Effects of Clipping on the Error Performance of OFDM in Frequency Selective Fading Channels

Kusha R. Panta and Jean Armstrong

Abstract—**Previous studies on the effect of the clipping noise on the error performance of orthogonal frequency-division multiplexing (OFDM) systems in frequency selective fading channels provide pessimistic results. They do not consider the effect of channel fading on the clipping noise. The clipping noise is added at the transmitter and hence fades with the signal. Here, the authors show that the "bad" subcarriers that dominate the error performance of the OFDM system are least affected by the clipping noise and, as a result, the degradation in the error performance of OFDM system in fading channels is very small.**

Index Terms—**Frequency selective fading channel, in-band distortion and clipping noise, orthogonal frequency-division multiplexing (OFDM), peak-to-average power ratio (PAPR).**

I. Introduction

A MAJOR disadvantage of orthogonal frequency division multiplexing (OFDM) is that it has a high peak-to-average power ratio (PAPR). Clipping of high amplitude peaks is a simple and effective way to reduce PAPR [1]. However, clipping causes distortion of the transmitted signal. In general, clipping will cause both in-band and out-of-band distortion. It has been shown that the out-of-band distortion can be removed using a simple filter [1]. The clipping and filtering technique presented in [1] clips the oversampled signal and uses a discrete Fourier transform (DFT)-based filter to remove the OOB power without adding any distortion to the signal. Recently, Dardari *et al.* [2] showed that the remaining in-band distortion has two effects on the transmitted signals: an overall shrinking of the signal constellation and an added noise-like component called *clipping noise*. The shrinking of the constellation affects all subcarriers equally and can be modeled as a reduction in the overall transmitted signal power. Clipping noise can be modeled as an additive noise component that is added at the transmitter. Earlier papers, which studied the effect of in-band distortion on the performance of OFDM systems [3], [4], overestimated the effect of clipping noise on the system's performance because they did not take into account the shrinking of the constellation or the fact that clipping noise is added at the transmitter rather than the receiver.

Clipping ratio (CR) is often used as a measure of clipping. CR is defined as the ratio of the maximum signal amplitude that is allowed to pass unclipped to the root-mean-square power of the unclipped signal. In this paper, we present a theoretical analysis of clipping noise in a frequency selective fading channel. The clipping noise is caused by the clipping technique used to reduce the PAPR of OFDM signals. A theoretical model of an OFDM subcarrier with transmitter-added clipping noise is presented. The effect of transmitter-added clipping noise on the received signal-to-noise ratio (SNR) of OFDM subcarriers is studied. Results show that the effect of transmitter-added noise on the received SNR on subcarriers subject to severe fading is minimal. Simulation results of the error performance of OFDM system subject to different levels of clipping are given for an additive white Gaussian noise (AWGN) channel and a two-path fading channel. We will show that the effect of clipping on the error performance of the OFDM system is negligible in frequency selective fading channels.

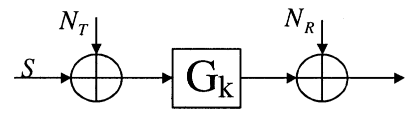

Fig. 1. Theoretical model of an OFDM subcarrier with noise added at the transmitter.

The signal peaks are clipped and filtered according to the scheme described in [1]. The oversampling of signals is performed before clipping is needed, as signal peaks that may occur in between the inverse DFT outputs will otherwise be missed during clipping [5]. In our simulations, signals are oversampled with a sampling factor of two. Increasing the sampling factor beyond two does not result in significant improvement in PAPR reduction [1].

II. Effect of Transmitter-Added Noise on the SNR of the Received Signal

The effect of fading on the clipping noise can be understood by considering Fig. 1, which shows the model of an OFDM subcarrier with clipping noise. A complete OFDM transmission with N subcarriers is modeled by N parallel channels of this form. The transmitted signal power per subcarrier is S. This value takes account of the overall shrinkage of the signal constellation caused by clipping. The clipping noise per subcarrier is modeled by N_T which is added to the signal at the transmitter. The effect of the channel on the subcarrier is represented by G_K. Noise added in the channel and/or the receiver is represented by N_R. This is the noise that would be present in an OFDM system without clipping. The values of S, N_T, and N_R are the same for each OFDM subcarrier. The value of G_k depends on the fading

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The authors are with the Department of Electronic Engineering, La Trobe University, Bundoora, Victoria 3086, Australia (e-mail: k.panta@ee.latrobe.edu.au).

