VANDERBILT UNIVERSITY School of Engineering

Discrete Structures

CS 2212

(Fall 2020)

1 – Introduction and Logic

Welcome

- Instructor: **Dr. Waseem Abbas**
- Email: waseem.abbas@vanderbilt.edu
- Office: **314 FGH**
  406 Institute for Software Integrated Systems
- Office Hours: **Thu. (1:00 – 3:00pm)** virtually via **Zoom**, or by appointment.

Textbook

zyBooks

1. Sign in or create an account at
   learn.zybooks.com
2. Enter zyBook code:

Be sure to:

1. Use your registered Brightspace name.
2. Use your VUNet ID for Student ID
3. Join the correct section (Abbas)

Class Platforms

brightspace

- Course Syllabus
- Course Calendar
- Lecture Slides
- TA Office Hours Link
- Homework Assignments
- Important Announcements
- Grades

https://brightspace.vanderbilt.edu

Class Platforms

- Where you submit HWs (stay tuned)
- HW/ Exam grading
- You should be automatically added to our class in Gradescope
- Cost = Free

- Discussion board / Q & A.
- Include your Professor's name for section specific question.
- You should have been added already. If not, use the below link:

Teaching Mode

In-class

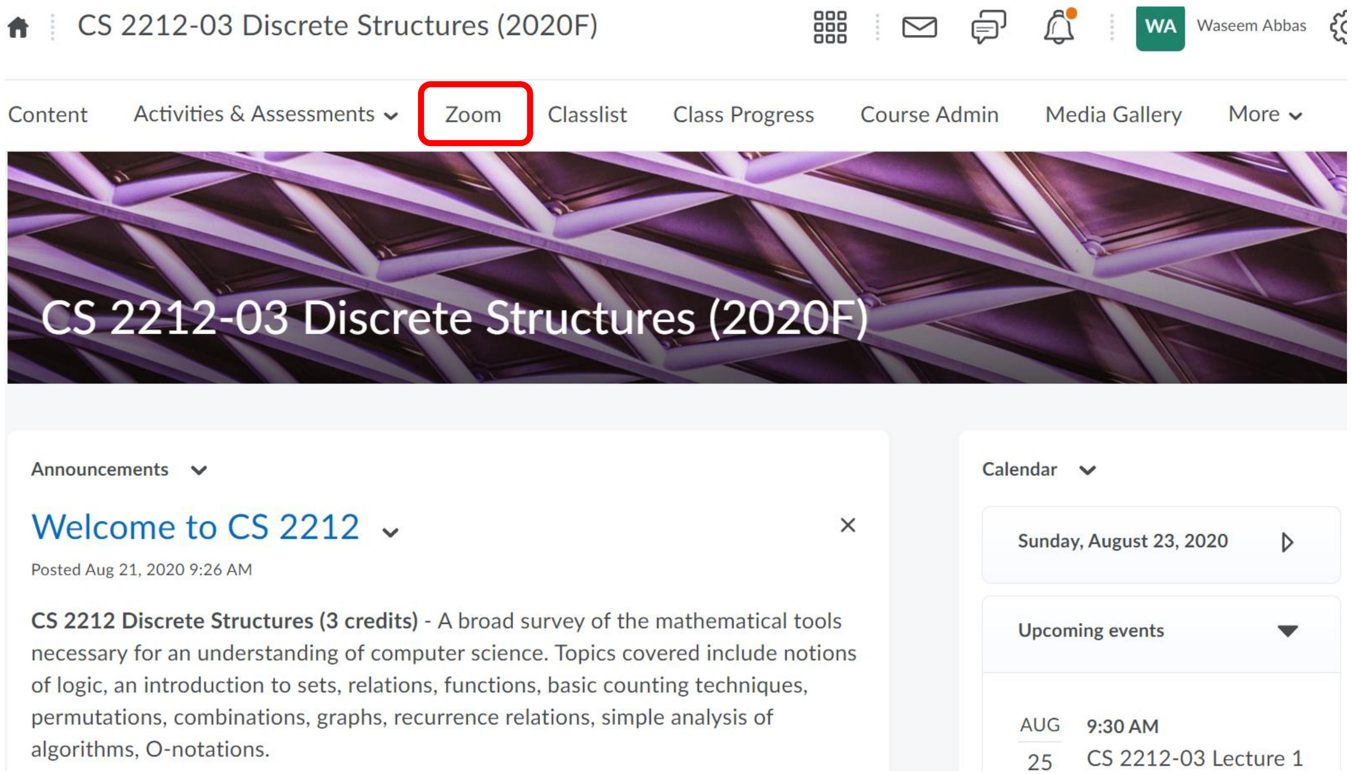
FGH 132

Remote

Zoom

Tue. and **Thu.**

9:35 – 10:50 am



Grading

Description	Weight
Participation: zyBooks Assignments (14)	**10%**
Exams (3)	**36%**
Homeworks (4)	**36%**
Final Exam	**18%**

ZyBook Assig.

- Weekly (14)
- Practice + Challenge
- Complete at least 90% the required weekly points during the semester to earn 100% participation points.
- **No late assignment** accepted.

Exams

- Almost monthly (3)
- About 30-40 minutes
- Covers current material.
- **No makeup** without excused absence from Dean of Students

Homeworks

- Monthly (4)
- No collaboration
- One free late day
- Late policy
  - 20% penalty (24 hrs)
  - 30% (24 – 48 hrs)
  - No credit after 48 hrs.
  - Advice: start early

Calendar

Please regularly check course website and calendar for due assessments.

