Fuzzy Reliability Evaluation of Loosely Coupled Parallel Computer Interconnection Networks

N.K. Barpanda[1], M.Panda[2]
[1]*Reader, Department of Electronics, Sambalpur University,Odisha,India*
[2]*Assistant Professor, Computer Science, Sambalpur University,Odisha,India*

Abstract- Uncertainty affects decision-making and appears in a number of different forms. The effects of uncertainty in a system can be handled in a better way by using fuzzy logic. This paper presents a new and simple method for evaluating the fuzzy reliability of loosely coupled interconnection networks. A new algorithm to evaluate the fuzzy reliability has been proposed. The algorithm enumerates all the path sets from the source node to destination node. Then the system fuzzy reliability is expressed in terms of fuzzy probability of the disjoint terms of all path sets. The proposed method is well illustrated through a simple example. Fuzzy reliabilities of some important loosely coupled interconnection networks have been evaluated by the proposed method.

Keywords- Reliability, Fuzzy set, Interconnection network

NOTATIONS

X	a set containing a space of points in the probability domain
x	an element of X
p_i	fuzzy probability of an event i
$\overline{p}_i$	complement of fuzzy probability of an event i
$\mu_{p_i}(p)$	membership function of fuzzy probability p_i
N	number of nodes of the MIN
R	fuzzy reliability of MIN
G	reliability logic graph
V	vertex set
E	edge set
S	system success containing all paths between the source node(s) to destination node (t)
S_k	*kth minimal path set*
R_k	*the fuzzy reliability at kth step*
P_i	path at the i^{th} step
$W, \overline{W}$	indicator variables

Assumptions

1. Initially, all components of the system are in good conditions.
2. The link failure and link success probability is assumed to be fuzzy numbers
3. Failures can not be determined with certainty.
4. Repair facility is not available.

I. INTRODUCTION

Uncertainty affects the computation process, prediction mechanism and decision-making as it appears in different forms. The uncertainty involved in any problem solving situation is a result of some information deficiency, which may be incomplete, imprecise, fragmentary, not fully reliable, vague, contradictory, or deficient in some other ways. So the effects of uncertainty in a system can be handled in a better way by using the concept of fuzzy set theory [1].

The analysis soft reliability is of paramount importance in an interconnection system where thousands of processors cooperate with each other to solve a complicated problem [2]. However, there lies a large degree of uncertainty in system failure and therefore, the conventional methods of reliability evaluation for large interconnection system may not be appropriate to get a realistic view. It is also well known that the conventional method of reliability analysis, using the concept of probabilities, has been found to be inadequate to handle uncertainty of failure data and modeling. To overcome this problem, the concept of fuzzy set approach is necessary to be used in the evaluation of the reliability of interconnection systems.

Fuzzy set theory was first introduced by Zadeh [1]. A fuzzy set can be defined as follows:

Let X be a space of points and an element of X be denoted by x, i.e. $X=\{x\}$. A fuzzy set A in X is characterized by a membership function $\mu_A(x)$, which is a real number in the interval [0,1] and represents the degree of membership of x in A. In the conventional methods [2-4], it is required to find the minimized expression of system reliability using Boolean algebra. However, these expressions cannot be used in fuzzy set theory because of non-applicability of complementary laws. The expression used for fuzzy reliability of parallel systems has to be different from the expression of conventional probability analysis for obvious reason.

Keller and Kara-Zaitri [5] presented a method for assessment of reliability of a non-series parallel network using fuzzy logic. Soman and Misra analysed fuzzy fault tree using resolution identity [6], Tanaka et al [7] and Misra and Weber [8] showed how fuzzification can be carried out for the quantitative analysis of fault tree. Chowdhury and Mishra [9] evaluated the reliability of a non-series parallel network. Bastani et al [10] considered the reliability modeling continuous process-control system. Patra et al [11] presents a method for evaluating fuzzy reliability of a communication network with fuzzy element capacities and probabilities. Tripathy et al [12] evaluate the fuzzy reliability of multistage interconnection network. But none of methods discussed above considers the cube based interconnection networks and suggests a general method of evaluating fuzzy reliability of the said networks where there lies a large degree of uncertainty in system failure. So, there is always a need to search for a general and efficient method to evaluate the fuzzy reliability of such systems.

In this paper, a general and efficient method has been proposed to find an expression of fuzzy system reliability of parallel systems taking into consideration the special requirements of fuzzy sets. This method is supported by an efficient algorithm.