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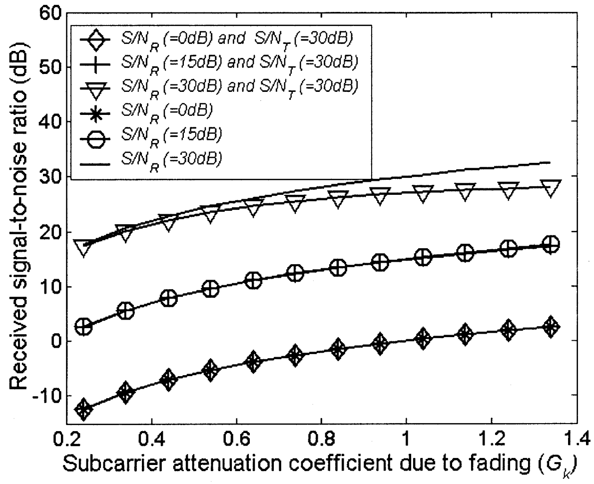

Fig. 2. SNR of a received signal in a two-path channel.

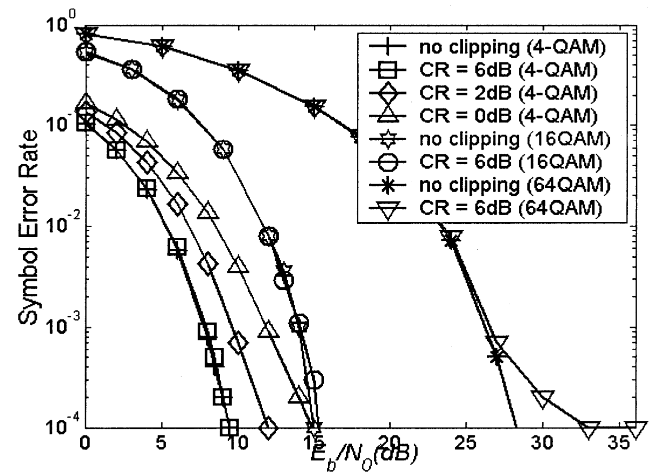

Fig. 3. SER versus E_b/N_b in an AWGN for varying clipping ratios.

characteristics of the channel. For AWGN and flat fading channels, G_k will be the same for all subcarriers. In a frequency selective fading channel, G_k will be relatively large for "good" subcarriers and relatively small for subcarriers in deep fades.

The signal-to-noise ratio (SNR) of the kth subcarrier at the receiver is given by

$$\mathrm{SNR}_k = \frac{G_k^2 S}{G_k^2 N_T + N_R} = \frac{S}{N_T + \frac{N_R}{G_k^2}}. \quad (1)$$

Equation (1) shows that there is an overall reduction of SNR at the receiver due to the addition of N_T to the signal. At the receiver input, the overall noise consists of faded noise $G_k^2 N_T$ and N_R. For a subcarrier that is subject to severe fading and therefore has a small G_k, $G_k^2 N_T$ becomes considerably smaller than N_R. As a result, the degradation of the received SNR caused by clipping will be very small. Previous studies [3], [4] miss the point that in channels with frequency selective fading the clipping noise that is added at the transmitter is subject to fading and hence is not as bad as channel or receiver added noise.

Fig. 2 shows the effect of transmitter-added noise on the SNR of the received signal in a frequency selective fading channel. To model a channel subject to a moderate level of frequency selective fading, we considered a two-path fading channel where signal power of the echo path is half of that of the direct path. Similar results would follow for channels subject to fading greater than this. The delay in the echo signal equals $T/4$ where T is the symbol period without the cyclic prefix (CP). The transmitted power is normalized so that the total received power will be independent of the power of the echo signal. In this case, G_k varies from 0.24 on bad subcarriers to 1.40 on good subcarriers. The plots show SNR_k as a function of G_k for a fading channel. Results are shown for systems with and without clipping. For the systems without clipping, N_T is zero. For systems with clipping a value of S/N_T of 30 dB was used. This value was chosen as the results in [1] show that practical levels of clipping (about 6–7 dB of CR) would result in S/N_T of around this value. The plots are obtained for three different levels of channel noise chosen to give S/N_R of 0, 15, and 30 dB. Fig. 2 shows that when N_R is significantly higher than N_T as in the case of $S/N_R = 0$ dB and $S/N_R = 15$ dB, the noise added at the transmitter does not degrade the received SNR significantly. The effect of the transmitter-added noise is only noticeable when its level is comparable to the channel noise. However, even in such cases the degradation of SNR is very small for subcarriers with small G_k.