September

Mon	Tue	Wed	Thu	Fri	Sat	Sun
	1	2	3	4	5	6 ZY 1A , ZY 1B
7	8 HW 1 given	9	10	11	12	13 ZY 2A
14	15	16	17 Exam 1	18	19	20 ZY 2B, ZY 3
21	22 HW 1 due	23	24	25	26	27 ZY 4
28	29 HW 2 given	30				

October

Mon	Tue	Wed	Thu	Fri	Sat	Sun
			1	2	3	4 ZY 5
5	6	7	8	9	10	11 ZY 6
12	13 HW 2 due	14	15	16	17	18 ZY 7A
19	20 Exam 2, HW 3 given	21	22	23	24	25 ZY 7B
26	27	28	29	30	31	

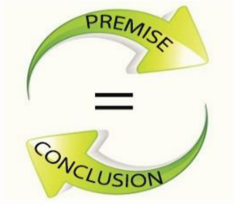
Grading

Average	Assigned Grade
>=90	A category *
80-89.9	B category *
70-79.9	C category *
60-69.9	D category *
< 60	F

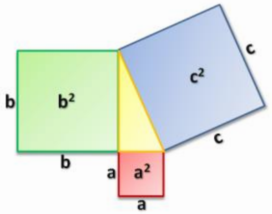
- This is just a "rough" distribution for your reference.
- Actual grades will be determined after the final exam.

What is this Course All About?

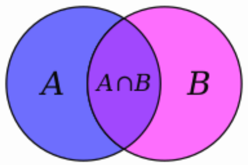
Discrete + **Structures**



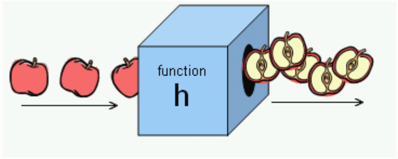
Logic



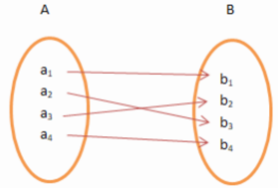
Proofs



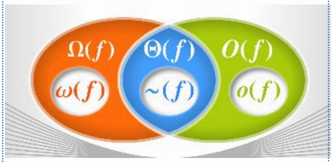
Sets



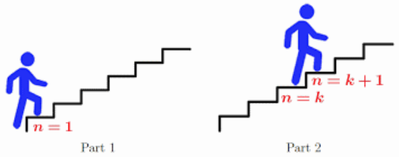
Functions



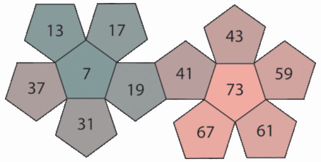
Binary Relations



Computation



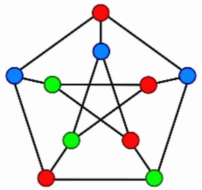
Induction



Integer Properties

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Counting



Graphs & Trees

Example: Product Marketing at Minimum Cost

- **Market** a new product, say cell phone.
- **Strategy:** Give away free cell phones to few individuals, who will be the brand ambassadors and advertise the product to their friends.
- **Our goal:** Give away **minimum number** of cell phones, while ensuring that the **whole community** knows about the phone

Is this a **Discrete Math** problem?

If **yes**, where are

- Graphs?
- Computation?
- Counting?
- Sets?
- Proofs?
- ….

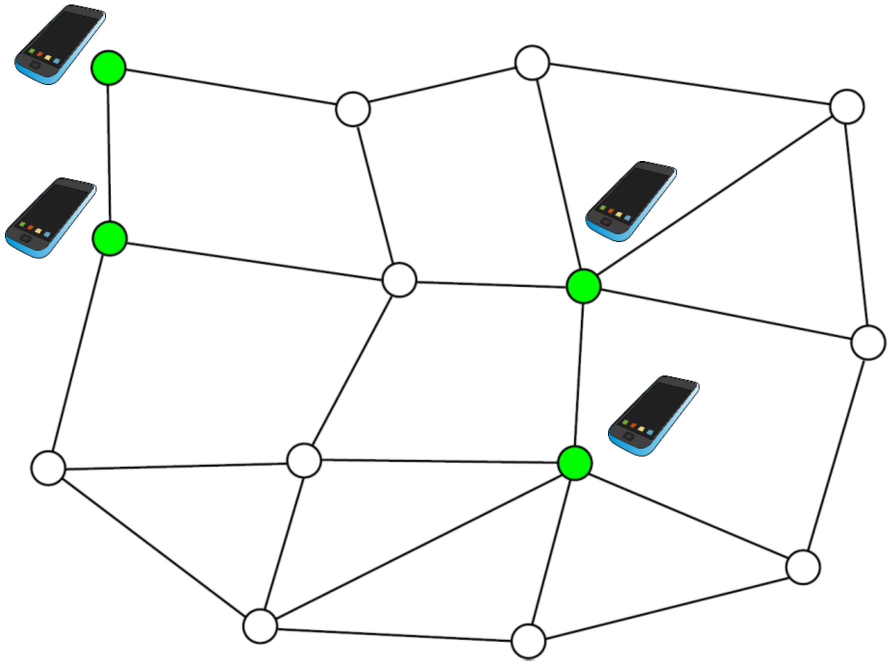
Example: Product Marketing at Minimum Cost

We can model individuals and their friendships as **graphs**.

How many free cell phones are needed? (3,4,5?) **(computation)**

Who should get the cell phone?
How many possibilities are there?
(counting)

Is this a best solution? **(proof)**



Five free phones should be sufficient? Can we do better?
Yes, **four** are sufficient.

Our Approach in the Class

Keep a track of the

Big Picture

What?
as well as
Why?

