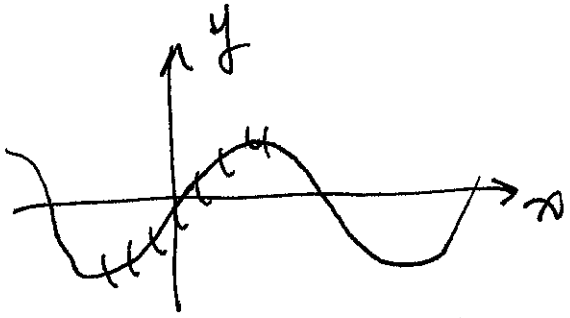
Math 1496 - Calc I

Inverse Trig Fcts

Now we've seen the 6 trig fcts & inverse we now consider inverses of the trig fcts

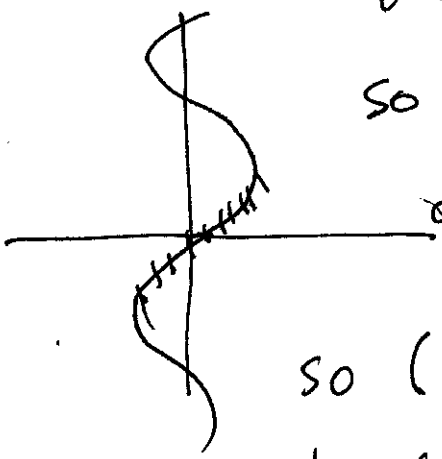
$y = \sin x$

Here is the $f(x) = \sin x$ curve



if we interchang x & y then we have $x = \sin y$

which is clearly not a function



so we consider only a section of $y = \sin x$ from $-\pi/2 \le x \le \pi/2$

so $(-\pi/2, -1) \to (-1, -\pi/2)$

& $(\pi/2, 1) \to (1, \pi/2)$

This is seen in the ++++ part of the curve

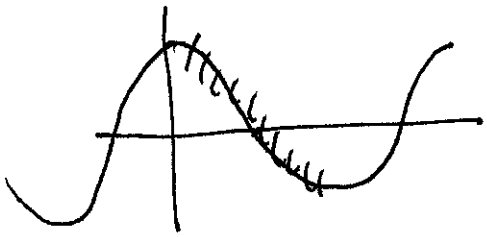
with $x = \sin y \iff y = \arcsin x$

$= \sin^{-1} x$

This latter form $\sin^{-1} x \neq (\sin x)^{-1} = \frac{1}{\sin x}$

so $f(x) = \sin^{-1} x \quad -1 \leq x \leq 1, \quad -\pi/2 \leq y \leq \pi/2$

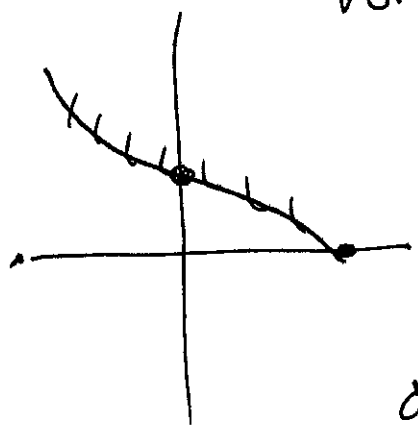
$y = \cos x$



The part we consider is

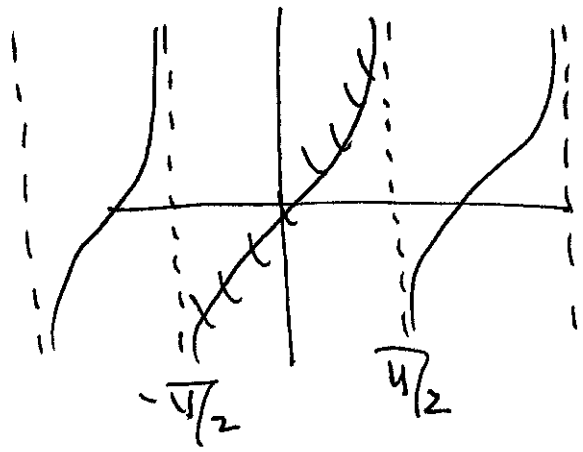
$0 \leq x \leq \pi$

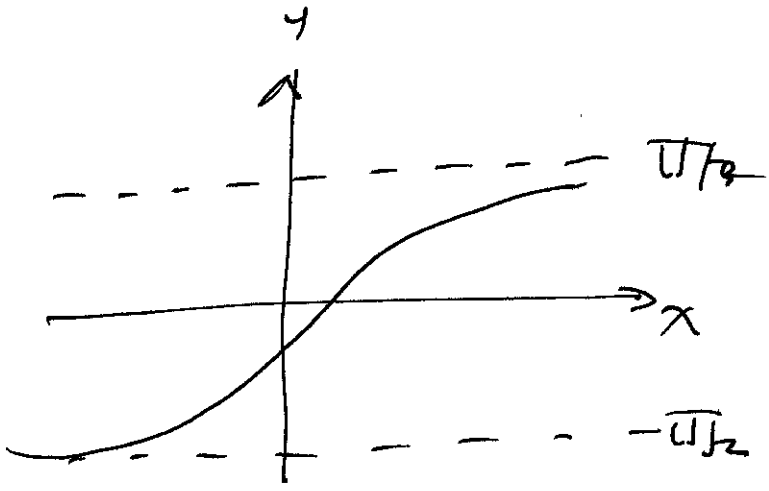
so $(0,1) \to (1,0)$ & $(\pi, -1) \to (-1, \pi)$



Note the point $(\pi/2, 0)$ on the $\cos x$ curve becomes $(0, \pi/2)$ on the $\arccos(x)$ or $\cos^{-1} x$ curve.