III. Effect of Clipping Noise on the Error Performance of an OFDM System

In the case of an OFDM system subject to frequency selective fading, the error rate before error correction depends mainly on the performance of the subcarriers which lie in deep fades [6]. In Section II, we have shown that the overall SNR degradation due to the clipping noise at the deep fades is very small. In this section, it will be shown that the clipping noise will have a minimal effect on the error performance of an OFDM system in frequency selective fading channels. The shrinking of the signal constellation caused by clipping is corrected by adjusting the average transmitted power. No symbol-by-symbol adjustment to gain is required at either the receiver or the transmitter and the receiver is not assumed to have any knowledge of whether clipping is used in the system or whether an individual symbol is clipped.

Fig. 3 shows the symbol error rate (SER) plots for an OFDM system in AWGN with varying levels of clipping and different QAM constellations. The ratio of the transmitted energy per bit to noise E_b/N_o is used as a measure of SNR. The simulations are for a system with $N = 64$ but other values of N would give similar results. The length of the CP is $T/4$. Clipping with a CR of 6–7 dB is often enough in practical OFDM systems [7]. Fig. 3 shows that the degradation in the SER performance is negligible for clipping with a CR of 6 dB for 4-, 16-, and 64-QAM. The effect of clipping noise is more obvious for 64-QAM at high E_b/N_o. For 64-QAM, the SER plot plateaus for E_b/N_o above 30 dB. This is because the clipping noise becomes the dominant noise as the channel added noise becomes smaller. Fig. 3 also shows SER plots for 4-QAM when the signal peaks are clipped with a CR of 0 and 2 dB. The plots show that even extreme clipping results in a comparatively small SER degradation. However, larger constellations would be more sensitive.

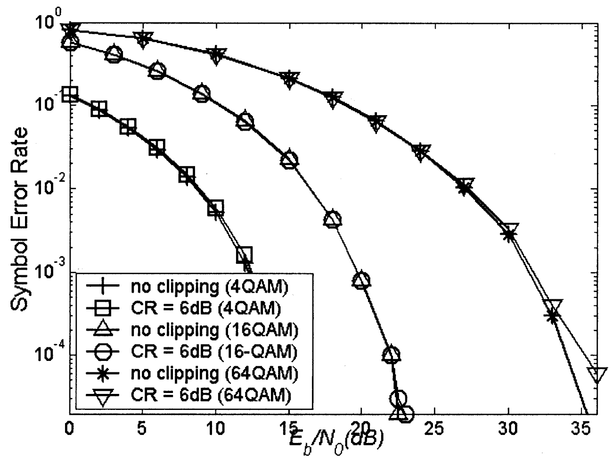

Fig. 4. SER versus E_b/N_o in a two-path channel for varying clipping ratios when CP = delay.

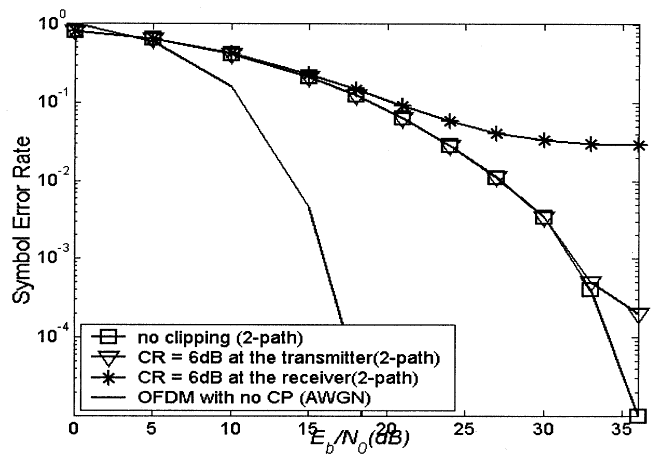

Fig. 5. SER versus E_b/N_o for 64 QAM with clipping at the transmitter and at the receiver when CP = delay.