Connect
information

Interactive

Some Tips...

The course is *mile wide and foot deep*. There will be a lot of new concepts/topic almost every week. So,

Do not fall behind.

Participate, do not be shy to ask questions, So,

Be active and interactive.

Often students say *"I understood everything in class, but am unable to solve problems"*. The secret is

Practice, practice, and more practice.

Some Tips…

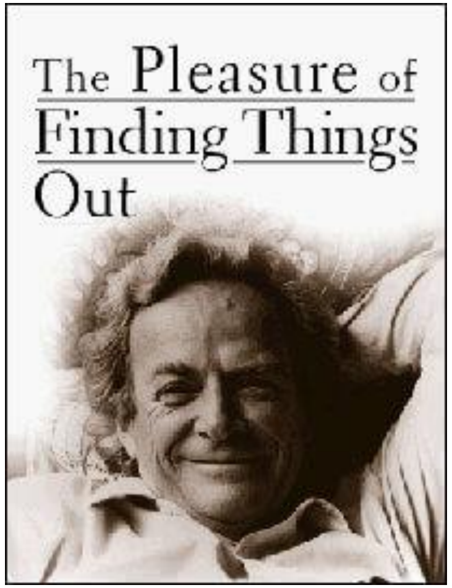
Do not wait till the last minute to start HW.
So,

Start early …

Do not study just for grades, focus on *learning*, and importantly *enjoy* learning beautiful things. So,

Enjoy the "pleasure of finding things out"

(as Richard Feynman said)

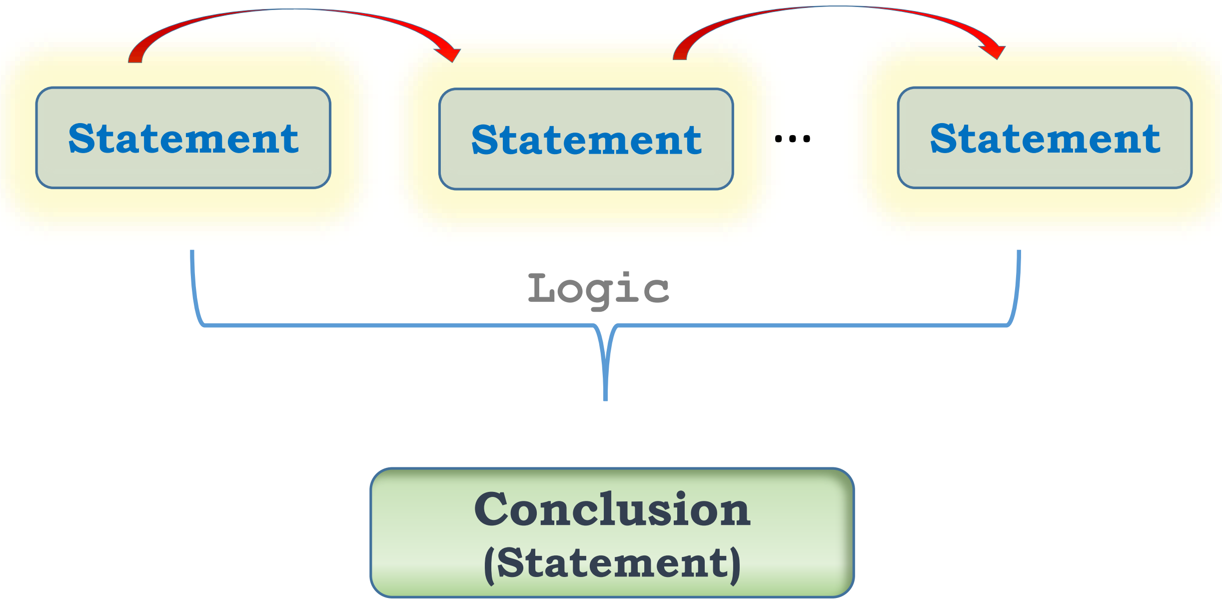


1 - Logic

Logic – Study of Reasoning

(Loosely speaking)

- Given pieces of **information** (statements/facts) that may be **related** to each other, how can we **accurately** and **systematically** draw more **conclusions**.



- From known facts and premises, how can we infer new statement (conclusion), and construct an argument?

Drawing Conclusions from Given Information

Example: A Crime Detection Problem

We know:

- One of them is **thief**
- Exactly one of them is speaking the truth

A.

I am not a thief

B.

A is the thief

C.

I am not a thief

Who is the thief ?

Drawing Conclusions from Given Information

Example: A Crime Detection Problem

Suppose **A** is the thief,

A.

I am not a thief

B.

A is the thief

C.

I am not a thief

then both **B** and **C** are speaking the truth.

But, we know exactly one of them is speaking the truth. So, a contradiction. Hence,

<u>Conclusion:</u> **A** can't be the thief.

Drawing Conclusions from Given Information

Example: A Crime Detection Problem

Suppose **B** is the thief,

A.

I am not a thief

B.

A is the thief

C.

I am not a thief

then both **A** and **C** are speaking the truth.

But, we know exactly one of them is speaking the truth. So, a contradiction. Hence,

Conclusion: **B** can't be the thief.

Drawing Conclusions from Given Information

Example: A Crime Detection Problem

Suppose **C** is the thief,

A.

I am not a thief

B.

A is the thief

C.

I am not a thief

then **A** is speaking the truth, whereas, **B** and **C** are lying.

Conclusion: **C** is the thief.

Drawing Conclusions from Given Information

What if we have ***n*** persons, and exactly ***k*** of them are speaking the truth? Who is the thief?

