Math 1496 - Calc 1

Review

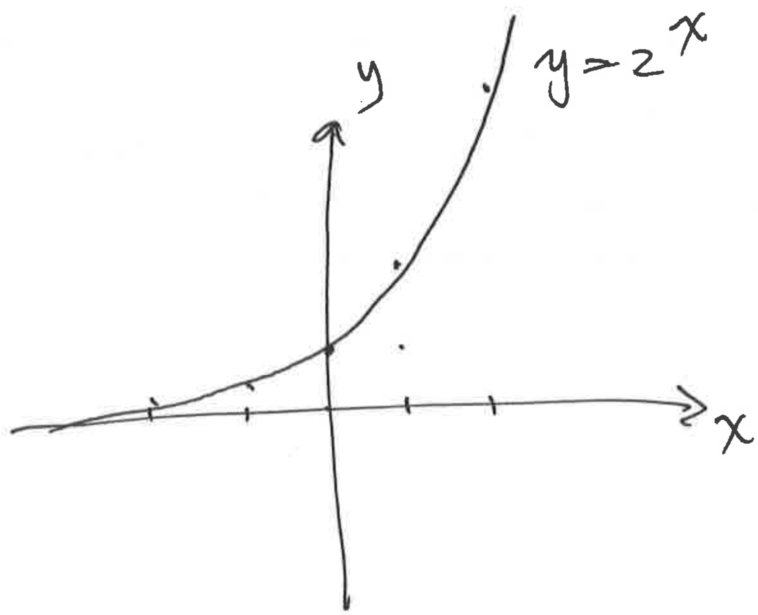
Exponential Functions

$y = a^x$ or $f(x) = a^x$ where $a \neq 1$

For example $f(x) = 2^x$

Table of values

x	$f(x)$
-2	$2^{-2} = \frac{1}{4}$
-1	$2^{-1} = 1/2$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$



Suppose we had $f(x) = 4^x$

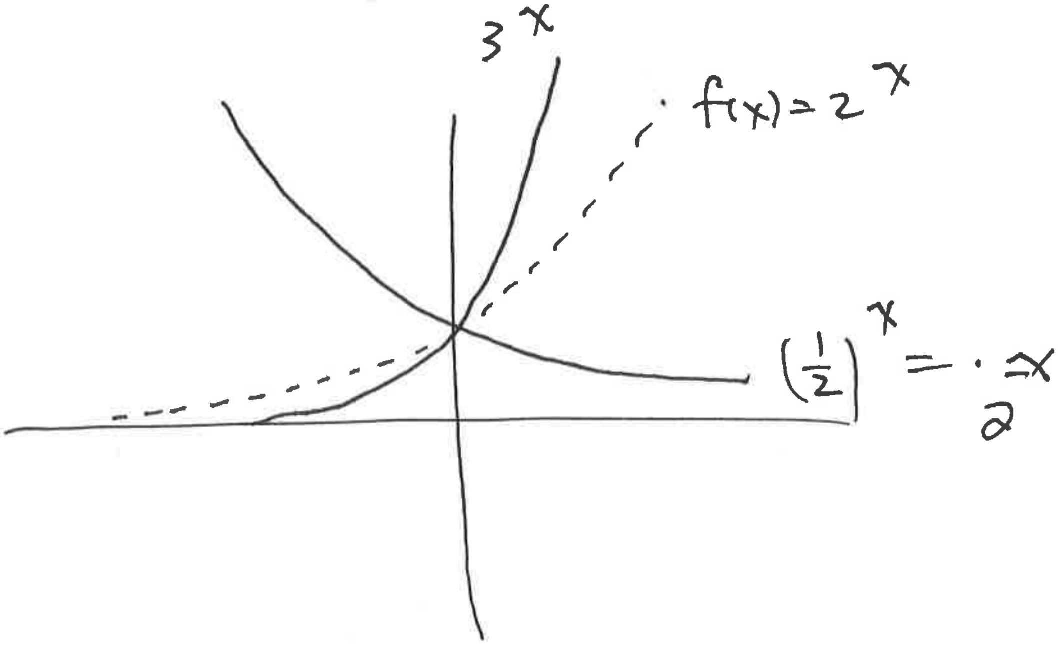
so what is $f(3/2) = 4^{3/2} = (2^2)^{3/2} = 2^3 = 8$

