

Math 4315/5315 - PDEs Home Work 7

Due: Fri. Nov. 19, 2021

1. Solve the heat equation

$$u_t = u_{xx}, \quad 0 < x < 4,$$

subject to the initial condition

$$u(x, 0) = 4x - x^2,$$

and subject to the following boundary conditions

$$\begin{aligned} (i) \quad & u(0, t) = 0, \quad u(4, t) = 0, \\ (ii) \quad & u_x(0, t) = 0, \quad u_x(4, t) = 0. \end{aligned}$$

1b. Solve the heat equation

$$u_t = u_{xx}, \quad 0 < x < 1,$$

subject to the initial condition

$$u(x, 0) = 1 + 2x - x^2,$$

and subject to the boundary conditions

$$u(0, t) = 1, \quad u(1, t) = 2,$$

2. Solve Laplace's equation

$$u_{xx} + u_{yy} = 0, \quad 0 < x < 1, \quad 0 < y < 1,$$

subject to the boundary conditions

$$\begin{aligned} (i) \quad & u(x, 0) = 0 \quad u(0, y) = 0 \quad u(x, 1) = x - x^2 \quad u(1, y) = 0 \\ (ii) \quad & u(x, 0) = 0 \quad u(0, y) = 0 \quad u(x, 1) = 0 \quad u(1, y) = y \end{aligned}$$