II. CONCEPT OF FUZZY PROBABILITY

Fuzzy probability represents a fuzzy number between zero and one, assigned to the probability of an event. One can choose different types of membership functions for fuzzy probability. For instance, a fuzzy probability may have a trapezoidal membership function. The fuzzy probabilities of an event i can then be denoted by a four parameter function i.e.

$$p_i = (\alpha_{i1}, \alpha_{i2}, \beta_{i2}, \beta_{i1}) \qquad (1)$$

The membership function is given by

$$\mu_{p_i}(p) = \begin{cases} 0, & 0 \le p \le \alpha_{i1} \\ 1 - \dfrac{\alpha_{i2} - p}{\alpha_{i2} - \alpha_{i1}} & \alpha_{i1} \le p \le \alpha_{i2} \\ 1 & \alpha_{i2} \le p \le \beta_{i2} \\ 1 - \dfrac{p - \beta_{i2}}{\beta_{i1} - \beta_{i2}} & \beta_{i2} \le p \le \beta_{i1} \\ 0 & \beta_{i1} \le p \le 1 \end{cases} \qquad (2)$$

Operation used in computing fuzzy reliability

Let pi and pj be two fuzzy sets that have membership functions given by $\mu(p_i) \& \mu(p_j)$, respectively. The operations used in fuzzy reliability evaluation, i.e. multiplication and complementation can be defined as follows:

1. *Multiplication-*

$p_i.p_j$ = *product of* p_i *and* p_j

$$= \mu_{p_i p_j}(p) = \mu_{p_i}(p).\mu_{p_j}(p) \qquad (3)$$

How ever, Tanka et al [8] provided an approximation of the multiplication procedure by defining

$$p_{ij} = p_i.p_j = (\alpha_{i1}.\alpha_{j2}, \alpha_{i2}.\alpha_{j2}, \beta_{i2}.\beta_{j2}, \beta_{i1}\beta_{j1}) \qquad (4)$$

2. *Complementation*

The complementation of any fuzzy set p_i will be given by

$$\bar{p}_i = 1 - \mu_{p_i} \qquad (5)$$

for example, in case of trapezoidal membership function, one could obtain

$$\bar{p}_i = (1-\alpha_{i1}, 1-\alpha_{i2}, 1-\beta_{i2}, 1-\beta_{i1}) \qquad (6)$$

III. PROPOSED METHOD FOR FUZZY RELIABILITY EVALUATION

First, the loosely coupled (cube based) interconnection network is converted into the equivalent reliability logic graph G {V, E}, where V is the vertex set and E is the link (edge) set. The edge (link) success and edge failure probability is assumed to be fuzzy numbers. Let S_k denote the k^{th} minimal path set and p_k be the fuzzy probability associated with S_k. Also let R_k be the fuzzy reliability at k^{th} step of the sum of fuzzy path probabilities. The reliability expression can be found out by a recursive formula:

$$R_k = R_{k-1} + \Pr\left\{ S_k \cap \overline{\left(\bigcup_{i}^{k-1} S_i \right)} \right\}$$
$$= R_{k-1} + \Pr\{ S_k \cap \bar{S}_1 \cap \bar{S}_2 \cap \cdots \cap \bar{S}_{k-1} \} \qquad (7)$$

Proposed algorithm

1. Convert the loosely coupled parallel computer interconnection system to a probabilistic graph.
2. Enumerate all the minimal path sets from the source to destination node.
3. Rearrange the total number of path sets (h) according to increasing order of cardinality.
4. Set k=1 and R_0=0
5. Repeat 6-7 for k=1 to h
6. The reliability expression can be given as
$$R_k = (R_{k-1} + S_k)_{dis}$$
7. $p_k = R_k - R_{k-1}$
8. Compute the fuzzy reliability of the system as
$$R = 1 - \prod_{k=1}^{h} \bar{p}_k$$

Efficiency:
Finding the minimal paths using the above proposed algorithm requires $O(N^3)$ operations. So, the running time of proposed algorithm is $O(N^3)$ which is polynomial.

IV. RESULTS AND DISCUSSIONS

The fuzzy reliability of three important parallel computer interconnection networks viz. hypercube (HC), Crossed cube (CQ) and Fault-tolerant hypercube (FTH), have been evaluated by the proposed method. The membership functions of the said parallel computer interconnection networks are plotted against the probability (Figs. 1-3). The parameter functions of the said parallel computers are presented in Table 1. The inference that can be drawn from Table 1 is that the fuzzy reliability of hypercube lies between the limits 0.71-0.95 with a 100% possibility. Similarly the fuzzy reliability of FTH and CQ between the limits 0.74-0.97, 0.76-0.96 respectively with a 100% possibility.