I am not a thief | **A is the thief** | **I am not a thief** | **X is the thief** | **Y is the thief** | … | **I am not a thief**

Takeaways:

- We can infer new statements (conclusions) by carefully considering the given statements and premise.
- Things can get complex quickly, so we need to **formalize** and **systemize** our method of reasoning.

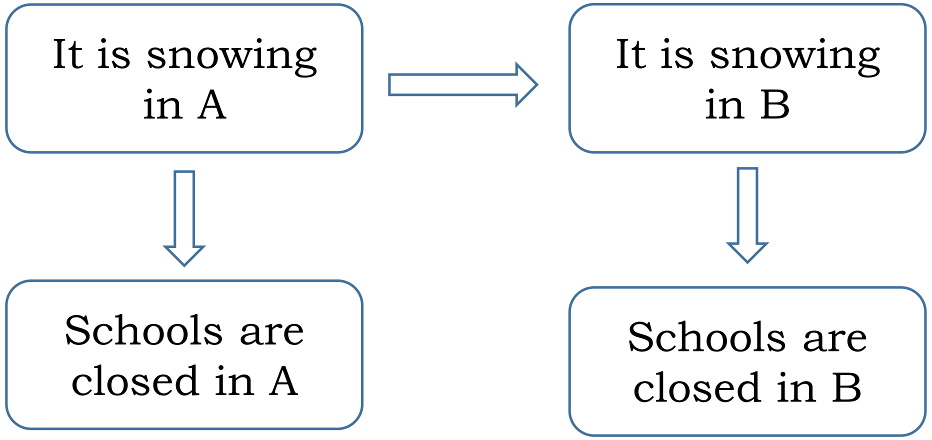
Example – How to Reason?

Lets look at another example. Our focus now is on the **process of reasoning**.

Consider two cities A and B.

- It is snowing in city A.
- If it snows in a city, then its schools are closed.
- If it snows in city A, then it snows in city B.

Can I conclude the following? **"Schools in city B are closed."**



1. Statements
2. Relation between statements (structure)

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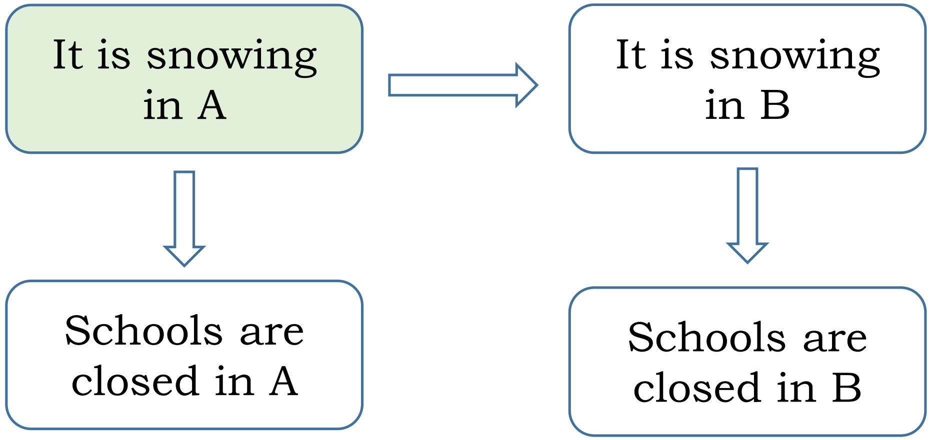

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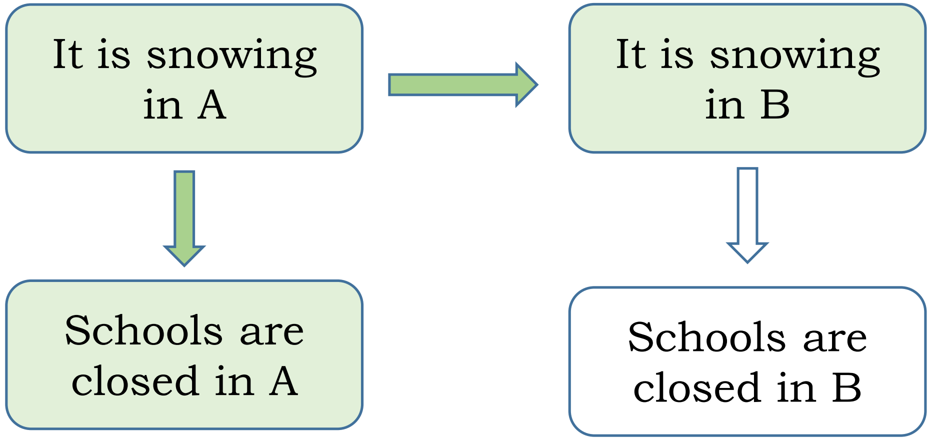

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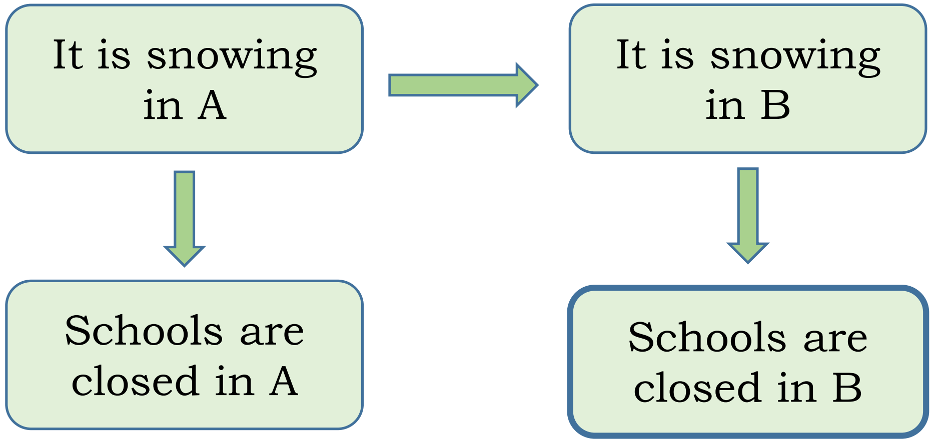