$$(a^x)^y = a^{xy}$$

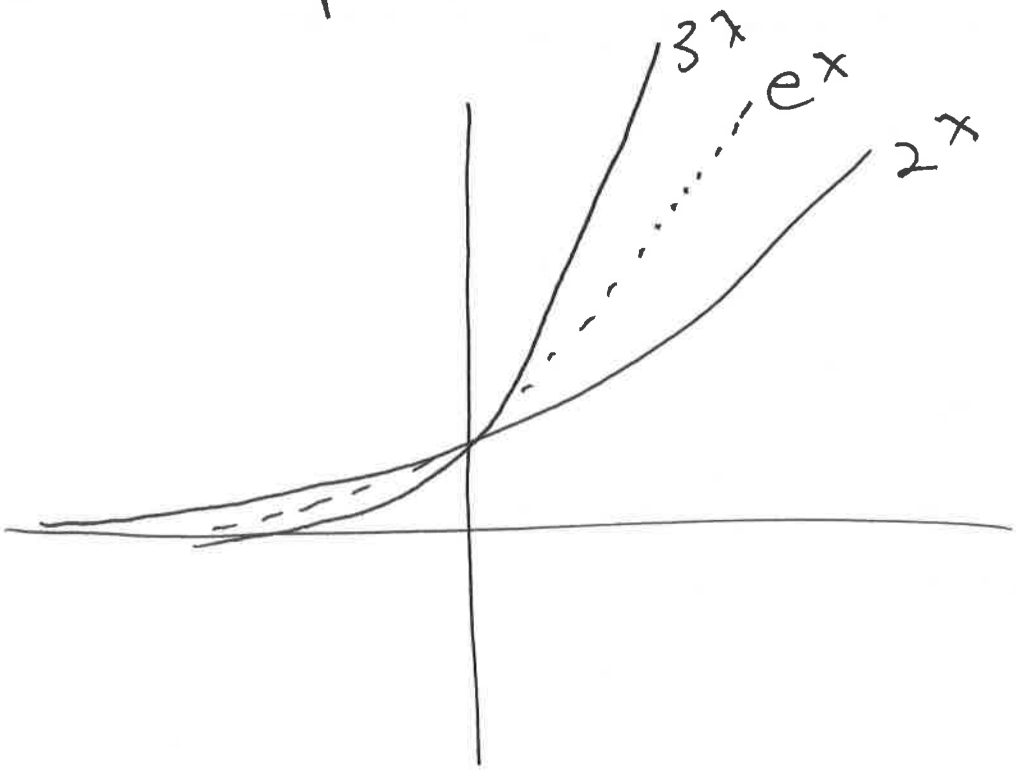
Now let's change the base (a)