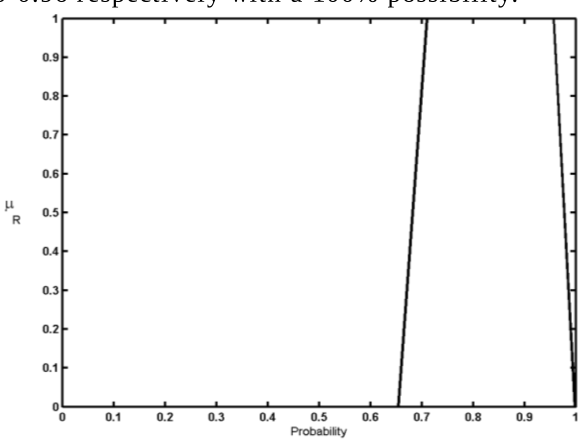

Fig.1: Fuzzy reliability of Hypercube

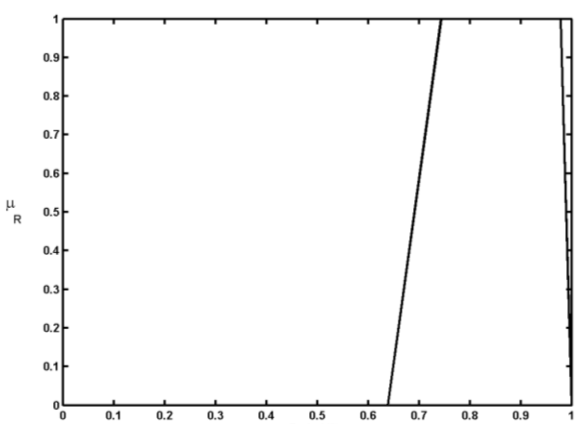

Fig.2: Fuzzy reliability of Fault-Tolerant Hypercube

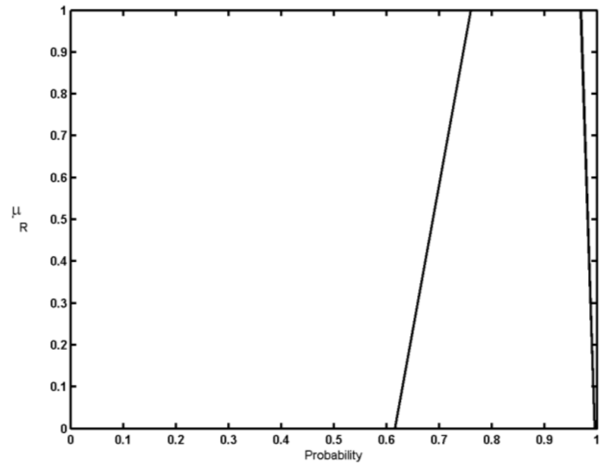

Fig.3: Fuzzy reliability of Crossed Cube

Table1. Parameter functions of the Cube based interconnection networks

Cubes	α_1	α_2	β_2	β_1
HC	0.6544	0.7105	0.9566	0.9982
CQ	0.6166	0.7604	0.9689	0.9954
FTH	0.6388	0.7440	0.9781	0.9998

V. CONCLUSION

In this paper, a general and efficient method has been proposed to analyze the fuzzy reliabilities of loosely coupled parallel computer interconnection networks. Basically, the proposed method uses the path enumeration technique in evaluating fuzzy reliability. The link failure and link success probabilities are assumed to be fuzzy numbers. The method is followed by mathematical basis and algorithm.
Using the proposed techniques, the fuzzy reliabilities of three number of loosely coupled parallel computer interconnection networks are evaluated. The method proposed here can also be extended further to evaluate all categories of parallel computer networks.

VI. REFERENCES

[1]. L.A. Zadeh, “Fuzzy sets”, Information control, vol. 1, no.1, pp. 338-353, 1965.
[2]. Ali M. Rushdi, King Abdul Aziz, “On reliability evaluation by network decomposition”, IEEE Transactions on Reliability, vol.R-33, no.5, pp.379-384, Dec 1984.
[3]. J.A. Abraham, “An Improved Algorithm for Network Reliability”, IEEE Transactions on Reliability, vol R-28, no. 1, pp.58-61, 1979.

[4]. C.R. Tripathy, R.N. Mahapatra, R.B.Mishra, “Reliability Analysis of Hypercube Multicomputer”, Microelectronics & Reliability An International Journal, vol.37 no.6, pp.885-891, 1997.

[5]. A.Z. Keller and C. Kara-Zaitri, “Further Applications of fuzzy logic to Reliability Assessment and Safety Analysis”, Microelectronics & Reliability-An International Journal, 29, 399-404, 1989.

[6]. K. P. Soman and K. B. Misra, “Fuzzy-tree analysis using resolution identity”, Journal of Fuzzy Math., vol. 1, pp. 193-212, 1993.

[7]. H. Tanaka, L. T. Fan, F. S. Lai and K. Toguchi, “Fault tree analysis by fuzzy probability”, IEEE Transaction on Computer, vol. 32, pp.455-457, 1983.

[8]. K. B. Misra and G. G. Weber, Use of fuzzy set theory for level-I studies in probabilistic risk assessment, Fuzzy Sets and systems, 37, 139-160, 1990.

[9]. S. G. Chowdhury and K.B. Mishra, “Evaluation or fuzzy reliability of a non-series parallel network”, Microelectronics & Reliability, vol.32, pp.1-4 1992.

[10]. F.B. Bastani, I.R. Chen and T. Tsao, “Reliability of systems with fuzzy-failure criterion”, 1994 Proceeding of Annual Reliability and Maintenance. Symposium, 1994, pp.442-448.

[11]. S. Patra et. al., “Reliability evaluation of flow networks considering multistate modeling of network elements”, Microelectronics & Reliability, vol.33(14), pp. 2161-2164, 1993.

[12]. C.R. Tripathy, R.N. Mahapatra and R.B. Misra, “A method for evaluation of fuzzy reliability of multistage interconnection networks”, Comp. Sc. and Informatics Journal, vol. 25(4), pp. 17-19, 1995.