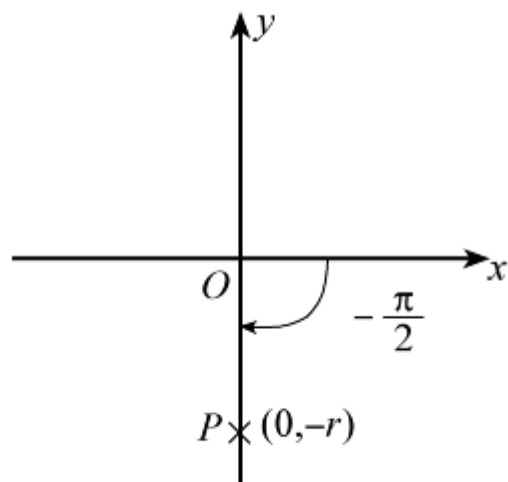
Solution:

(a)



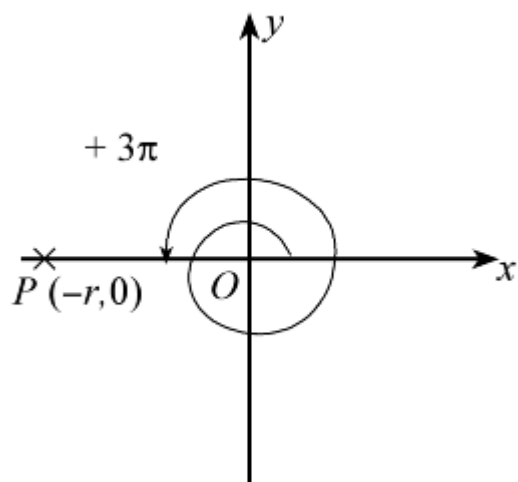
$$\sin \frac{3\pi}{2} = \frac{-r}{r} = -1$$

(b)



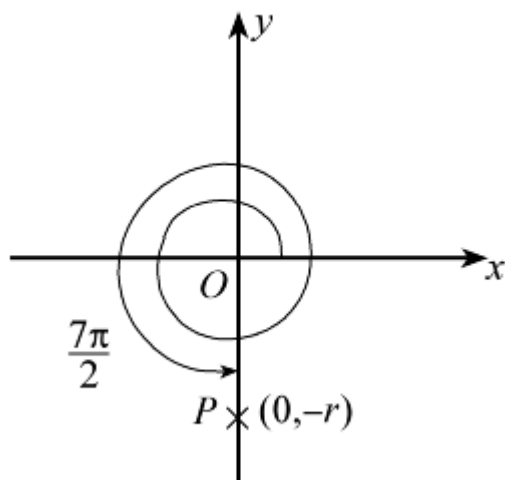
$$\sin \left(\frac{-\pi}{2} \right) = \frac{-r}{r} = -1$$

(c)



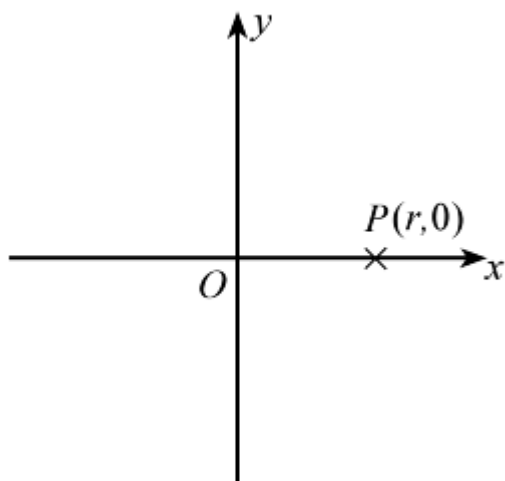
$$\sin 3\pi = \frac{0}{r} = 0$$

(d)



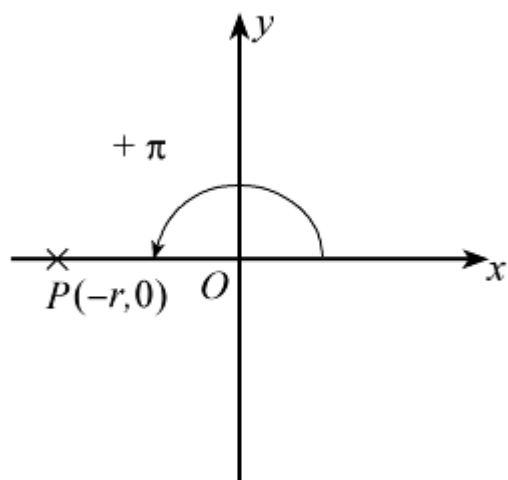
$$\sin \frac{7\pi}{2} = \frac{-r}{r} = -1$$

(e)



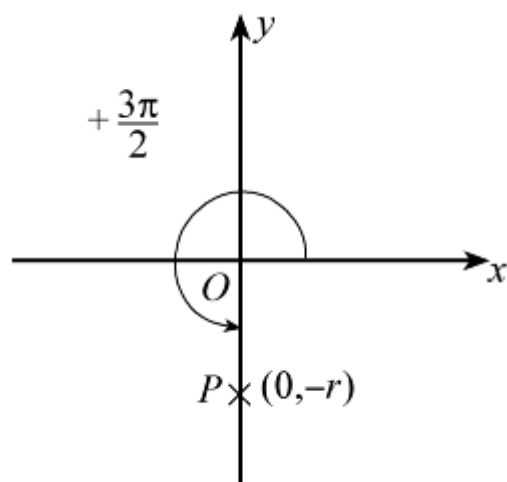
$$\cos 0^\circ = \frac{r}{r} = 1$$

(f)



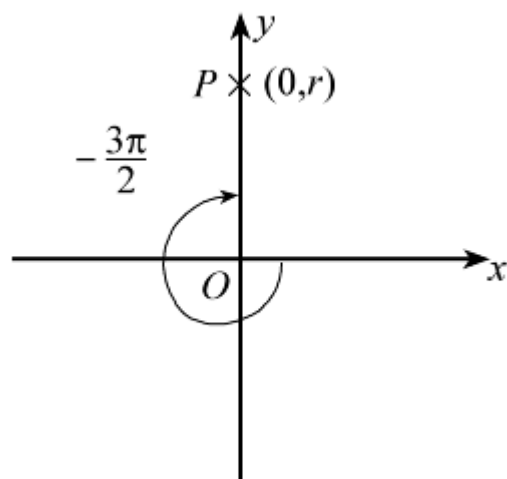
$$\cos \pi = \frac{-r}{r} = -1$$

(g)



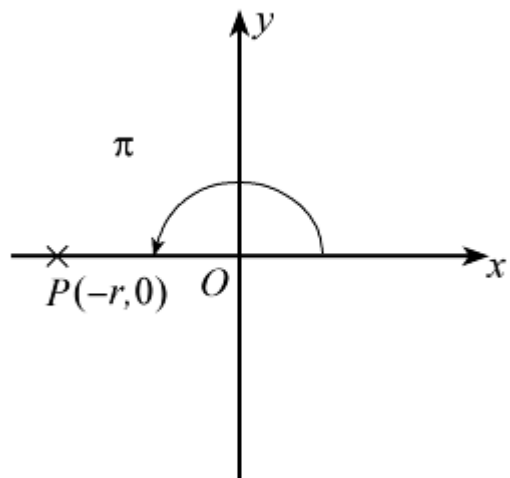
$$\cos \frac{3\pi}{2} = \frac{0}{r} = 0$$

(h)



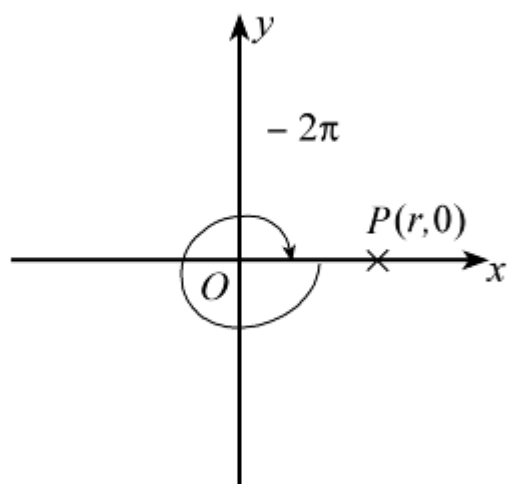
$$\cos \left(-\frac{3\pi}{2} \right) = \frac{0}{r} = 0$$

(i)



$$\tan \pi = \frac{0}{-r} = 0$$

(j)



$$\tan \left(-2\pi \right) = \frac{0}{r} = 0$$

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Graphics of trigonometric functions

Exercise C, Question 1

Question:

(Note: Do not use a calculator.)

By drawing diagrams, as in Example 6, express the following in terms of trigonometric ratios of acute angles:

(a) $\sin 240^\circ$

(b) $\sin (-80)^\circ$

(c) $\sin (-200)^\circ$

(d) $\sin 300^\circ$

(e) $\sin 460^\circ$

(f) $\cos 110^\circ$

(g) $\cos 260^\circ$

(h) $\cos (-50)^\circ$

(i) $\cos (-200)^\circ$

(j) $\cos 545^\circ$

(k) $\tan 100^\circ$

(l) $\tan 325^\circ$

(m) $\tan (-30)^\circ$

(n) $\tan (-175)^\circ$

(o) $\tan 600^\circ$

(p) $\sin \frac{7\pi}{6}$

(q) $\cos \frac{4\pi}{3}$

(r) $\cos \left(-\frac{3\pi}{4} \right)$

(s) $\tan \frac{7\pi}{5}$

(t) $\tan \left(-\frac{\pi}{3} \right)$

(u) $\sin \frac{15\pi}{16}$

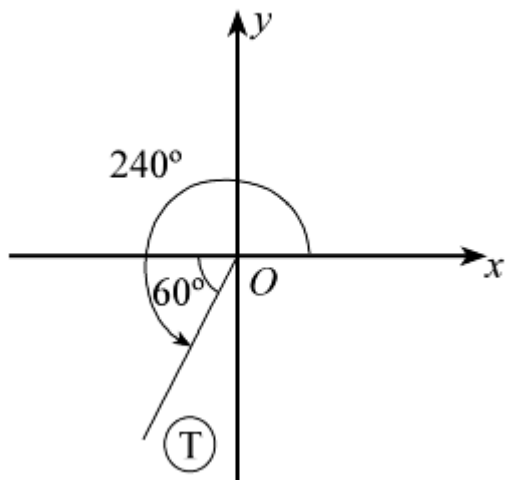
(v) $\cos \frac{8\pi}{5}$

(w) $\sin \left(-\frac{6\pi}{7} \right)$

(x) $\tan \frac{15\pi}{8}$

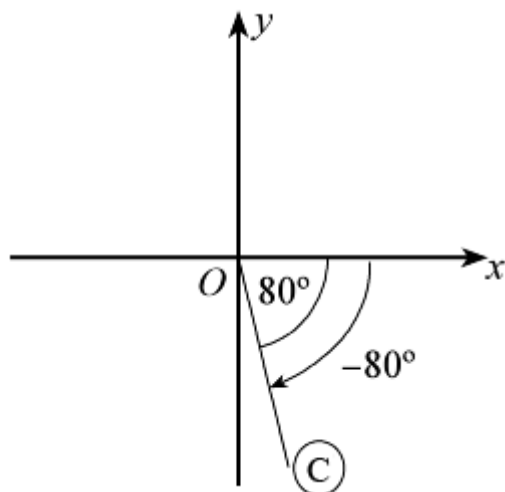
Solution:

(a)



60° is the acute angle.
In third quadrant sin is - ve.
So $\sin 240^\circ = -\sin 60^\circ$

(b)

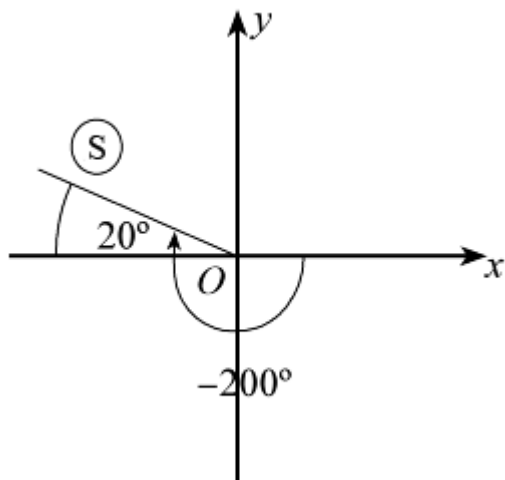


80° is the acute angle.