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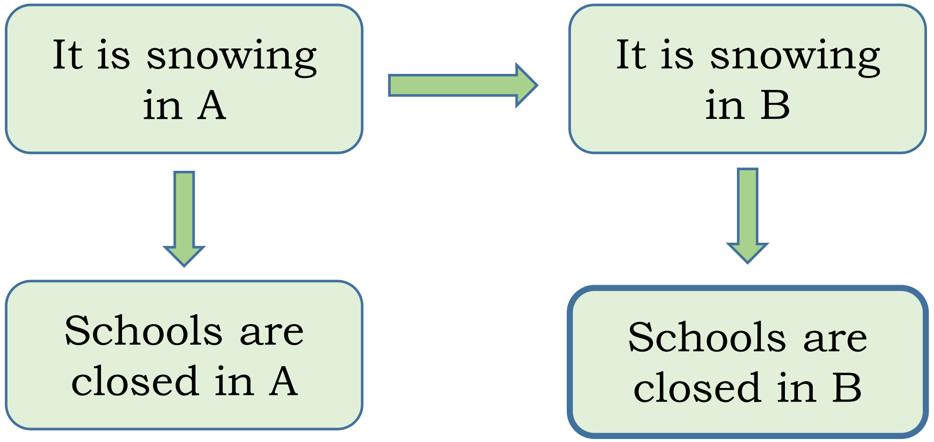
3. Drawing conclusion

Formalizing Reasoning



- Do you notice anything about the *"form"* of statements and conclusion.
- They are all **Yes/No** statements.
- How does this help?

Our goal will be to formalize the process of reasoning.

- What do we mean by **statements (mathematically)**?
- What can be a good way to capture **relations** between statements?
- How can we write new statements from known ones, such as by performing some **"operations"** on them?

Why Formal Reasoning?

This formalization is key to

- Constructing precise mathematical arguments,
- Proving (disproving) complex statements,
- Verifying correctness of computer programs,
- Designing computer circuits
- …
- **(And to passing the CS 2212 course.)**

1.1 Propositions and Logical Operations

Proposition: A statement that is either **true** or **false**, but not both.

Propositions are basic **building blocks** of logical reasoning.

Statement	Proposition	Truth Value
29 is a prime number		
Open the door		
$x + y > 5$,		
Earth is the only planet where life exists		
For every positive integer n, there is a prime number larger than n		

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Open the door	✗	
$x + y > 5$,	✗	
Earth is the only planet where life exists	✓	??
For every positive integer n, there is a prime number larger than n		

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Statement	Proposition	Truth Value
29 is a prime number	✓	Yes
Open the door	✗	
$x + y > 5$,	✗	
Earth is the only planet where life exists	✓	??
For every positive integer n, there is a prime number larger than n	✓	Yes

Why we defined our **basic building block** this particular way?

- **Simplest, concise,** and the **most un-ambiguous** way of declaring a fact / information
- Has a **definite truth value**

1.1 Propositions and Logical Operations

- Sometimes simple statements are not enough (to express complicated ideas).
- Combine propositions to get **compound propositions** using certain composition rules called **logical operations**

It is snowing	It is cold	Simple propositions
It is snowing **and** it is cold		Compound proposition

5 is a prime number	5 is an even number	Simple propositions
5 is a prime **or** even number		Compound proposition

- What determines the **truth value** of a **compound proposition**?
- Let’s see some basic logical operations

1.1 Propositions and Logical Operations

Conjunction:

Let **p** and **q** be simple propositions, then a **conjunction** of **p** and **q** is a new proposition, whose truth value is ***true*** only when **both p** and **q** are true, and is ***false*** otherwise.

Written as: **p ∧ q**

Read as: **p and q**

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

p: 5 is an even number (**F**)
q: 5 is a prime number (**T**)
p ∧ q: 5 is an even and a prime number ( )

p: 29 is not an even number ( )
q: 29 is a prime number ( )
What is p ∧ q?

1.1 Propositions and Logical Operations

Disjunction:

Disjunction of propositions **p** and **q** is a new proposition, whose truth value is ***false*** only when **both p** and **q** are false, and is ***true*** otherwise.

Written as: **p ∨ q**

Read as: **p or q**

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

p: 5 is an even number (**F**)
q: 5 is a prime number (**T**)
p ∨ q: 5 is an even or a prime number ( )

p: 29 is an even number ( )
q: 29 is not a prime number ( )
What is p ∨ q?

1.1 Propositions and Logical Operations

Exclusive-or:

Exclusive-or of propositions **p** and **q** is a new proposition whose truth value is ***true*** if **exactly one** of the propositions **p** and **q** is true but not both, and is ***false*** otherwise.

Written as: **p ⊕ q**

Read as: **p xor q**

p	q	p ⊕ q
T	T	F
T	F	T
F	T	T
F	F	F

p: 5 is an even number (**F**)
q: 5 is a prime number (**T**)
p ⊕ q: 5 is an even number exclusively or a prime number (**T**)

p: 29 is an odd number ( )
q: 29 is a prime number ( )
What is p ⊕ q?