Consider

$$f(x) = 3^x \quad \text{\&} \quad f(x) = \left(\tfrac{1}{2}\right)^x$$



Natural Exponential Function $f(x) = e^x$

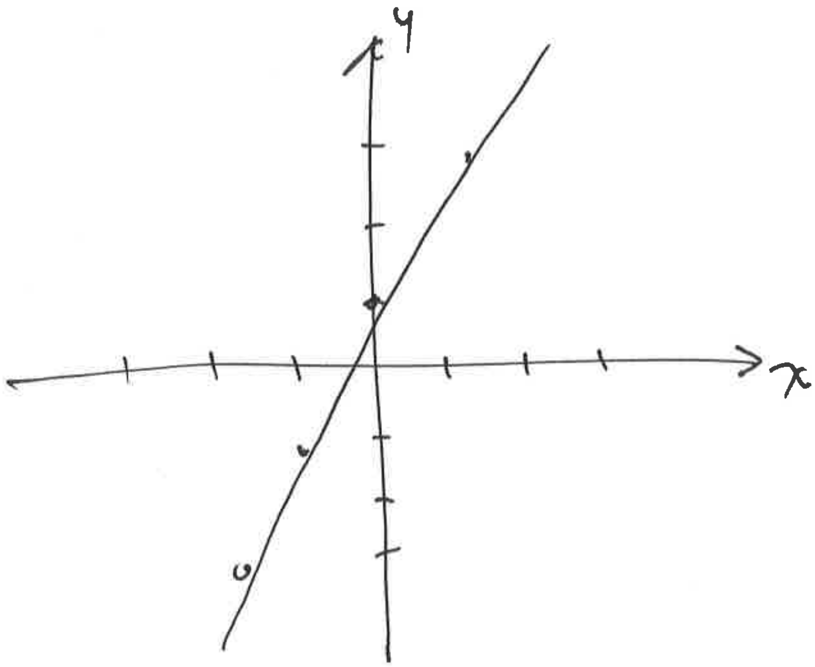


$e = 2.71828$

Inverse Functions

Consider $f(x) = 2x+1$

x	$f(x)$
-2	-3
-1	-1
0	1
1	3
2	5
3	7



so can we undo this operation

x	
-3	-2
-1	-1
1	0
3	1
5	2
7	3

the undoing is the "inverse function" and denoted by

$f^{-1}(x)$

so given $y = f(x)$

then $x = f(y) \iff y = f^{-1}(x)$

inverse inverse

ex $y = 2x+1$

$x = 2y+1$ (inverse)

Solve for $y = \dfrac{x-1}{2}$

so $f(x) = 2x+1$, $f^{-1}(x) = \dfrac{x-1}{2}$

Now compose

$$f(f^{-1}(x)) = 2(f^{-1}(x))+1 = 2\left(\frac{x-1}{2}\right)+1 = x-1+1$$
$$= x$$

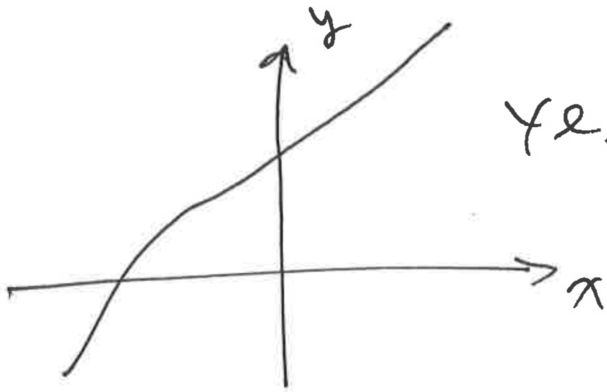
$$f^{-1}(f(x)) = \frac{f(x)-1}{2} = \frac{2x+1-1}{2} = \frac{2x}{2} = x$$