In fourth quadrant sin is - ve.

$$\text{So } \sin (-80)^\circ = -\sin 80^\circ$$

(c)

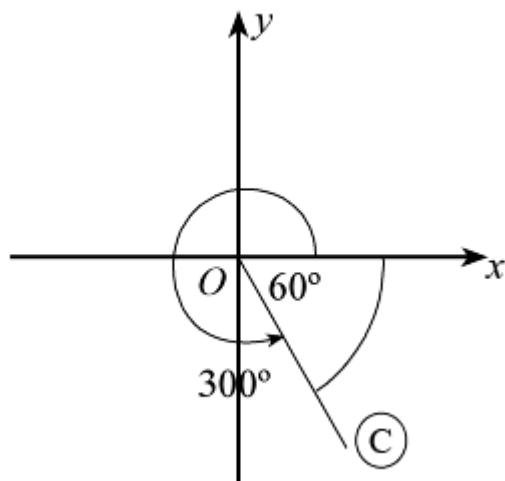


20° is the acute angle.

In second quadrant sin is +ve.

$$\text{So } \sin (-200)^\circ = +\sin 20^\circ$$

(d)

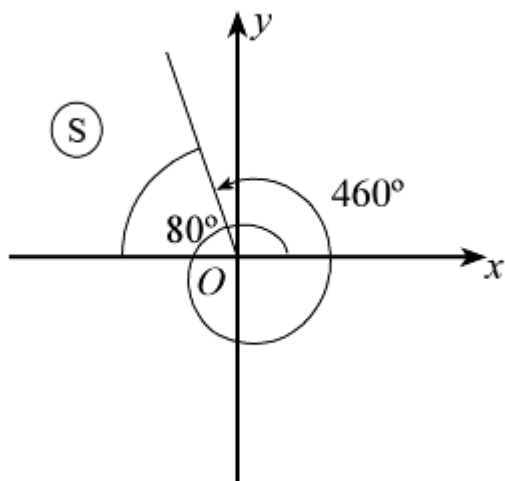


60° is the acute angle.

In fourth quadrant sin is - ve.

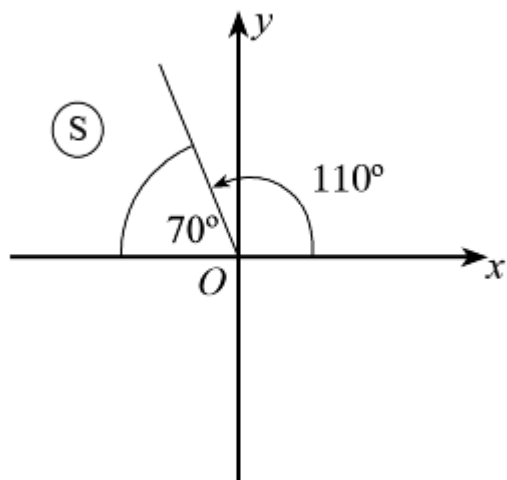
$$\text{So } \sin 300^\circ = -\sin 60^\circ$$

(e)



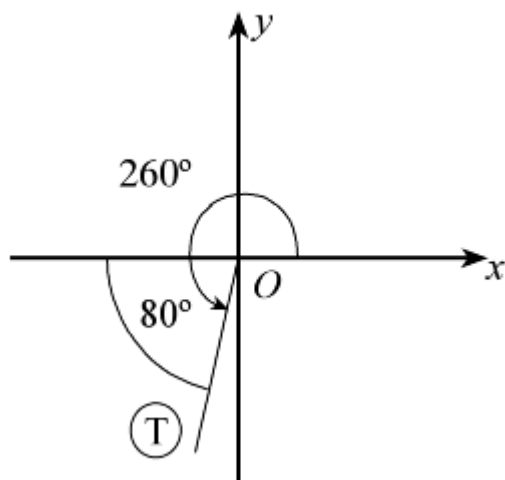
80° is the acute angle.
 In second quadrant sin is +ve.
 So $\sin 460^\circ = +\sin 80^\circ$

(f)



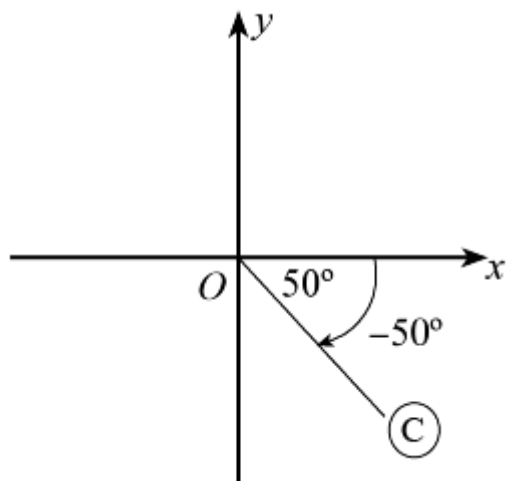
70° is the acute angle.
 In second quadrant cos is - ve.
 So $\cos 110^\circ = -\cos 70^\circ$

(g)



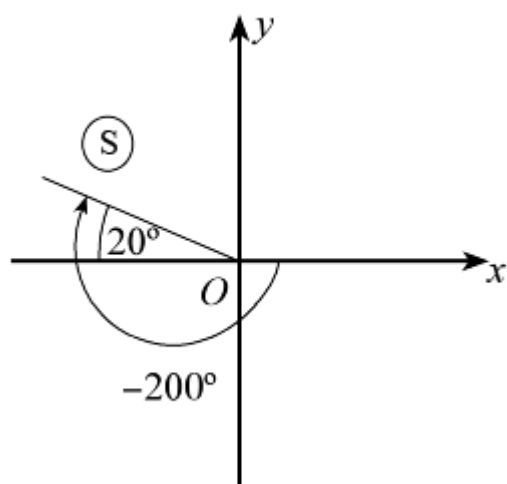
80° is the acute angle.
 In third quadrant cos is - ve.
 So $\cos 260^\circ = -\cos 80^\circ$

(h)



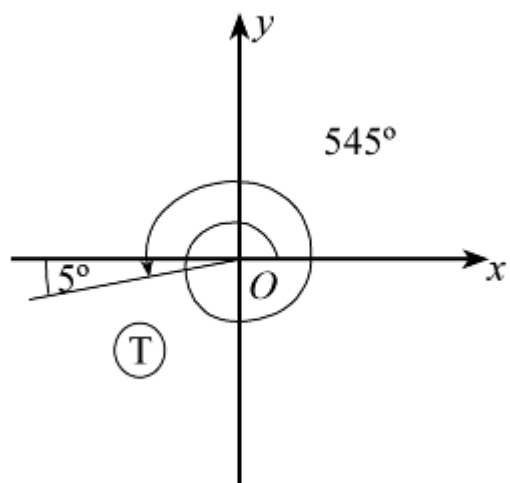
50° is the acute angle.
 In fourth quadrant cos is +ve.
 So $\cos (-50)^\circ = +\cos 50^\circ$

(i)



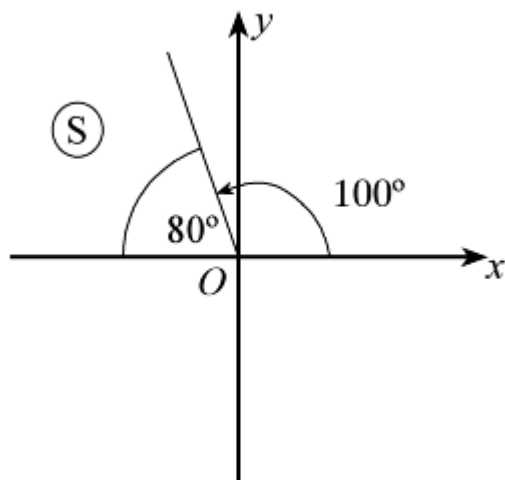
20° is the acute angle.
 In second quadrant cos is - ve.
 So $\cos (-200)^\circ = -\cos 20^\circ$

(j)



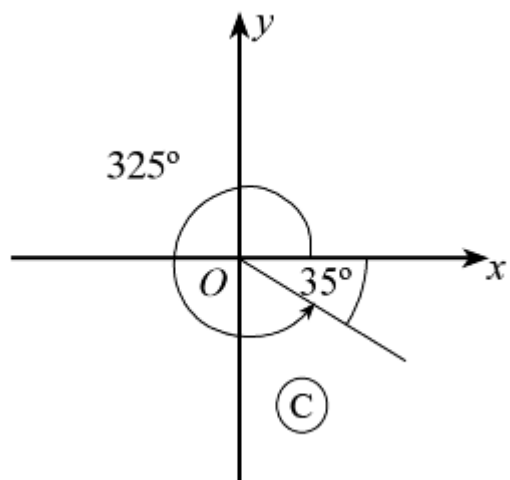
5° is the acute angle.
 In third quadrant cos is - ve.
 So $\cos 545^\circ = -\cos 5^\circ$

(k)



80° is the acute angle.
 In second quadrant tan is - ve.
 So $\tan 100^\circ = -\tan 80^\circ$

(l)

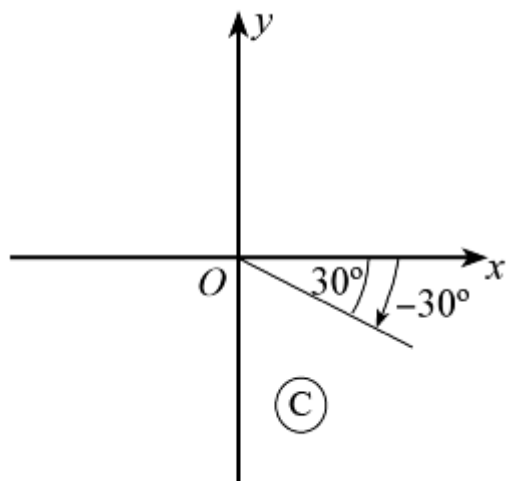


35° is the acute angle.

In fourth quadrant tan is - ve.

So $\tan 325^\circ = -\tan 35^\circ$

(m)

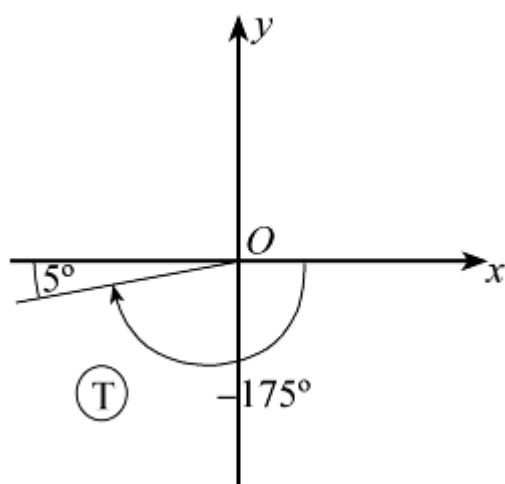


30° is the acute angle.

In fourth quadrant tan is - ve.

So $\tan (-30)^\circ = -\tan 30^\circ$

(n)

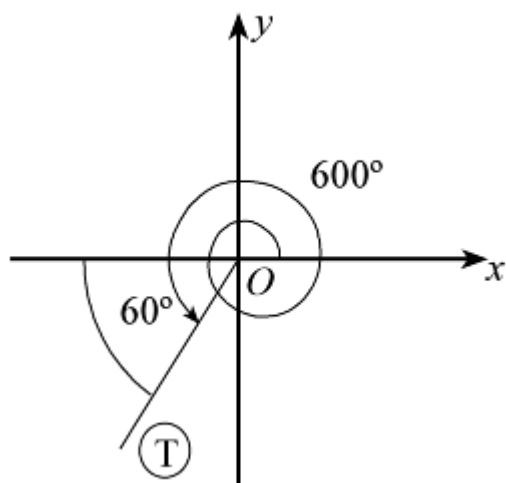


5° is the acute angle.

In third quadrant tan is +ve.