Fig. 4 shows the results for the two-path fading channel previously described in Section II. The CP equals the delay in the echo path, but any delay spread less than the CP would give similar results. Fig. 4 shows that the degradation in the error rates is very small for clipping with the CR of 6 dB. Fig. 4 shows that the SER plot for 64-QAM plateaus at a higher value of E_b/N_o than for AWGN channels. The overall effect of channel fading is to minimize the effect of transmitter-added noise on the error performance of the OFDM system. The transmitter noise is subject to fading and becomes very small compared to channel/receiver added noise. The actual effect depends on the fading characteristics of the individual subcarriers, particularly the ones that lie in deep fades.

Fig. 5 shows the SER plots for 64-QAM with clipping applied at the transmitter or at the receiver. This is used to demonstrate that in fading channels, performance prediction based on the level of clipping noise can be misleading due to the fact that clipping usually occurs at the transmitter rather than the receiver is considered. Results are shown for the two-path channel described earlier in Section II. Clipping of 6 dB is considered. Fig. 5 shows that the degradation of the error performance of OFDM in the two-path channel is minimal when the clipping is performed at the transmitter. However, the degradation will be significant when the clipping is performed at the receiver. This is because in this case, only the transmitted signal is subject to fading, not the clipping noise.

One of the reviewers pointed out that a related study has been presented in [8] while this paper was under review. Results in [8] are for a noiseless flat fading channel. However, practical OFDM systems suffer from fading that is not constant over all subcarriers and it has been established that the overall error performance of OFDM depends mainly on the error performance of the subcarriers that are in deep fades [6], [7]. Moreover, in [8] clipping has been considered both at the transmitter and the receiver. In the absence of channel noise, the clipping noise added at the receiver will be the dominant noise. However, for the values of E_b/N_o at which most practical systems operate, the channel added noise will be the dominant noise. Therefore, results obtained without considering noise introduced by the channel can be misleading.

IV. Conclusion

In this paper, it has been shown that the effect of the clipping at the transmitter on the error performance of the OFDM system subject to frequency selective fading is minimal. Clipping of the signal peaks occurs rarely and, therefore, results in relatively small clipping noise. The clipping noise is added to the transmitted signal and in a frequency selective fading channel fades along with the signal. Its effect on the received SNR is very small for subcarriers that lie in deep fades. As a result, the degradation in the error performance of these bad subcarriers is also very small. This paper presents a more accurate study of the effect of the clipping noise on the error performance of OFDM system by including the effect of frequency selective channel fading. The analysis takes into account the effect of the shrinking of the signal constellation caused by clipping.

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Kusha R. Panta received the B.S. degree in electronic engineering (with first class honors) from The University of Melbourne, Melbourne, Australia, in 1997 and the M.S. degree from La Trobe University, Australia, in 2002.

He worked as a Software Engineer for Fujitsu Singapore, Ltd., from 1998 to 2000. He has been working as a Research Assistant at the Department of Electronic Engineering, La Trobe University since March 2002. His current research interests include multicarrier modulation techniques (OFDM technology), signal processing, and statistical modeling of communication networks.

Jean Armstrong received the B.Sc. in electrical engineering from the University of Edinburgh, U.K., in 1974, the M.Sc. degree in digital techniques from Heriot-Watt University, Edinburgh, U.K., in 1980, and the Ph.D. degree in digital communications from Monash University, Melbourne, Australia, in 1993.

From 1974 to 1977, she worked as a Design Engineer at Hewlett-Packard Ltd., U.K. In 1977, she was appointed as a Lecturer in electrical engineering at the University of Melbourne, Australia. Since 1977, she has held a variety of academic positions at the University of Melbourne, Monash University, and La Trobe University, Australia. Her research interests include digital communications, engineering education, and women in engineering and she has published over 50 research papers. She is currently an Associate Professor at La Trobe University.