1.1 Propositions and Logical Operations

Let p and q be propositions, then under what conditions

1) $p \oplus q \neq p \vee q$

2) $p \vee q = p \wedge q$

3) $p \oplus q = p \wedge q$

1.1 Propositions and Logical Operations

Negation:

Negation of a proposition p is a proposition, whose truth value is the *opposite* of the truth value of p.

Written as: **¬p**

Read as: **not p**

p	¬p
T	F
F	T

p: 5 is an even number (**F**)

¬p : 5 is not an even number (T)

1.2 Evaluating Compound Propositions

Order of operations is important.

Example: **p** = True, **q** = False,

$$r = \neg p \wedge q \;\; ??$$

- If $\neg$ is first, then **r** = False
- If $\wedge$ is first, then **r** = True

Order of operations (in the absence of parentheses):

Operator	Order
$\neg$	1
$\wedge$	2
$\vee$	3

1.2 Evaluating Compound Propositions

s = (p ∨ q) ∧ ¬ (p ∧ q)

p	q			
T	T			
T	F			
F	T			
F	F			

1.2 Evaluating Compound Propositions

s = (p ∨ q) ∧ ¬ (p ∧ q)

p	q	p ∨ q		
T	T	T		
T	F	T		
F	T	T		
F	F	F		

1. Evaluate **p ∨ q**

1.2 Evaluating Compound Propositions

s = (p ∨ q) ∧ ¬ (p ∧ q)

p	q	p ∨ q	¬ (p ∧ q)	
T	T	T	F	
T	F	T	T	
F	T	T	T	
F	F	F	T	

1. Evaluate **p ∨ q**
2. Evaluate **p ∧ q**
3. Evaluate **¬ (p ∧ q)**

1.2 Evaluating Compound Propositions

$s = (p \lor q) \land \neg (p \land q)$

p	q	p ∨ q	¬ (p ∧ q)	s
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	T	F

1. Evaluate $p \lor q$
2. Evaluate $p \land q$
3. Evaluate $\neg (p \land q)$
4. Evaluate the or of step 1 and step 3.

- Note that $s = p \oplus q$
- $p \oplus q$ and $(p \lor q) \land \neg (p \land q)$ are **logically equivalent.**

(same truth tables)

1.2 Evaluating Compound Propositions

- **Truth table** supplies all possible truth values of a compound proposition for various truth values of its constituent proposition.
- If there are n variables, how many rows are in the truth table? 2^n

Example: $(p \land q) \lor \neg r$

How many rows in the truth table?

n = 3 variables

8 rows.

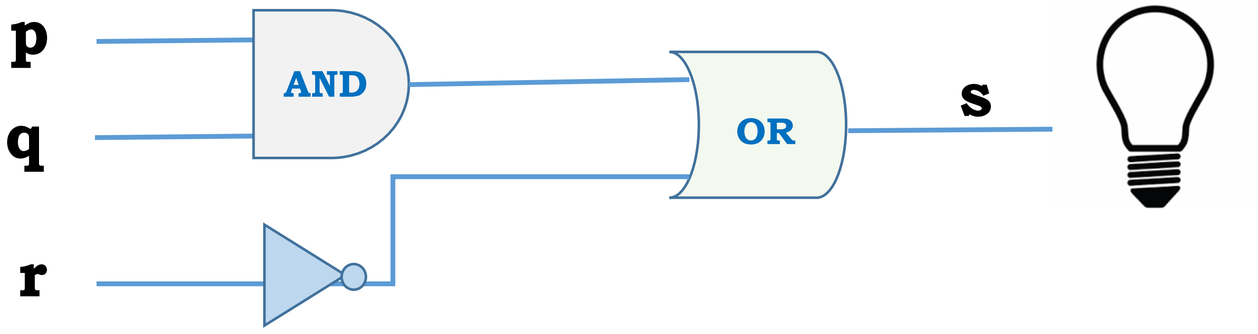
- Compound statements also represent digital logic circuits.