so $f(f^{-1}(x)) = x$

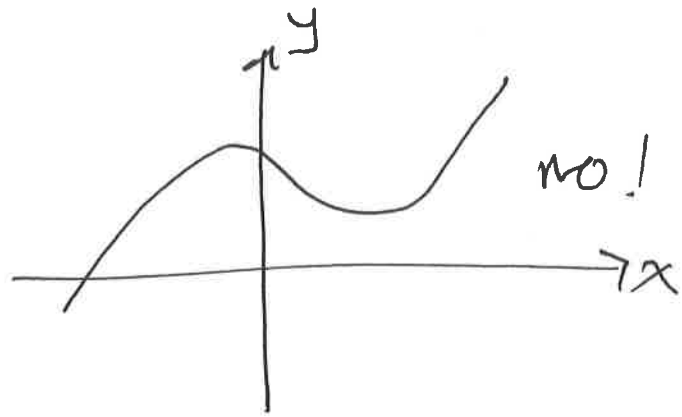
undoes the operation

One-to-one functions

For each x there is exactly 1 y

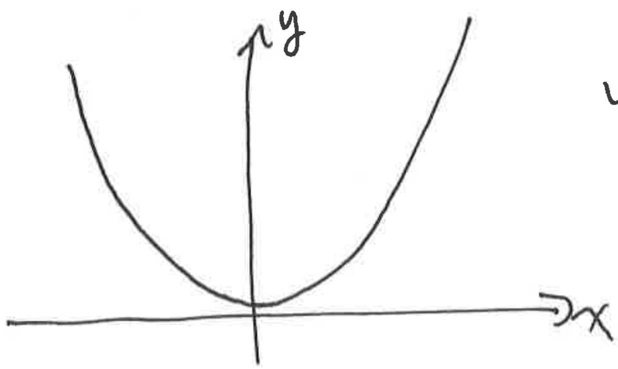


Yes!



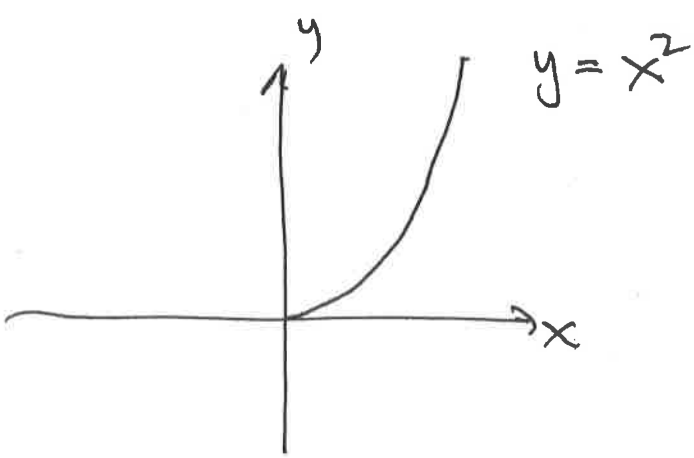
no!