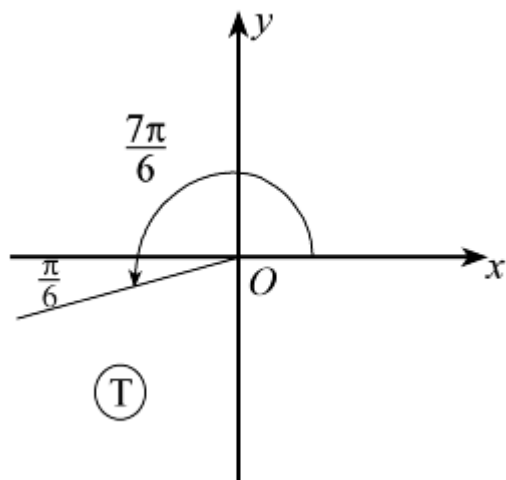
So $\tan (-175)^\circ = +\tan 5^\circ$

(o)



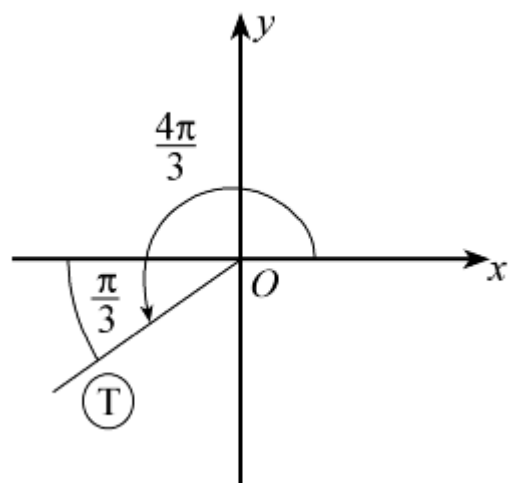
60° is the acute angle.
 In third quadrant tan is +ve.
 So $\tan 600^\circ = +\tan 60^\circ$

(p)



$\frac{\pi}{6}$ is the acute angle.
 In third quadrant sin is -ve.
 So $\sin \frac{7\pi}{6} = -\sin \frac{\pi}{6}$

(q)

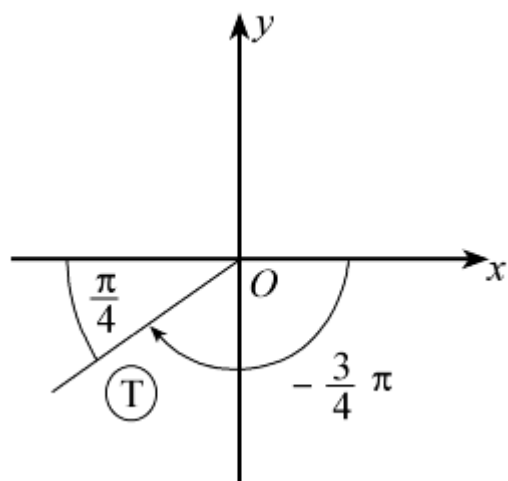


$\frac{\pi}{3}$ is the acute angle.

In third quadrant cos is - ve.

$$\text{So } \cos \frac{4\pi}{3} = -\cos \frac{\pi}{3}$$

(r)

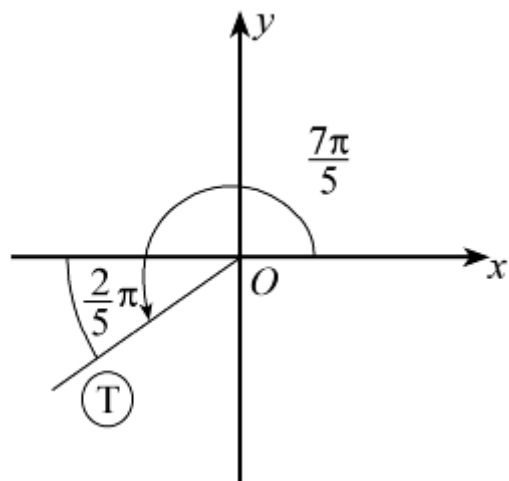


$\frac{\pi}{4}$ is the acute angle.

In third quadrant cos is - ve.

$$\text{So } \cos \left(-\frac{3}{4}\pi \right) = -\cos \frac{\pi}{4}$$

(s)

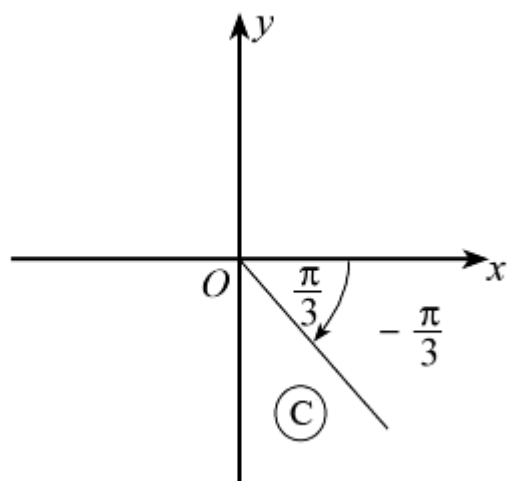


$\frac{2\pi}{5}$ is the acute angle.

In third quadrant tan is +ve.

$$\text{So } \tan \frac{7\pi}{5} = + \tan \frac{2\pi}{5}$$

(t)

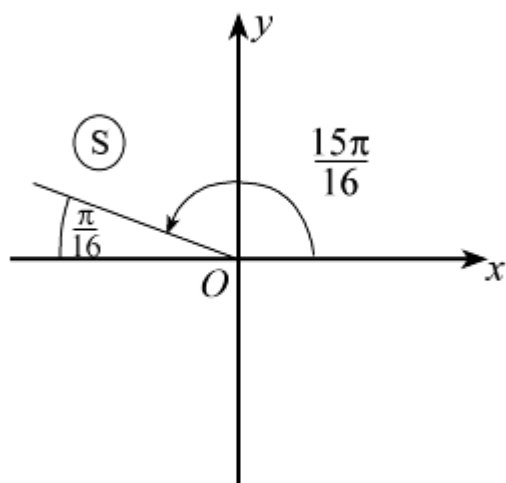


$\frac{\pi}{3}$ is the acute angle.

In fourth quadrant tan is - ve.

$$\text{So } \tan \left(-\frac{\pi}{3} \right) = - \tan \frac{\pi}{3}$$

(u)

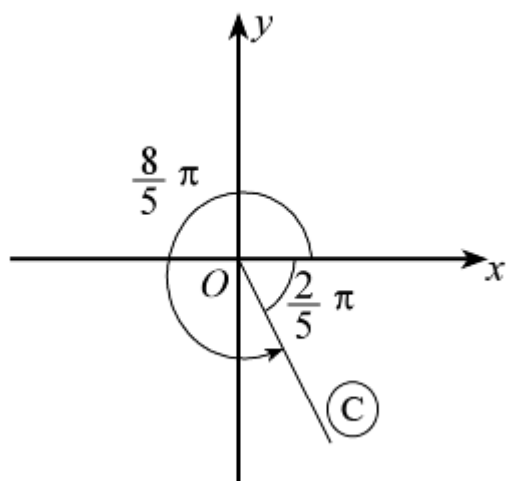


$\frac{\pi}{16}$ is the acute angle.

In second quadrant sin is +ve.

$$\text{So } \sin \frac{15\pi}{16} = + \sin \frac{\pi}{16}$$

(v)

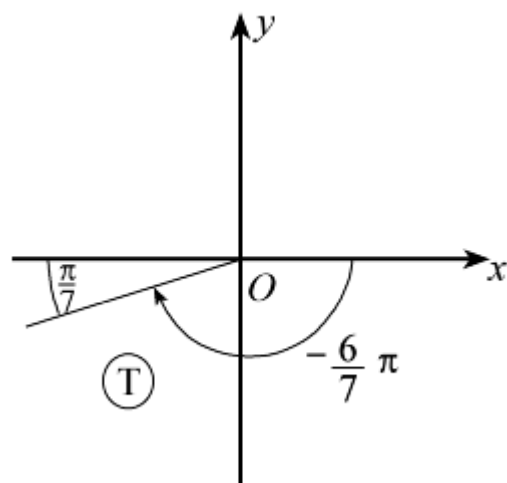


$\frac{2}{5}\pi$ is the acute angle.

In fourth quadrant cos is +ve.

$$\text{So } \cos \frac{8}{5}\pi = + \cos \frac{2}{5}\pi$$

(w)

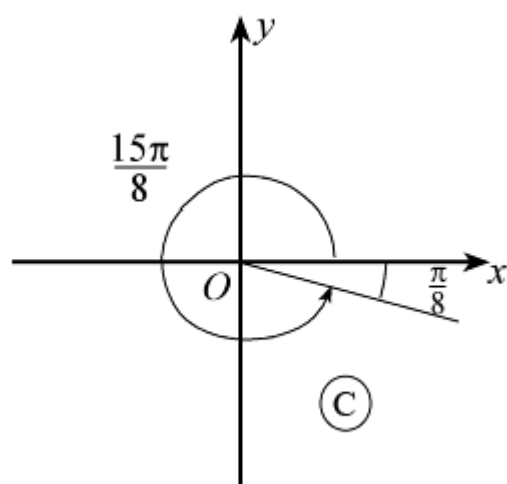


$\frac{\pi}{7}$ is the acute angle.

In third quadrant sin is - ve.

$$\text{So sin} \left(-\frac{6\pi}{7} \right) = -\text{sin} \frac{\pi}{7}$$

(x)



$\frac{\pi}{8}$ is the acute angle.

In fourth quadrant tan is - ve.

$$\text{So tan} \frac{15\pi}{8} = -\text{tan} \frac{\pi}{8}$$

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Exercise C, Question 2

Question:

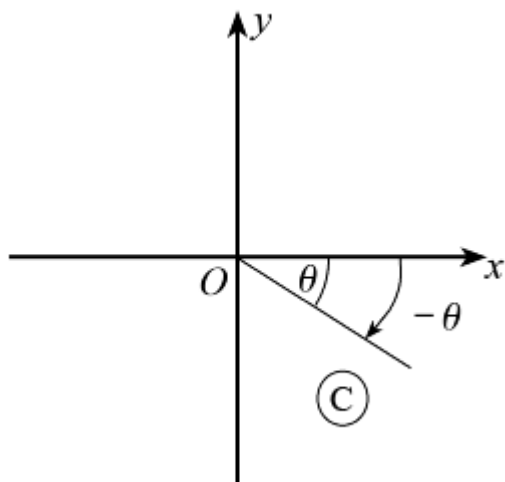
(Note: Do not use a calculator.)

Given that θ is an acute angle measured in degrees, express in terms of $\sin \theta$:

- (a) $\sin (-\theta)$
- (b) $\sin (180^\circ + \theta)$
- (c) $\sin (360^\circ - \theta)$
- (d) $\sin - (180^\circ + \theta)$
- (e) $\sin (-180^\circ + \theta)$
- (f) $\sin (-360^\circ + \theta)$
- (g) $\sin (540^\circ + \theta)$
- (h) $\sin (720^\circ - \theta)$
- (i) $\sin (\theta + 720^\circ)$

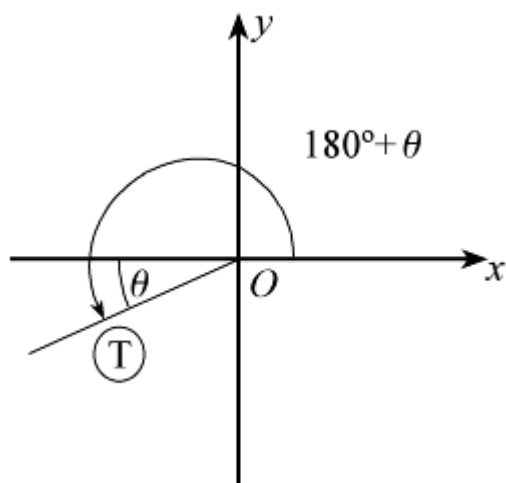
Solution:

(a)