1.2 Evaluating Compound Propositions

$s = (p \land q) \lor \neg r$

True = **1**

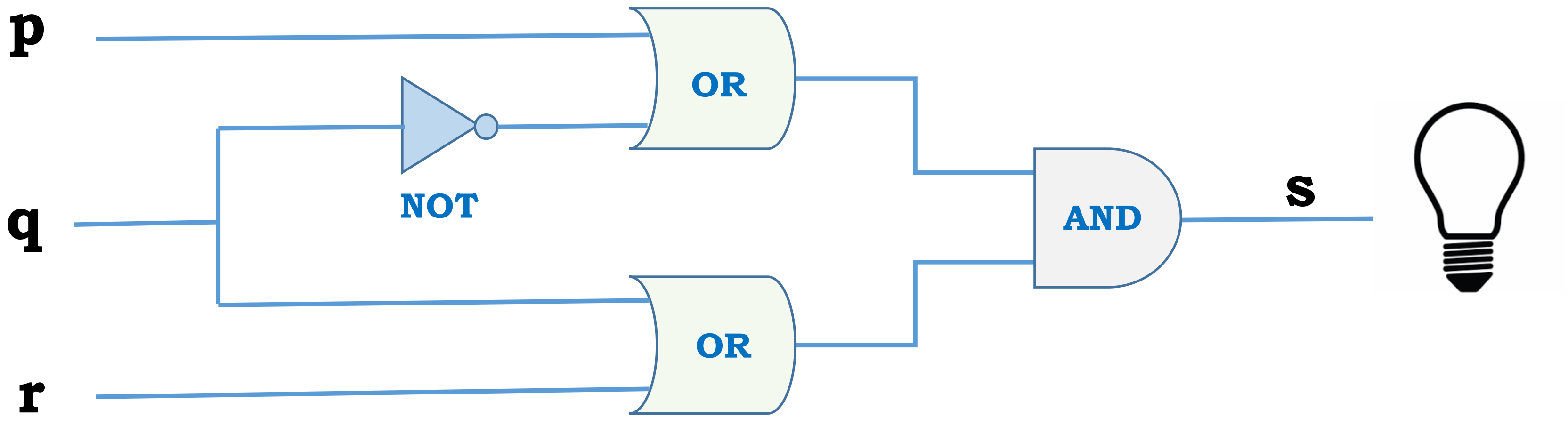
False = **0**



Example

True = **1**

False = **0**



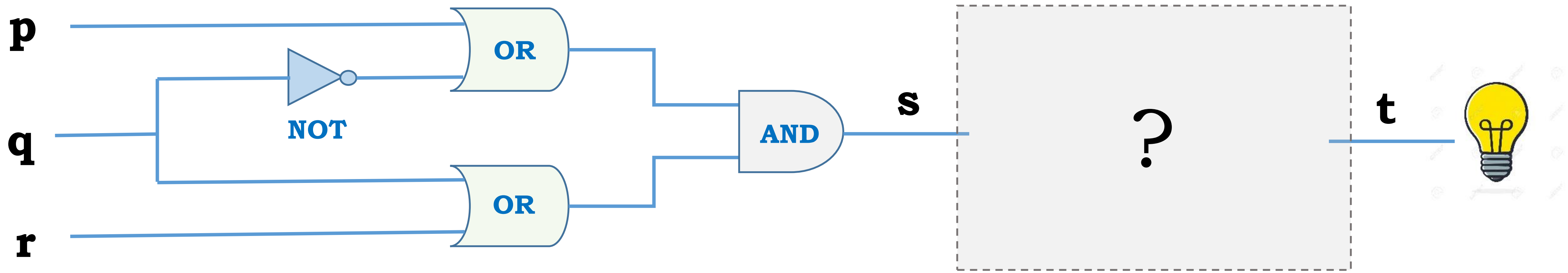
- For what values of **p**, **q**, and **r**, the bulb lights up (**s** = 1)?

$$s = (\neg q \vee p) \wedge (q \vee r)$$

- A solution is: **p = 1**, **r = 1**, **q = 0**

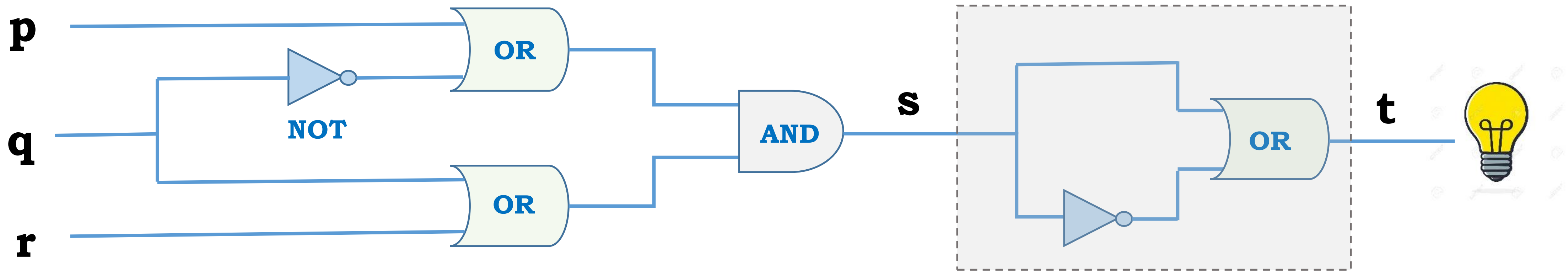
Example

What can we do to ensure that bulb **always** lights up (**s = 1** irrespective of **p, q, r**) ?



Example

What can we do to ensure that bulb **always** lights up (**s = 1** irrespective of **p, q, r**) ?



$t = (\neg s \lor s)$

s	t
1	1
0	1

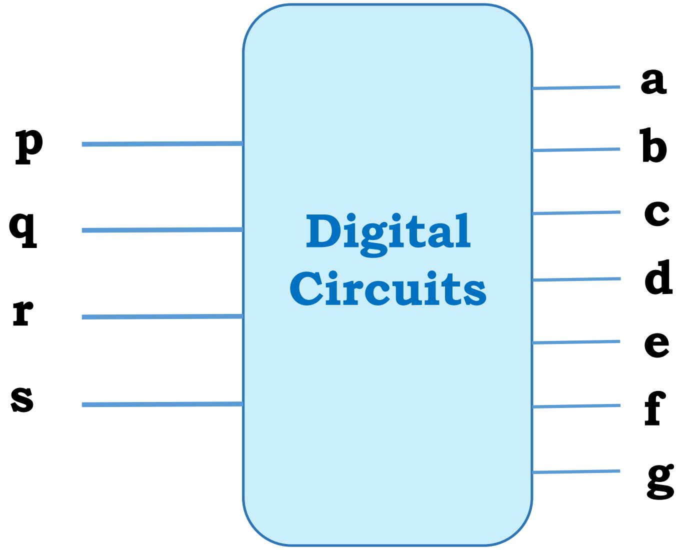
Tautology

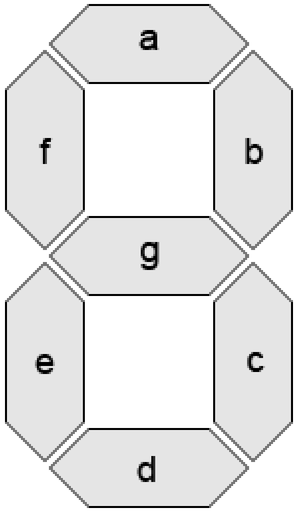
Applications – Digital Circuit Design

Lets look at a practical example.