Is $y = x^2$



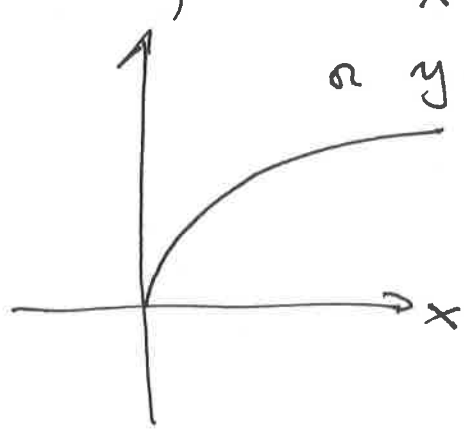
in general no

but what about on $[0, \infty)$ Yes



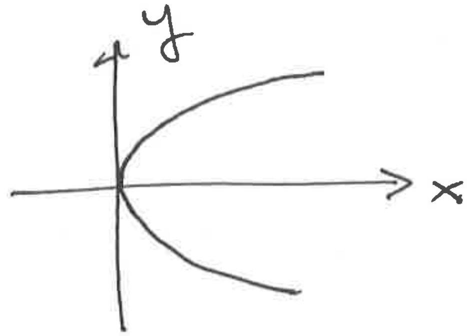
$x = y^2$

or $y = \sqrt{x}$



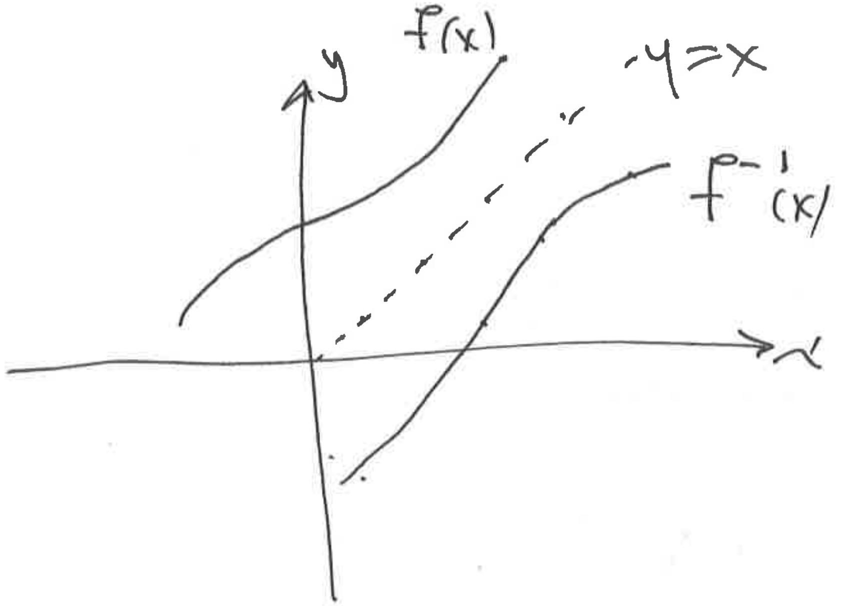
Note $x = y^2$ has

but this is not a function!



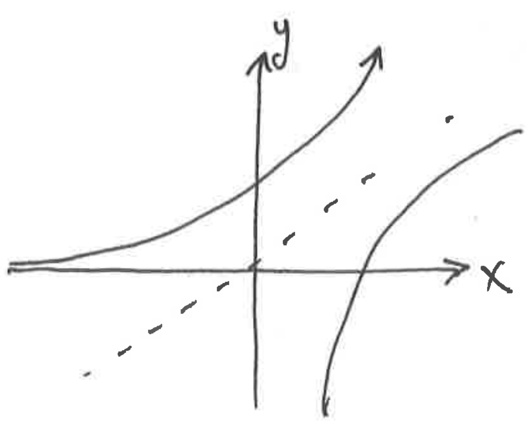
If $f(x)$ is 1-1 on some domain D then f has a unique inverse such that

$$f\left(f^{-1}(x)\right) = x$$



a reflection in the line $y = x$

what is the inverse of $y = a^x$ or $y = e^x$



so they are

$$x = a^y \quad \text{and} \quad x = e^y$$

we solve for y giving

$$y = \log_a x \quad \text{\&} \quad y = \ln x \qquad (\log_e x = \ln x)$$

if $f(x) = a^x$ $\quad f^{-1}(x) = \log_a x$

$$f(f^{-1}(x)) = a^{\log_a x} = x \qquad x > 0$$

$$f^{-1}(f(x)) = \log_a a^x = x \qquad \text{for all real } x$$

<u>Properties</u> $a > 0$, x, y real numbers

(1) $a^0 = 1$ $\qquad$ (4) $(e^x)^y = e^{xy}$

(2) $a^{x+y} = a^x \cdot a^y$

(3) $a^{-x} = \dfrac{1}{a^x}$

Properties of Log

(i) $a^{\log_a x} = x$ (ii) $\log_a a^x = x$

(iii) $\log_a 1 = 0$ (iv) $\log_a a = 1$

(v) $\log_a (xy) = \log_a x + \log_a y$

(vi) $\log_a (x/y) = \log_a x - \log_a y$

(vii) $\log_a x^y = y \log_a x$

Change of Base

$\log_a x$

let $y = \log_a x$ so $a^y = x$

$\log_b a^y = \log_b x \Rightarrow y \log_b a = \log_b x$

$$\boxed{y = \frac{\log_b x}{\log_b a} = \log_a x}$$