\sin is $-ve$ in this quadrant.
So $\sin (-\theta) = -\sin \theta$

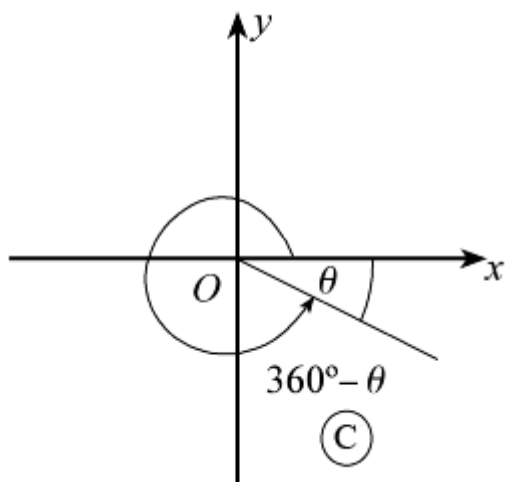
(b)



sin is -ve in this quadrant.

$$\text{So } \sin (180^\circ + \theta) = -\sin \theta$$

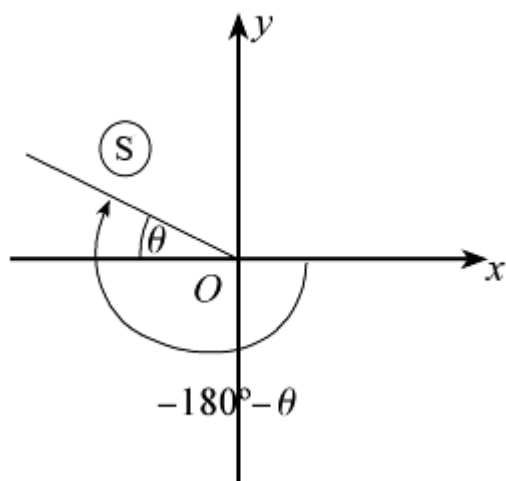
(c)



sin is -ve in this quadrant.

$$\text{So } \sin (360^\circ - \theta) = -\sin \theta$$

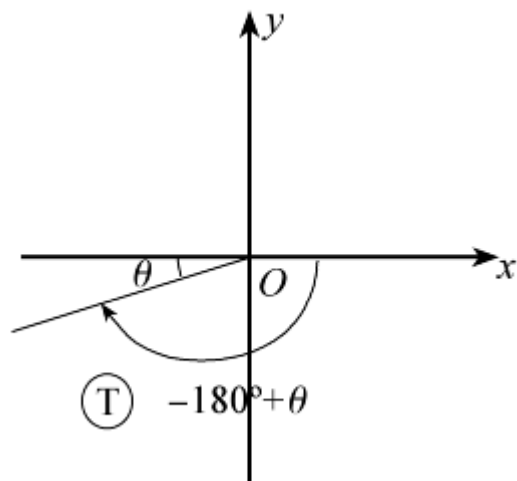
(d)



sin is +ve in this quadrant.

$$\text{So } \sin - (180^\circ + \theta) = +\sin \theta$$

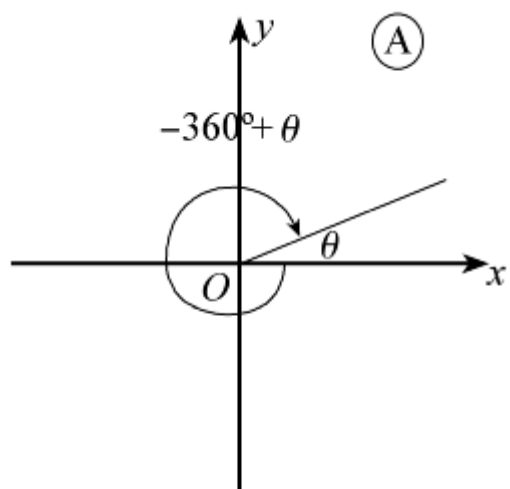
(e)



sin is -ve in this quadrant.

So $\sin (-180^\circ + \theta) = -\sin \theta$

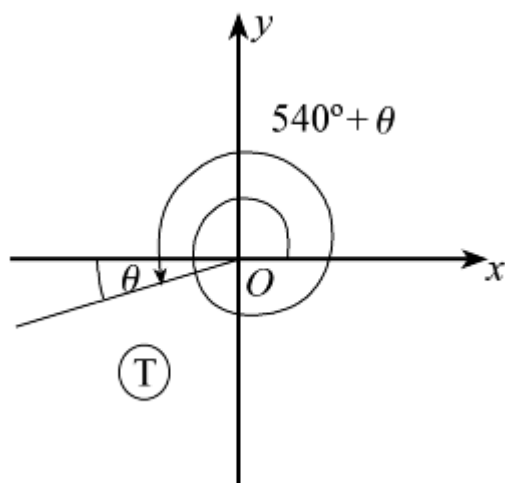
(f)



sin is +ve in this quadrant.

So $\sin (-360^\circ + \theta) = +\sin \theta$

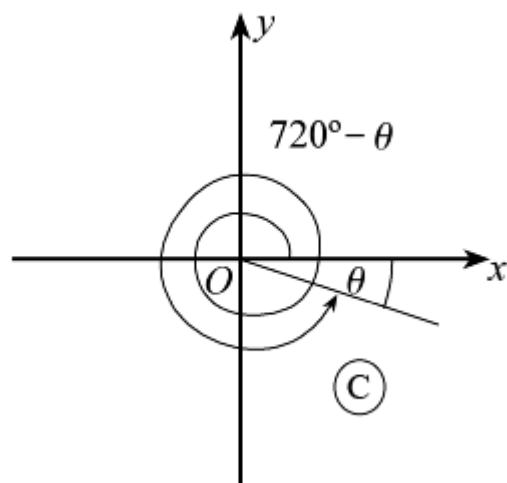
(g)



sin is -ve in this quadrant.

So $\sin (540^\circ + \theta) = -\sin \theta$

(h)



sin is - ve in this quadrant.

$$\text{So } \sin (720^\circ - \theta) = -\sin \theta$$

(i) $\theta + 720^\circ$ is in the first quadrant with θ to the horizontal.

$$\text{So } \sin (\theta + 720^\circ) = +\sin \theta$$

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Exercise C, Question 3

Question:

(Note: Do not use a calculator.)

Given that θ is an acute angle measured in degrees, express in terms of $\cos \theta$ or $\tan \theta$:

(a) $\cos (180^\circ - \theta)$

(b) $\cos (180^\circ + \theta)$

(c) $\cos (-\theta)$

(d) $\cos -(180^\circ - \theta)$

(e) $\cos (\theta - 360^\circ)$

(f) $\cos (\theta - 540^\circ)$

(g) $\tan (-\theta)$

(h) $\tan (180^\circ - \theta)$

(i) $\tan (180^\circ + \theta)$

(j) $\tan (-180^\circ + \theta)$

(k) $\tan (540^\circ - \theta)$

(l) $\tan (\theta - 360^\circ)$

Solution:

(a) $180^\circ - \theta$ is in the second quadrant where \cos is $-ve$, and the angle to the horizontal is θ , so
 $\cos (180^\circ - \theta) = -\cos \theta$

(b) $180^\circ + \theta$ is in the third quadrant, at θ to the horizontal, so
 $\cos (180^\circ + \theta) = -\cos \theta$

(c) $-\theta$ is in the fourth quadrant, at θ to the horizontal, so
 $\cos (-\theta) = +\cos \theta$

(d) $-180^\circ + \theta$ is in the third quadrant, at θ to the horizontal, so
 $\cos (-180^\circ + \theta) = -\cos \theta$

(e) $\theta - 360^\circ$ is in the first quadrant, at θ to the horizontal, so
 $\cos (\theta - 360^\circ) = +\cos \theta$

(f) $\theta - 540^\circ$ is in the third quadrant, at θ to the horizontal, so
 $\cos (\theta - 540^\circ) = -\cos \theta$

(g) $\tan (-\theta) = -\tan \theta$ as $-\theta$ is in the fourth quadrant.

(h) $\tan (180^\circ - \theta) = -\tan \theta$ as $(180^\circ - \theta)$ is in the second quadrant.

(i) $\tan (180^\circ + \theta) = +\tan \theta$ as $(180^\circ + \theta)$ is in the third quadrant.

(j) $\tan (-180^\circ + \theta) = +\tan \theta$ as $(-180^\circ + \theta)$ is in the third quadrant.

(k) $\tan (540^\circ - \theta) = -\tan \theta$ as $(540^\circ - \theta)$ is in the second quadrant.

(l) $\tan (\theta - 360^\circ) = +\tan \theta$ as $(\theta - 360^\circ)$ is in the first quadrant.

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Exercise C, Question 4

Question:

(Note: Do not use a calculator.)

A function f is an even function if $f(-\theta) = f(\theta)$.

A function f is an odd function if $f(-\theta) = -f(\theta)$.

Using your results from questions 2(a), 3(c) and 3(g), state whether $\sin \theta$, $\cos \theta$ and $\tan \theta$ are odd or even functions.

Solution:

As $\sin(-\theta) = -\sin \theta$ (question 2a)
 $\sin \theta$ is an odd function.

As $\cos(-\theta) = +\cos \theta$ (question 3c)
 $\cos \theta$ is an even function.

As $\tan(-\theta) = -\tan \theta$ (question 3g)
 $\tan \theta$ is an odd function.

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Exercise D, Question 1

Question:

Express the following as trigonometric ratios of either 30° , 45° or 60° , and hence find their exact values.

(a) $\sin 135^\circ$

(b) $\sin (-60^\circ)$

(c) $\sin 330^\circ$

(d) $\sin 420^\circ$

(e) $\sin (-300^\circ)$

(f) $\cos 120^\circ$

(g) $\cos 300^\circ$

(h) $\cos 225^\circ$

(i) $\cos (-210^\circ)$

(j) $\cos 495^\circ$

(k) $\tan 135^\circ$

(l) $\tan (-225^\circ)$

(m) $\tan 210^\circ$

(n) $\tan 300^\circ$

(o) $\tan (-120^\circ)$

Solution:

(a) $\sin 135^\circ = +\sin 45^\circ$ (135° is in the second quadrant at 45° to the horizontal)

So $\sin 135^\circ = \frac{\sqrt{2}}{2}$

(b) $\sin (-60^\circ) = -\sin 60^\circ$ (-60° is in the fourth quadrant at 60° to the horizontal)

So $\sin \left(-60^\circ \right) = -\frac{\sqrt{3}}{2}$

(c) $\sin 330^\circ = -\sin 30^\circ$ (330° is in the fourth quadrant at 30° to the horizontal)

So $\sin 330^\circ = -\frac{1}{2}$

(d) $\sin 420^\circ = +\sin 60^\circ$ (on second revolution)

$$\text{So } \sin 420^\circ = \frac{\sqrt{3}}{2}$$

$$\text{(e) } \sin (-300)^\circ = +\sin 60^\circ \text{ (} -300^\circ \text{ is in the first quadrant at } 60^\circ \text{ to the horizontal)}$$

$$\text{So } \sin \left(-300 \right)^\circ = \frac{\sqrt{3}}{2}$$

$$\text{(f) } \cos 120^\circ = -\cos 60^\circ \text{ (} 120^\circ \text{ is in the second quadrant at } 60^\circ \text{ to the horizontal)}$$

$$\text{So } \cos 120^\circ = -\frac{1}{2}$$

$$\text{(g) } \cos 300^\circ = +\cos 60^\circ \text{ (} 300^\circ \text{ is in the fourth quadrant at } 60^\circ \text{ to the horizontal)}$$

$$\text{So } \cos 300^\circ = \frac{1}{2}$$

$$\text{(h) } \cos 225^\circ = -\cos 45^\circ \text{ (} 225^\circ \text{ is in the third quadrant at } 45^\circ \text{ to the horizontal)}$$

$$\text{So } \cos 225^\circ = -\frac{\sqrt{2}}{2}$$

$$\text{(i) } \cos (-210)^\circ = -\cos 30^\circ \text{ (} -210^\circ \text{ is in the second quadrant at } 30^\circ \text{ to the horizontal)}$$

$$\text{So } \cos \left(-210^\circ \right) = -\frac{\sqrt{3}}{2}$$

$$\text{(j) } \cos 495^\circ = -\cos 45^\circ \text{ (} 495^\circ \text{ is in the second quadrant at } 45^\circ \text{ to the horizontal)}$$

$$\text{So } \cos 495^\circ = -\frac{\sqrt{2}}{2}$$

$$\text{(k) } \tan 135^\circ = -\tan 45^\circ \text{ (} 135^\circ \text{ is in the second quadrant at } 45^\circ \text{ to the horizontal)}$$

$$\text{So } \tan 135^\circ = -1$$

$$\text{(l) } \tan (-225)^\circ = -\tan 45^\circ \text{ (} -225^\circ \text{ is in the second quadrant at } 45^\circ \text{ to the horizontal)}$$

$$\text{So } \tan (-225)^\circ = -1$$

$$\text{(m) } \tan 210^\circ = +\tan 30^\circ \text{ (} 210^\circ \text{ is in the third quadrant at } 30^\circ \text{ to the horizontal)}$$

$$\text{So } \tan 210^\circ = \frac{\sqrt{3}}{3}$$

$$\text{(n) } \tan 300^\circ = -\tan 60^\circ \text{ (} 300^\circ \text{ is in the fourth quadrant at } 60^\circ \text{ to the horizontal)}$$

$$\text{So } \tan 300^\circ = -\sqrt{3}$$

$$\text{(o) } \tan (-120)^\circ = +\tan 60^\circ \text{ (} -120^\circ \text{ is in the third quadrant at } 60^\circ \text{ to the horizontal)}$$

$$\text{So } \tan (-120)^\circ = \sqrt{3}$$

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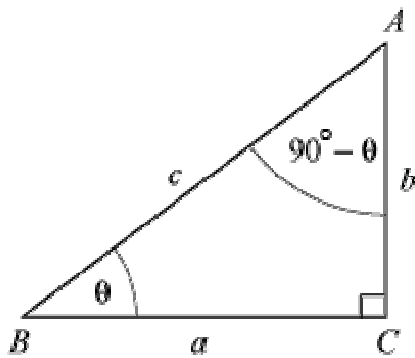
Graphics of trigonometric functions