Circuit for 7-Segment Display

Binary representation
of digits
from **0** to **9**

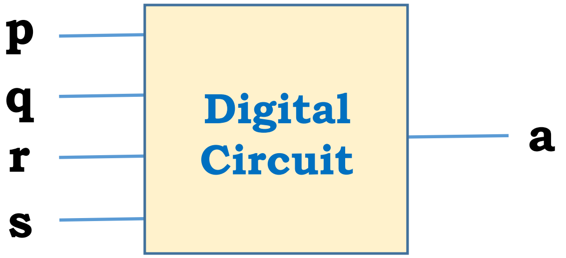


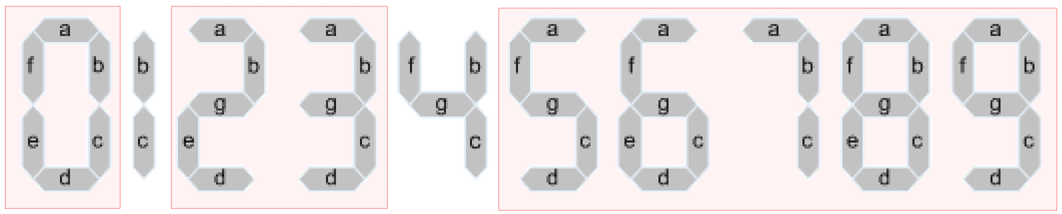


- Our circuit takes **4 input variables (propositions)** and displays the digit on right.
- Seven **output variables** (each corresponding to a **compound proposition**).
- There is a circuit for each output variable.

Applications – Digital Circuit Design

Lets consider circuit for an LED segment **a**.



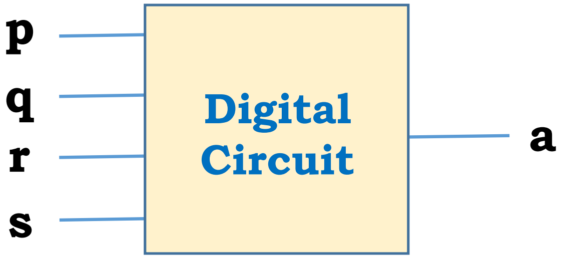


	Binary representation				LED
Digit	**p**	**q**	**r**	**s**	**a**
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	

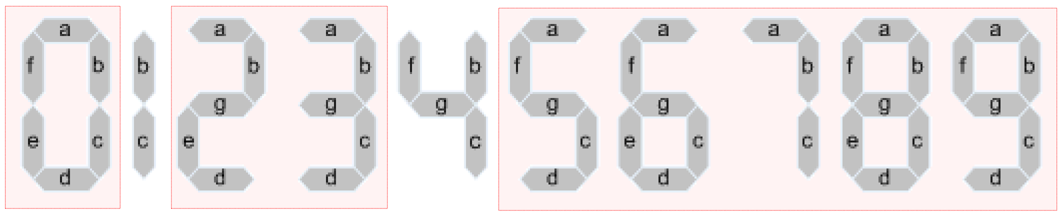
a lights up only when highlighted digits are at the input.

Applications – Digital Circuit Design

Lets consider circuit for an LED segment **a**.



	Binary representation				LED
Digit	**p**	**q**	**r**	**s**	**a**
0	0	0	0	0	**1**
1	0	0	0	1	**0**
2	0	0	1	0	**1**
3	0	0	1	1	**1**
4	0	1	0	0	**0**
5	0	1	0	1	**1**
6	0	1	1	0	**1**
7	0	1	1	1	**1**
8	1	0	0	0	**1**
9	1	0	0	1	**1**



a lights up only when highlighted digits are at the input.

$$\mathbf{a = (p \lor r) \lor (q \land s) \lor (\neg\, q \land \neg\, s)}$$

- Lets verify.
- Can you design a circuit for **b**?

1.3 Conditional Statement

Conditional Proposition:

Hypothesis → Conclusion
$p \rightarrow q$
If p, then q

Example: If it rains, then I will have an umbrella

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

1.3 Conditional Statement

Conditional Proposition:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

If the conclusion is always true regardless of the hypothesis part, the conditional statement is **trivially true.**

p: "whatever"
q: $3 < 4$ True
$p \to q$: If (whatever), then $(3 < 4)$ True

If the hypothesis is false, then the conditional statement is **vacuously true** regardless of the conclusion part.

p: $0 = 1$ False
q: "whatever"
$p \to q$: If $(0 = 1)$, then (whatever) True

1.3 Conditional Statement

Is $p \rightarrow q$ same as (equivalent to) $q \rightarrow p$?

p	q	$p \rightarrow q$	$q \rightarrow p$
T	T	T	
T	F	F	
F	T	T	
F	F	T	

1.3 Conditional Statement

Is $p \to q$ same as (equivalent to) $q \to p$?

p	q	$p \to q$	$q \to p$
T	T		T
T	F		T
F	T		F
F	F		T

1.3 Conditional Statement

Is $p \rightarrow q$ same as (equivalent to) $q \rightarrow p$?

p	q	$p \rightarrow q$	$q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

1.3 Conditional Statement

Conditional Proposition:

Is the truth value of

$$(p \vee q) \rightarrow r$$

always same as the truth value of

$$(p \rightarrow r) \wedge (q \rightarrow r)$$