Math 3331 - ODE's

Last class we considered the differential

$$dz = f_x\,dx + f_y\,dy$$

where f_x, f_y are partial derivatives

ex $\quad z = x^2y + y^2$

$$f_x = 2xy\,,\quad f_y = x^2 + 2y$$

so $\quad dz = 2xy\,dx + (x^2+2y)\,dy$

Suppose $z = c$, a const

then $dz = 0$ so

$$2xy\,dx + (x^2+2y)\,dy = 0 \quad \leftarrow \text{solve for } \frac{dy}{dx}$$

$$(x^2+2y)\,dy = -2xy\,dx$$

$$\frac{dy}{dx} = -\frac{2xy}{x^2+2y} \quad \leftarrow \text{an ODE.}$$

ex2 $\quad z = x^2 - 3xy + y^2$

$$dz = f_x\,dx + f_y\,dy$$

$$f_x = 2x - 3y \qquad f_y = -3x + 2y$$

So $\quad dz = (2x-3y)\,dx + (-3x+2y)\,dy$

if $z = c$ then

$$(2x-3y)\,dx + (-3x+2y)\,dy = 0$$

$$(-3x+2y)\,dy = -(2x-3y)\,dx$$

$$\frac{dy}{dx} = -\frac{(2x-3y)}{-3x+2y}$$

or

$$\frac{dy}{dx} = \frac{2x-3y}{3x-2y}$$ an ODE (Homogeneous)

Start with $\quad \dfrac{dy}{dx} = \dfrac{2x-3y}{3x-2y}$

get $\quad (2x-3y)\,dx + (-3x+2y)\,dy = 0 \quad$ "Alternate ODE Form"

Suppose we knew that there was some z

Such that $z = f(x,y)$

$$dz = (2x-3y)dx + (-3x+2y)dy = 0$$

then $dz = 0 \Rightarrow z = c \leftarrow$ the sol^n of
or $f(x,y) = c$ the ODE.

In general

if $\frac{dy}{dx} = F(x,y)$

an alternate form is

$$M(x,y)dx + N(x,y)dy = 0$$

we say this ODE is exact if z exits

Such that

$$dz = Mdx + Ndy$$

and if so $z = c$ is the sol^n of the

ODE.

Test for Exactness $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Previous ex.

$$(2x-3y)\,dx + (-3x+2y)\,dy = 0$$

$$M\,dx + N\,dy = 0$$

so $\quad M = 2x-3y \quad N = -3x+2y$

$$\frac{\partial M}{\partial y} = -3 \qquad \frac{\partial N}{\partial x} = -3 \qquad \text{same so yes exact}$$

so how to find z

Note $\quad dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$

compare

$$\frac{\partial f}{\partial x} = 2x-3y \Rightarrow f = x^2-3xy + A(y)$$

$$\frac{\partial f}{\partial y} = -3x+2y \Rightarrow f = -3xy + y^2 + B(x)$$

choose $A(y)$ & $B(x)$ as

$A = y^2 \quad B = x^2$

Sol^n $\quad f = c \quad$ or $\quad \boxed{x^2-3xy+y^2 = c}$

<u>Ex 2</u> $\dfrac{dy}{dx} = \dfrac{-y^3}{3xy^2-4y+1}$

So $-y^3dx - (3xy^2-4y+1)dy = 0$

or $y^3dx + (3xy^2-4y+1)dy = 0$

$M_y = 3y^2 \quad N_x = 3y^2$ same so exact

$f_x = M = y^3 \Rightarrow f = xy^3 + A(y)$

$f_y = N = 3xy^2 - 4y + 1 \quad f = xy^3 - 2y^2 + y + B(x)$

$f = xy^3 - 2y^2 + y = c$

Sol^n $\boxed{xy^3 - 2y^2 + y = c_1}$