Exercise D, Question 2

Question:

In Section 8.3 you saw that $\sin 30^\circ = \cos 60^\circ$, $\cos 30^\circ = \sin 60^\circ$, and $\tan 60^\circ = \frac{1}{\tan 30^\circ}$. These are particular examples of the general results: $\sin (90^\circ - \theta) = \cos \theta$, and $\cos (90^\circ - \theta) = \sin \theta$, and $\tan (90^\circ - \theta) = \frac{1}{\tan \theta}$, where the angle θ is measured in degrees. Use a right-angled triangle ABC to verify these results for the case when θ is acute.

Solution:



With $\angle B = \theta$, $\angle A = (90^\circ - \theta)$

$$\sin \theta = \frac{b}{c}, \cos (90^\circ - \theta) = \frac{b}{c}$$

So $\cos (90^\circ - \theta) = \sin \theta$

$$\cos \theta = \frac{a}{c}, \sin (90^\circ - \theta) = \frac{a}{c}$$

So $\sin (90^\circ - \theta) = \cos \theta$

$$\tan \theta = \frac{b}{a}, \tan (90^\circ - \theta) = \frac{a}{b} = \frac{1}{\left(\frac{b}{a}\right)} = \frac{1}{\tan \theta}$$

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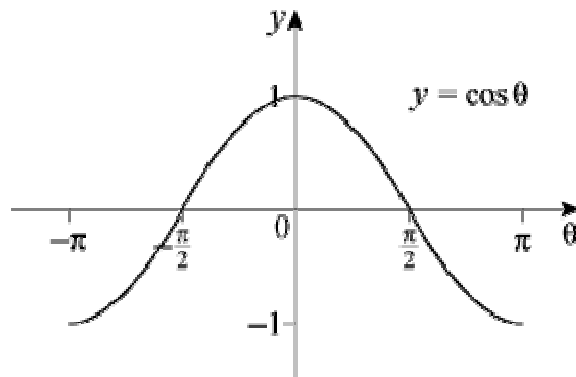
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Exercise E, Question 1

Question:

Sketch the graph of $y = \cos \theta$ in the interval $-\pi \leq \theta \leq \pi$.

Solution:



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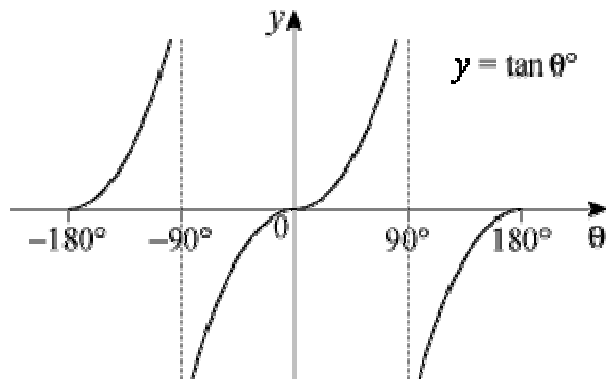
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Exercise E, Question 2

Question:

Sketch the graph of $y = \tan \theta^\circ$ in the interval $-180 \leq \theta \leq 180$.

Solution:



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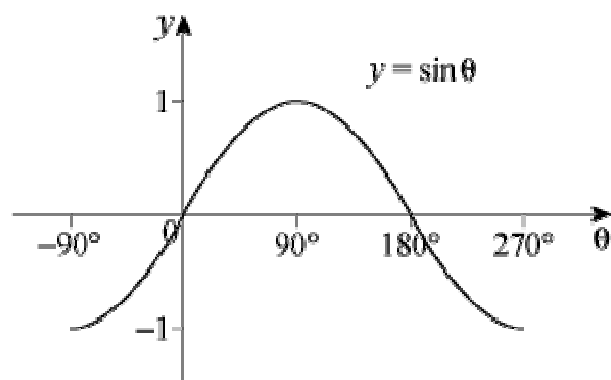
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Exercise E, Question 3

Question:

Sketch the graph of $y = \sin \theta^\circ$ in the interval $-90 \leq \theta \leq 270$.

Solution:



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Exercise F, Question 1

Question:

Write down (i) the maximum value, and (ii) the minimum value, of the following expressions, and in each case give the smallest positive (or zero) value of x for which it occurs.

(a) $\cos x^\circ$

(b) $4 \sin x^\circ$

(c) $\cos(-x)^\circ$

(d) $3 + \sin x^\circ$

(e) $-\sin x^\circ$

(f) $\sin 3x^\circ$

Solution:

(a) (i) Maximum value of $\cos x^\circ = 1$, occurs when $x = 0$.

(ii) Minimum value is -1 , occurs when $x = 180$.

(b) (i) Maximum value of $\sin x^\circ = 1$, so maximum value of $4 \sin x^\circ = 4$, occurs when $x = 90$.

(ii) Minimum value of $4 \sin x^\circ$ is -4 , occurs when $x = 270$.

(c) The graph of $\cos(-x)^\circ$ is a reflection of the graph of $\cos x^\circ$ in the y -axis.

This is the same curve; $\cos(-x)^\circ = \cos x^\circ$.

(i) Maximum value of $\cos(-x)^\circ = 1$, occurs when $x = 0$.

(ii) Minimum value of $\cos(-x)^\circ = -1$, occurs when $x = 180$.

(d) The graph of $3 + \sin x^\circ$ is the graph of $\sin x^\circ$ translated by $+3$ vertically.

(i) Maximum = 4 , when $x = 90$.

(ii) Minimum = 2 , when $x = 270$.

(e) The graph of $-\sin x^\circ$ is the reflection of the graph of $\sin x^\circ$ in the x -axis.

(i) Maximum = 1 , when $x = 270$.

(ii) Minimum = -1 , when $x = 90$.

(f) The graph of $\sin 3x^\circ$ is the graph of $\sin x^\circ$ stretched by $\frac{1}{3}$ in the x direction.

(i) Maximum = 1 , when $x = 30$.

(ii) Minimum = -1 , when $x = 90$.

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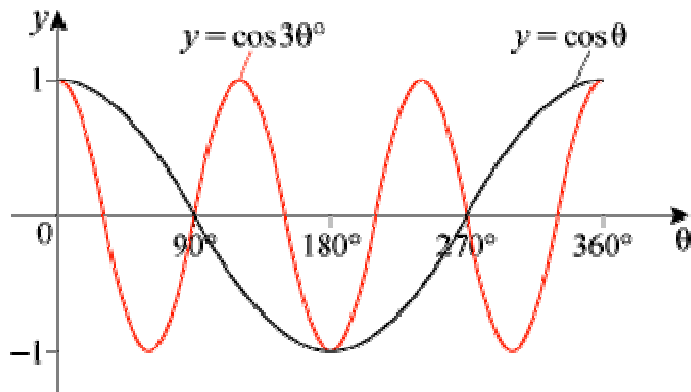
Graphics of trigonometric functions

Exercise F, Question 2

Question:

Sketch, on the same set of axes, in the interval $0 \leq \theta \leq 360^\circ$, the graphs of $\cos \theta$ and $\cos 3\theta$.

Solution:



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Exercise F, Question 3

Question:

Sketch, on separate axes, the graphs of the following, in the interval $0 \leq \theta \leq 360^\circ$. Give the coordinates of points of intersection with the axes, and of maximum and minimum points where appropriate.

(a) $y = -\cos \theta$

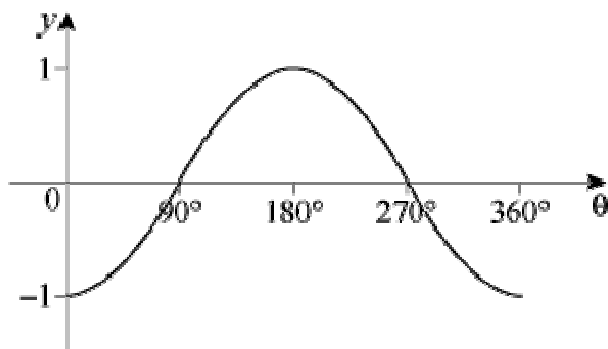
(b) $y = \frac{1}{3} \sin \theta$

(c) $y = \sin \frac{1}{3} \theta$

(d) $y = \tan (\theta - 45^\circ)$

Solution:

(a) The graph of $y = -\cos \theta$ is the graph of $y = \cos \theta$ reflected in the θ -axis.



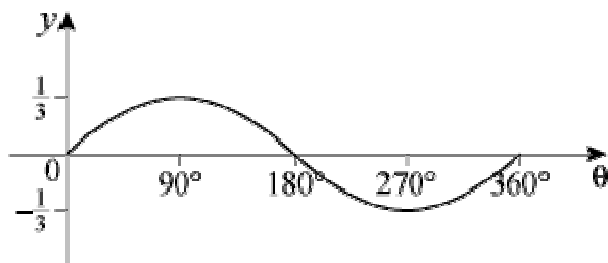
Meets θ -axis at $(90^\circ, 0)$, $(270^\circ, 0)$

Meets y -axis at $(0^\circ, -1)$

Maximum at $(180^\circ, 1)$

Minima at $(0^\circ, -1)$ and $(360^\circ, -1)$

(b) The graph of $y = \frac{1}{3} \sin \theta$ is the graph of $y = \sin \theta$ stretched by scale factor $\frac{1}{3}$ in y direction.



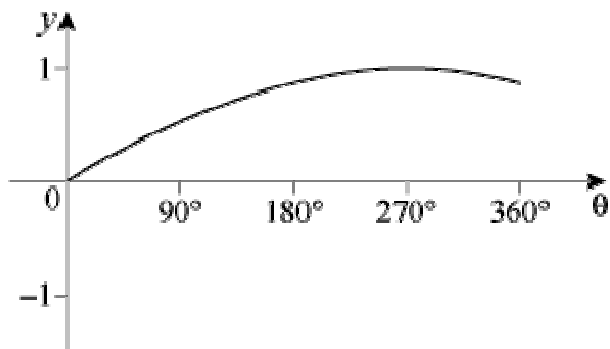
Meets θ -axis at $(0^\circ, 0)$, $(180^\circ, 0)$, $(360^\circ, 0)$

Meets y -axis at $(0^\circ, 0)$

Maximum at $\left(90^\circ, \frac{1}{3} \right)$

Minimum at $\left(270^\circ, -\frac{1}{3} \right)$

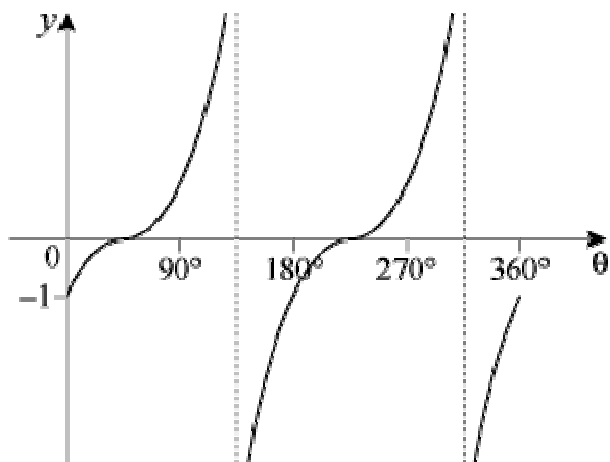
(c) The graph of $y = \sin \frac{1}{3}\theta$ is the graph of $y = \sin \theta$ stretched by scale factor 3 in θ direction.



Only meets axes at origin

Maximum at $(270^\circ, 1)$

(d) The graph of $y = \tan (\theta - 45^\circ)$ is the graph of $\tan \theta$ translated by 45° to the right.



Meets θ -axis at $(45^\circ, 0)$, $(225^\circ, 0)$

Meets y -axis at $(0^\circ, -1)$

(Asymptotes at $\theta = 135^\circ$ and $\theta = 315^\circ$)

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Graphics of trigonometric functions

Exercise F, Question 4

Question:

Sketch, on separate axes, the graphs of the following, in the interval $-180 \leq \theta \leq 180$. Give the coordinates of points of intersection with the axes, and of maximum and minimum points where appropriate.

(a) $y = -2 \sin \theta^\circ$

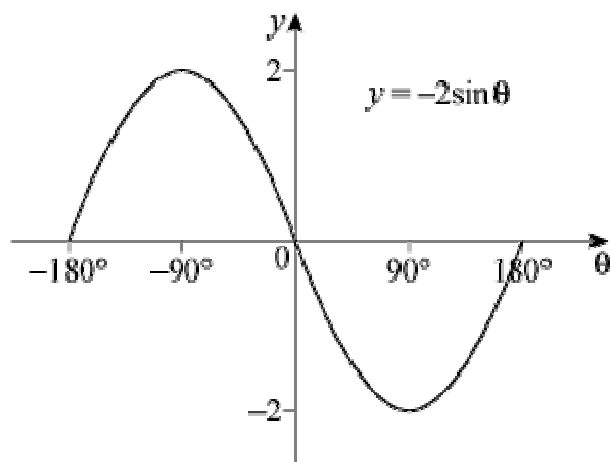
(b) $y = \tan (\theta + 180)^\circ$

(c) $y = \cos 4\theta^\circ$

(d) $y = \sin (-\theta)^\circ$

Solution:

(a) This is the graph of $y = \sin \theta^\circ$ stretched by scale factor -2 in the y direction (i.e. reflected in the θ -axis and scaled by 2 in the y direction).

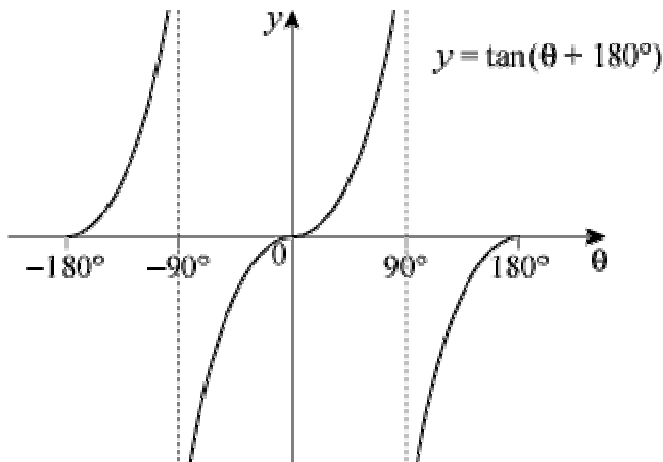


Meets θ -axis at $(-180^\circ, 0)$, $(0^\circ, 0)$, $(180^\circ, 0)$

Maximum at $(-90^\circ, 2)$

Minimum at $(90^\circ, -2)$

(b) This is the graph of $y = \tan \theta^\circ$ translated by 180° to the left.

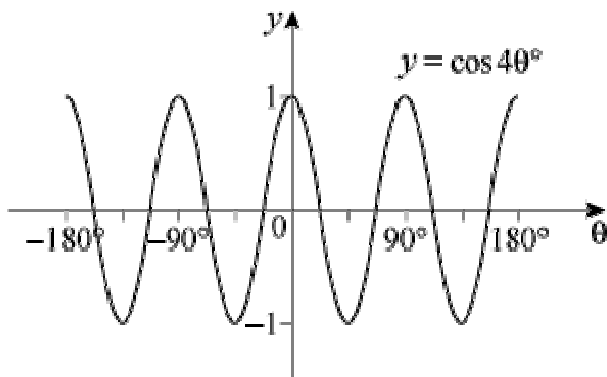


As $\tan \theta^\circ$ has a period of 180°

$$\tan(\theta + 180)^\circ = \tan \theta^\circ$$

Meets θ -axis at $(-180^\circ, 0)$, $(0^\circ, 0)$, $(180^\circ, 0)$

(c) This is the graph of $y = \cos \theta^\circ$ stretched by scale factor $\frac{1}{4}$ horizontally.



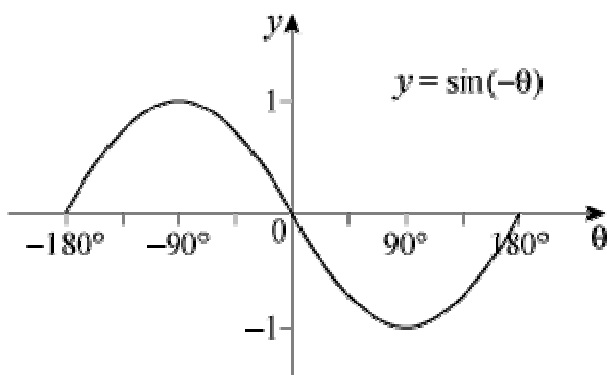
Meets θ -axis at $\left(-157\frac{1}{2}^\circ, 0\right)$, $\left(-112\frac{1}{2}^\circ, 0\right)$, $\left(-67\frac{1}{2}^\circ, 0\right)$, $\left(-22\frac{1}{2}^\circ, 0\right)$, $\left(22\frac{1}{2}^\circ, 0\right)$, $\left(67\frac{1}{2}^\circ, 0\right)$, $\left(112\frac{1}{2}^\circ, 0\right)$, $\left(157\frac{1}{2}^\circ, 0\right)$

Meets y -axis at $(0^\circ, 1)$

Maxima at $(-180^\circ, 1)$, $(-90^\circ, 1)$, $(0^\circ, 1)$, $(90^\circ, 1)$, $(180^\circ, 1)$

Minima at $(-135^\circ, -1)$, $(-45^\circ, -1)$, $(45^\circ, -1)$, $(135^\circ, -1)$

(d) This is the graph of $y = \sin \theta^\circ$ reflected in the y -axis. (This is the same as $y = -\sin \theta^\circ$.)



Meets θ -axis at $(-180^\circ, 0)$, $(0^\circ, 0)$, $(180^\circ, 0)$

Maximum at $(-90^\circ, 1)$

Minimum at $(90^\circ, -1)$

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Exercise F, Question 5

Question:

In this question θ is measured in radians. Sketch, on separate axes, the graphs of the following in the interval $-2\pi \leq \theta \leq 2\pi$. In each case give the periodicity of the function.

(a) $y = \sin \frac{1}{2}\theta$

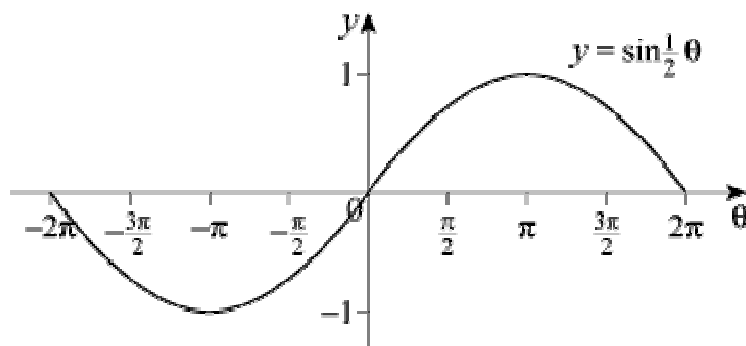
(b) $y = -\frac{1}{2}\cos \theta$

(c) $y = \tan \left(\theta - \frac{\pi}{2} \right)$

(d) $y = \tan 2\theta$

Solution:

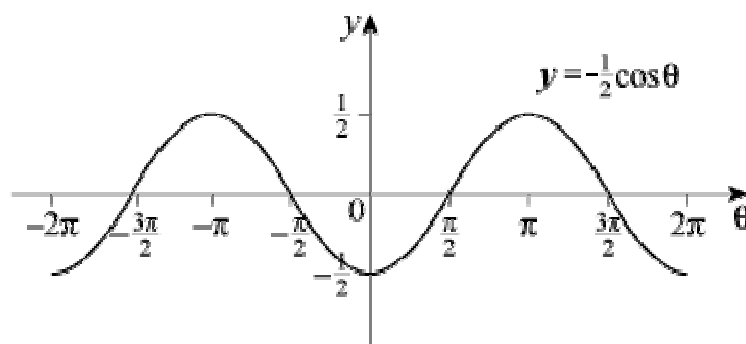
(a) This is the graph of $y = \sin \theta$ stretched by scale factor 2 horizontally.
Period = 4π



(b) This is the graph of $y = \cos \theta$ stretched by scale factor $-\frac{1}{2}$ vertically.

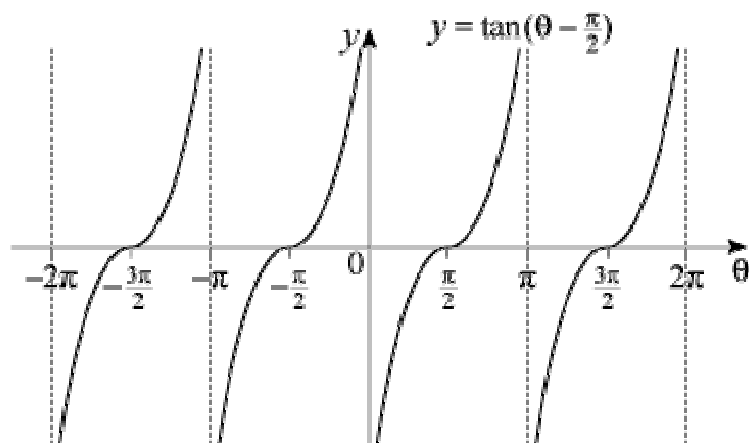
(Equivalent to reflection, in θ -axis and stretching vertically by $+\frac{1}{2}$.)

Period = 2π



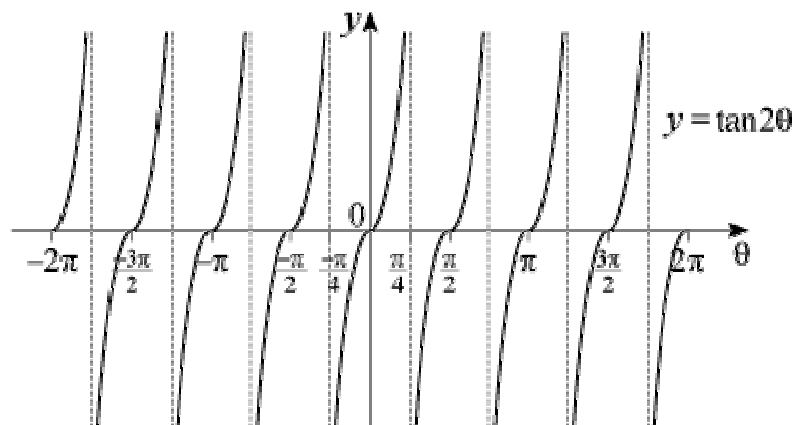
(c) This is the graph of $y = \tan \theta$ translated by $\frac{\pi}{2}$ to the right.

Period = π



(d) This is the graph of $y = \tan \theta$ stretched by scale factor $\frac{1}{2}$ horizontally.

Period = $\frac{\pi}{2}$



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Graphics of trigonometric functions

Exercise F, Question 6

Question:

(a) By considering the graphs of the functions, or otherwise, verify that:

(i) $\cos \theta = \cos (-\theta)$

(ii) $\sin \theta = -\sin (-\theta)$

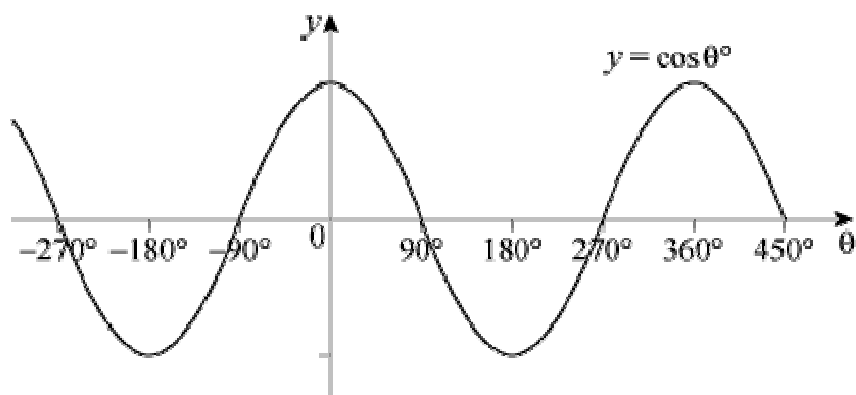
(iii) $\sin (\theta - 90^\circ) = -\cos \theta$

(b) Use the results in (a) (ii) and (iii) to show that $\sin (90^\circ - \theta) = \cos \theta$.

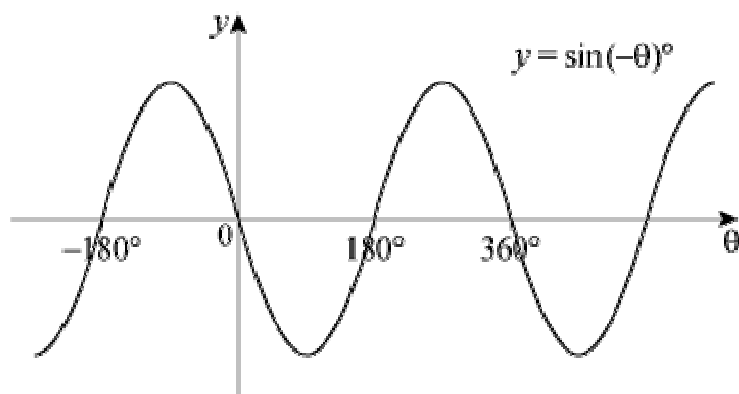
(c) In Example 11 you saw that $\cos (\theta - 90^\circ) = \sin \theta$. Use this result with part (a) (i) to show that $\cos (90^\circ - \theta) = \sin \theta$.

Solution:

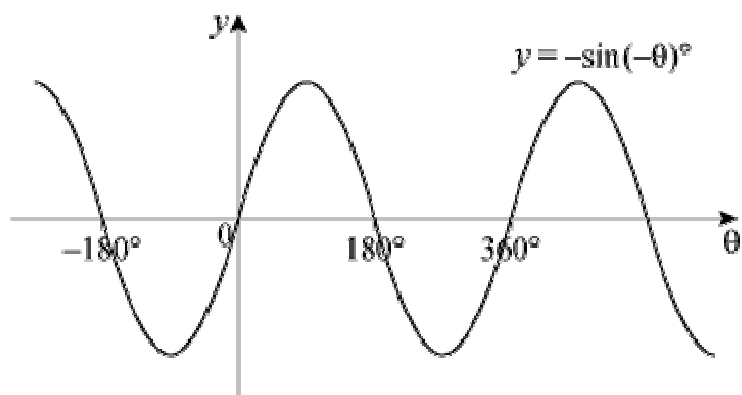
(a) (i) $y = \cos (-\theta)$ is a reflection of $y = \cos \theta$ in the y -axis, which is the same curve, so $\cos \theta = \cos (-\theta)$.



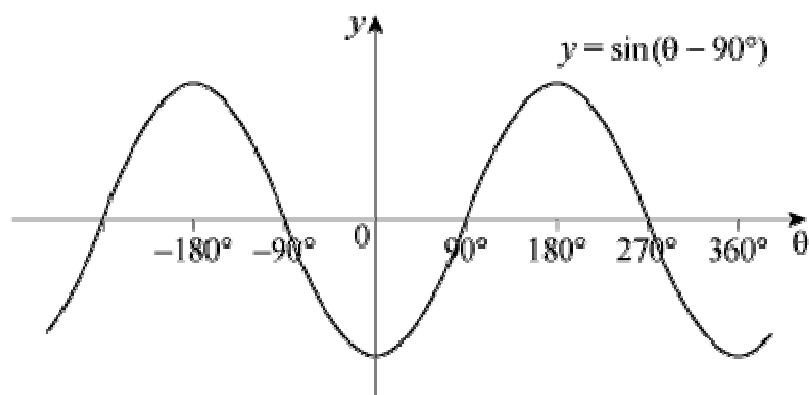
(ii) $y = \sin (-\theta)$ is a reflection of $y = \sin \theta$ in the y -axis



$y = -\sin (-\theta)$ is a reflection of $y = \sin (-\theta)$ in the θ -axis, which is the graph of $y = \sin \theta$, so $-\sin (-\theta) = \sin \theta$.



(iii) $y = \sin(\theta - 90^\circ)$ is the graph of $y = \sin \theta$ translated by 90° to the right, which is the graph of $y = -\cos \theta$, so $\sin(\theta - 90^\circ) = -\cos \theta$.



(b) Using (a) (ii), $\sin(90^\circ - \theta) = -\sin[-(90^\circ - \theta)] = -\sin(\theta - 90^\circ)$
 Using (a) (iii), $-\sin(\theta - 90^\circ) = -(-\cos \theta) = \cos \theta$
 So $\sin(90^\circ - \theta) = \cos \theta$.

(c) Using (a)(i), $\cos(90^\circ - \theta) = \cos(\theta - 90^\circ) = \sin \theta$, using Example 11.

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Graphics of trigonometric functions

Exercise G, Question 1

Question:

Write each of the following as a trigonometric ratio of an acute angle:

(a) $\cos 237^\circ$

(b) $\sin 312^\circ$

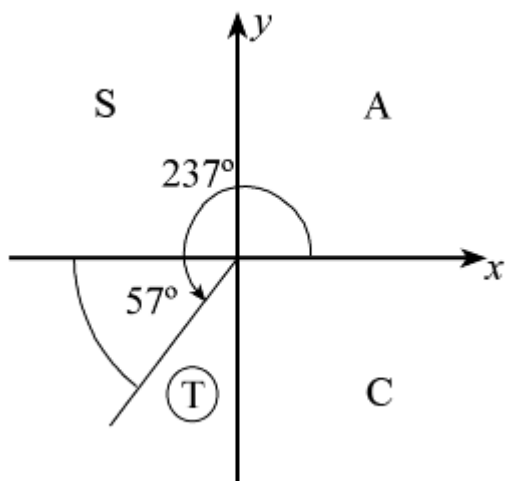
(c) $\tan 190^\circ$

(d) $\sin 2.3^\circ$

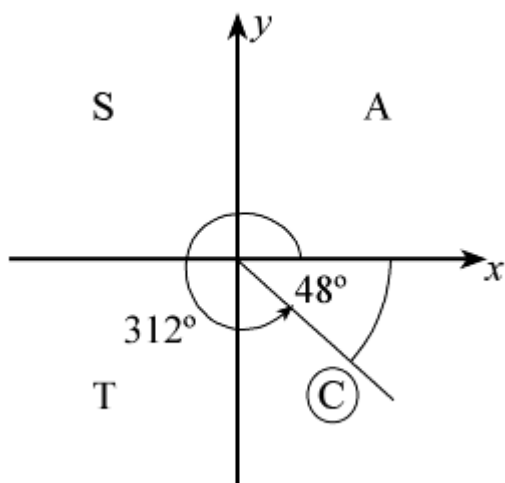
(e) $\cos \left(-\frac{\pi}{15} \right)$

Solution:

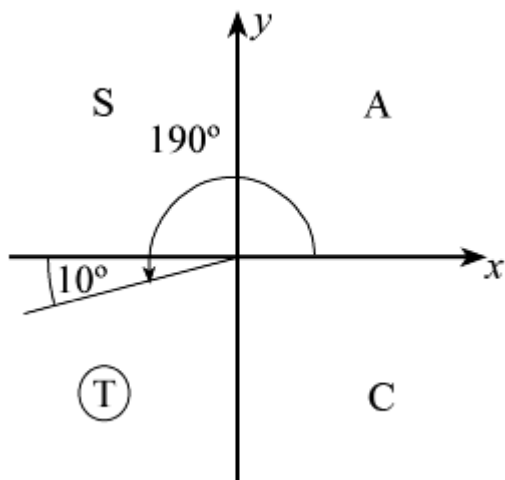
(a) 237° is in the third quadrant so $\cos 237^\circ$ is $-ve$.
The angle made with the horizontal is 57° .
So $\cos 237^\circ = -\cos 57^\circ$



(b) 312° is in the fourth quadrant so $\sin 312^\circ$ is $-ve$.
The angle to the horizontal is 48° .
So $\sin 312^\circ = -\sin 48^\circ$



(c) 190° is in the third quadrant so $\tan 190^\circ$ is +ve.
 The angle to the horizontal is 10° .
 So $\tan 190^\circ = +\tan 10^\circ$



(d) 2.3 radians ($131.78 \dots^\circ$) is in the second quadrant so $\sin 2.3^\circ$ is +ve.
 The angle to the horizontal is $(\pi - 2.3)$ radians = 0.84 radians (2 s.f.).
 So $\sin 2.3^\circ = +\sin 0.84^\circ$

(e) $-\left(\frac{\pi}{15}\right)$ is in the fourth quadrant so $\cos\left(-\frac{\pi}{15}\right)$ is +ve.

The angle to the horizontal is $\frac{\pi}{15}$.

So $\cos\left(-\frac{\pi}{15}\right) = +\cos\left(\frac{\pi}{15}\right)$

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Graphics of trigonometric functions

Exercise G, Question 2

Question:

Without using your calculator, work out the values of:

(a) $\cos 270^\circ$

(b) $\sin 225^\circ$

(c) $\cos 180^\circ$

(d) $\tan 240^\circ$

(e) $\tan 135^\circ$

(f) $\cos 690^\circ$

(g) $\sin \frac{5\pi}{3}$

(h) $\cos \left(-\frac{2\pi}{3} \right)$

(i) $\tan 2\pi$

(j) $\sin \left(-\frac{7\pi}{6} \right)$

Solution:

(a) $\sin 270^\circ = -1$ (see graph of $y = \sin \theta$)

(b) $\sin 225^\circ = \sin \left(180 + 45 \right)^\circ = -\sin 45^\circ = -\frac{\sqrt{2}}{2}$

(c) $\cos 180^\circ = -1$ (see graph of $y = \cos \theta$)

(d) $\tan 240^\circ = \tan (180 + 60)^\circ = +\tan 60^\circ$ (third quadrant)
So $\tan 240^\circ = +\sqrt{3}$

(e) $\tan 135^\circ = -\tan 45^\circ$ (second quadrant)
So $\tan 135^\circ = -1$

(f) $\cos 690^\circ = \cos (360 + 330)^\circ = \cos 330^\circ = +\cos 30^\circ$ (fourth quadrant)
So $\cos 690^\circ = +\frac{\sqrt{3}}{2}$

$$(g) \sin \frac{5\pi}{3} = -\sin \frac{\pi}{3} \text{ (fourth quadrant)}$$

$$\text{So } \sin \frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$(h) \cos \left(-\frac{2\pi}{3} \right) = -\cos \frac{\pi}{3} \text{ (third quadrant)}$$

$$\text{So } \cos \left(-\frac{2\pi}{3} \right) = -\frac{1}{2}$$

$$(i) \tan 2\pi = 0 \text{ (see graph of } y = \tan \theta \text{)}$$

$$(j) \sin \left(-\frac{7\pi}{6} \right) = +\sin \left(\frac{\pi}{6} \right) \text{ (second quadrant)}$$

$$\text{So } \sin \left(-\frac{7\pi}{6} \right) = +\frac{1}{2}$$

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Graphics of trigonometric functions

Exercise G, Question 3

Question:

Describe geometrically the transformations which map:

- (a) The graph of $y = \tan x^\circ$ onto the graph of $\tan \frac{1}{2}x^\circ$.
- (b) The graph of $y = \tan \frac{1}{2}x^\circ$ onto the graph of $3 + \tan \frac{1}{2}x^\circ$.
- (c) The graph of $y = \cos x^\circ$ onto the graph of $-\cos x^\circ$.
- (d) The graph of $y = \sin (x - 10)^\circ$ onto the graph of $\sin (x + 10)^\circ$.

Solution:

- (a) A stretch of scale factor 2 in the x direction.
- (b) A translation of $+3$ in the y direction.
- (c) A reflection in the x -axis
- (d) A translation of $+20$ in the negative x direction (i.e. 20 to the left).

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Graphics of trigonometric functions

Exercise G, Question 4

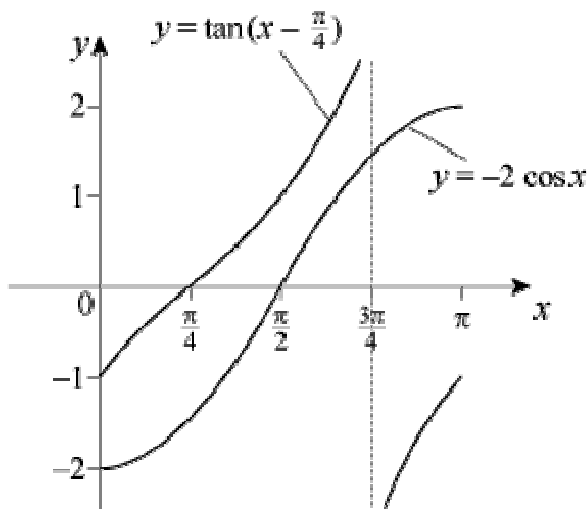
Question:

(a) Sketch on the same set of axes, in the interval $0 \leq x \leq \pi$, the graphs of $y = \tan\left(x - \frac{1}{4}\pi\right)$ and $y = -2 \cos x$, showing the coordinates of points of intersection with the axes.

(b) Deduce the number of solutions of the equation $\tan\left(x - \frac{1}{4}\pi\right) + 2 \cos x = 0$, in the interval $0 \leq x \leq \pi$.

Solution:

4 (a)



(b) There are no solutions of $\tan\left(x - \frac{\pi}{4}\right) + 2 \cos x = 0$ in the interval $0 \leq x \leq \pi$, since $y = \tan\left(x - \frac{\pi}{4}\right)$ and $y = -2 \cos x$ do not intersect in the interval.

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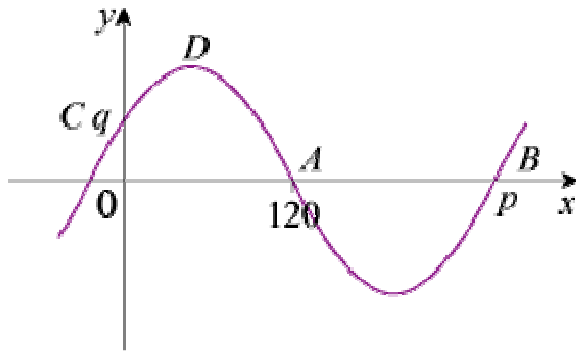
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Graphics of trigonometric functions

Exercise G, Question 5

Question:

The diagram shows part of the graph of $y = f(x)$. It crosses the x -axis at $A(120, 0)$ and $B(p, 0)$. It crosses the y -axis at $C(0, q)$ and has a maximum value at D , as shown.



Given that $f(x) = \sin(x + k)^\circ$, where $k > 0$, write down:

- the value of p
- the coordinates of D
- the smallest value of k
- the value of q

Solution:

(a) As it is the graph of $y = \sin x^\circ$ translated, the gap between A and B is 180, so $p = 300$.

(b) The difference in the x -coordinates of D and A is 90, so the x -coordinate of D is 30.
The maximum value of y is 1, so $D = (30, 1)$.

(c) For the graph of $y = \sin x^\circ$, the first positive intersection with the x -axis would occur at 180. The point A is at 120 and so the curve has been translated by 60 to the left.
 $k = 60$

(d) The equation of the curve is $y = \sin(x + 60)^\circ$.
When $x = 0$, $y = \sin 60^\circ = \frac{\sqrt{3}}{2}$, so $q = \frac{\sqrt{3}}{2}$.

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Graphics of trigonometric functions

Exercise G, Question 6

Question:

Consider the function $f(x) = \sin px$, $p \in \mathbb{R}$, $0 \leq x \leq 2\pi$.

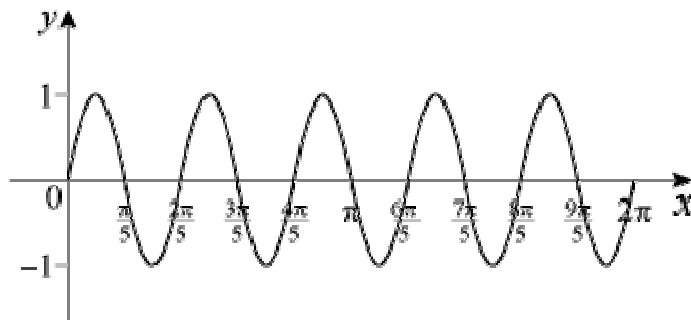
The closest point to the origin that the graph of $f(x)$ crosses the x -axis has x -coordinate $\frac{\pi}{5}$.

- (a) Sketch the graph of $f(x)$.
- (b) Write down the period of $f(x)$.
- (c) Find the value of p .

Solution:

(a) The graph is that of $y = \sin x$ stretched in the x direction.

Each 'half-wave' has interval $\frac{\pi}{5}$.



(b) The period is a 'wavelength', i.e. $\frac{2\pi}{5}$.

(c) The stretch factor is $\frac{1}{p}$.

As 2π has been reduced to $\frac{2\pi}{5}$, 2π has been multiplied by $\frac{1}{5}$ which is $\frac{1}{p} \Rightarrow p = 5$.

The curve is $y = \sin 5x$, there are 5 'waves' in 0 to 2π .

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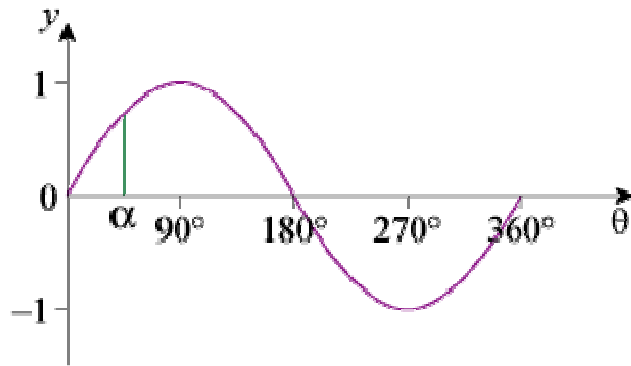
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Graphics of trigonometric functions

Exercise G, Question 7

Question:

The graph below shows $y = \sin \theta$, $0 \leq \theta \leq 360^\circ$, with one value of θ ($\theta = \alpha^\circ$) marked on the axis.

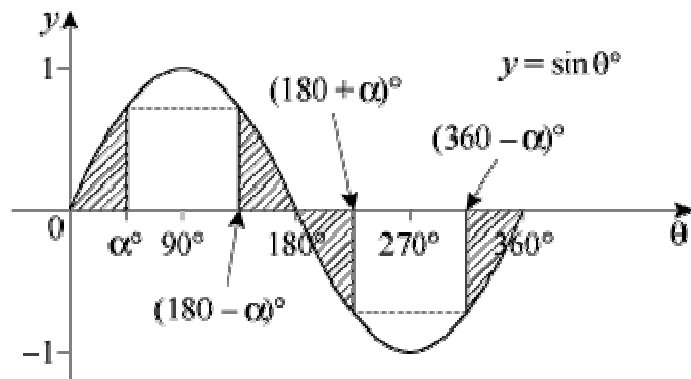


(a) Copy the graph and mark on the θ -axis the positions of $(180 - \alpha)^\circ$, $(180 + \alpha)^\circ$, and $(360 - \alpha)^\circ$.

(b) Establish the result $\sin \alpha^\circ = \sin (180 - \alpha)^\circ = -\sin (180 + \alpha)^\circ = -\sin (360 - \alpha)^\circ$.

Solution:

(a) The four shaded regions are congruent.



(b) $\sin \alpha^\circ$ and $\sin (180 - \alpha)^\circ$ have the same y value (call it k).

So $\sin \alpha^\circ = \sin (180 - \alpha)^\circ$

$\sin (180 + \alpha)^\circ$ and $\sin (360 - \alpha)^\circ$ have the same y value, which will be $-k$.

So $\sin \alpha^\circ = \sin (180 - \alpha)^\circ = -\sin (180 + \alpha)^\circ = -\sin (360 - \alpha)^\circ$

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Graphics of trigonometric functions

Exercise G, Question 8

Question:

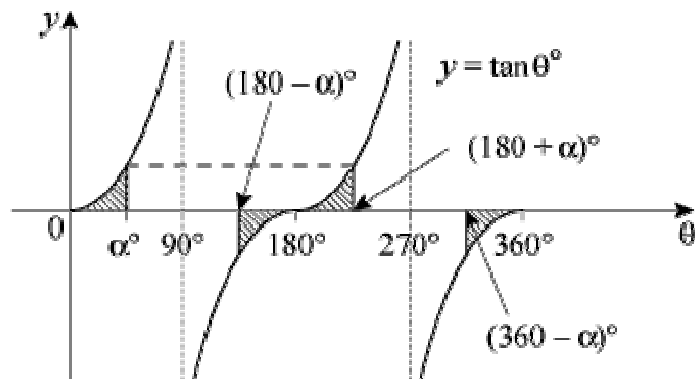
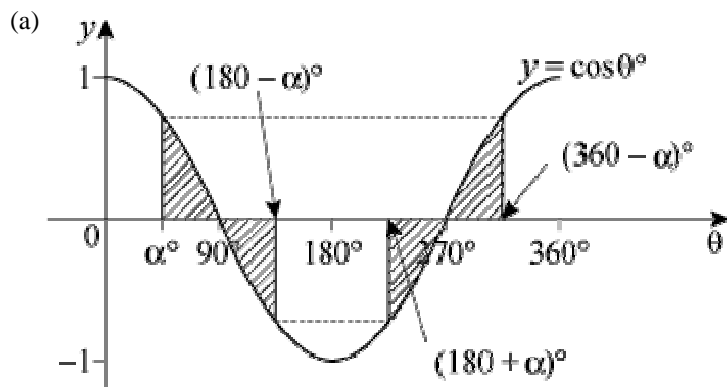
(a) Sketch on separate axes the graphs of $y = \cos \theta$ ($0 \leq \theta \leq 360^\circ$) and $y = \tan \theta$ ($0 \leq \theta \leq 360^\circ$), and on each θ -axis mark the point $(\alpha^\circ, 0)$ as in question 7.

(b) Verify that:

(i) $\cos \alpha^\circ = -\cos (180 - \alpha)^\circ = -\cos (180 + \alpha)^\circ = \cos (360 - \alpha)^\circ$.

(ii) $\tan \alpha^\circ = -\tan (180 - \alpha)^\circ = -\tan (180 + \alpha)^\circ = -\tan (360 - \alpha)^\circ$.

Solution:



(b) (i) From the graph of $y = \cos \theta^\circ$, which shows four congruent shaded regions, if the y value at α° is k , then y at $(180 - \alpha)^\circ$ is $-k$, y at $(180 + \alpha)^\circ$ is $-k$ and y at $(360 - \alpha)^\circ$ is $+k$.

So $\cos \alpha^\circ = -\cos (180 - \alpha)^\circ = -\cos (180 + \alpha)^\circ = \cos (360 - \alpha)^\circ$

(ii) From the graph of $y = \tan \theta^\circ$, if the y value at α° is k , then at $(180 - \alpha)^\circ$ it is $-k$, at $(180 + \alpha)^\circ$ it is $+k$ and at $(360 - \alpha)^\circ$ it is $-k$.

So $\tan \alpha^\circ = -\tan (180 - \alpha)^\circ = +\tan (180 + \alpha)^\circ = -\tan (360 - \alpha)^\circ$