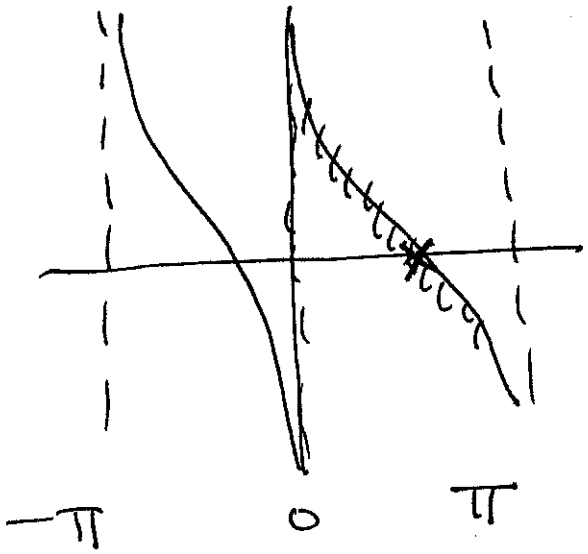
$y = \tan x$ Note the VA becomes HA

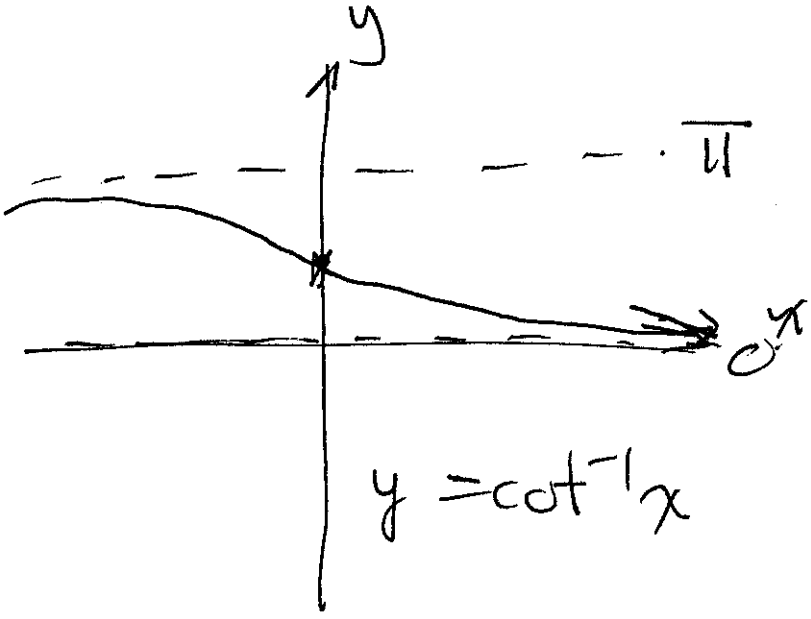




$y = \tan^{-1} x$

$y = \cot x$



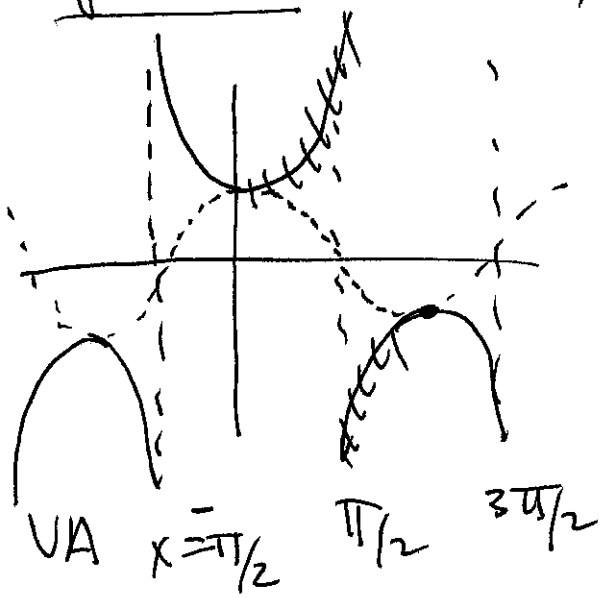


$y = \cot^{-1} x$

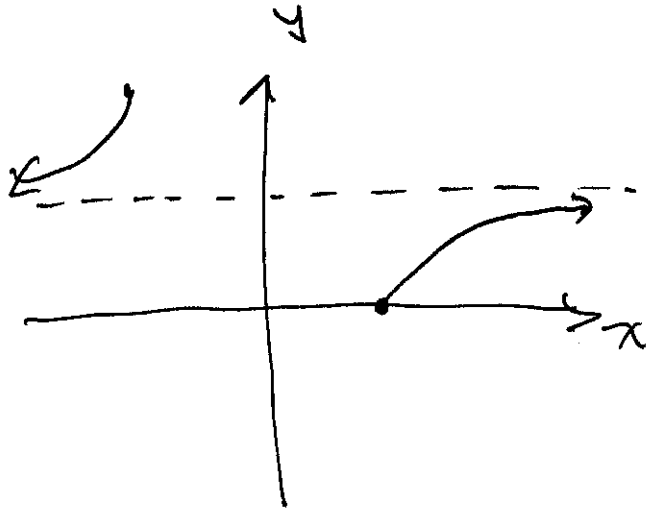
when x gets close to zero, $y \to +\infty$ in $\cot x$ graph

pt $(\pi/2, 0) \to (0, \pi/2)$

$y = \sec x \left(= \frac{1}{\cos x}\right)$



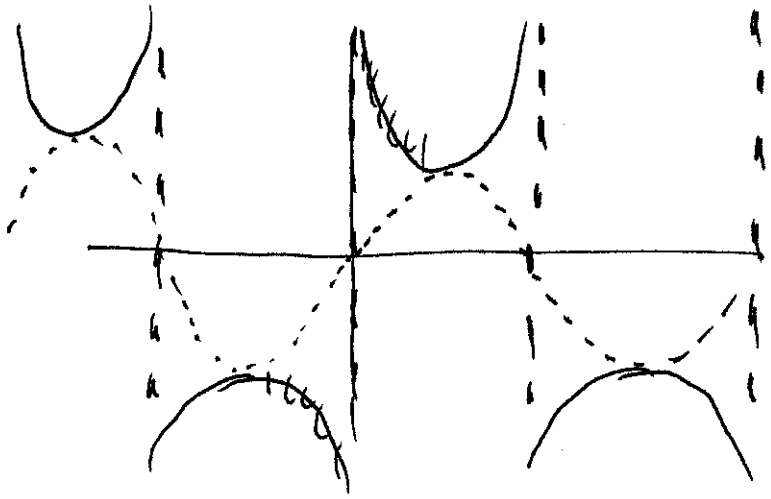
For the inverse of $y = \sec x$ we consider $0 \le x \le \pi$ denoted in the ~~++++~~ curve
The VA $x = \pi/2$ becomes the Hz



Now when x is between 0 & $\pi/2$ then inverse is > 0
& $(0,1) \to (1,0)$
& as x increase y approaches $+\infty$ as $x \to \pi/2$ $y \to +\infty$
so in the inverse
as $x \to \infty$ $y \to \pi/2$

Similarly for
$\pi/2 \le x \le \pi$
$y = \sec x$ is negative and as $x \to \pi/2$ $y \to -\infty$
so in the inverse $x \to -\infty$ $y \to \pi/2$
& the pt $(\pi, -1) \to (-1, \pi)$ (see the picture)

$y = \csc x \quad \left(= \frac{1}{\sin x} \right)$



The dashed lines are the VA. The dotted curve is $y = \sin x$ and the solid curve $y = \csc x$.

For the inverse $y = \cancel{\text{arc}}\cdot\cancel{\text{arccos}}\, x$ $\operatorname{arccsc} x$

$= \csc^{-1} x$

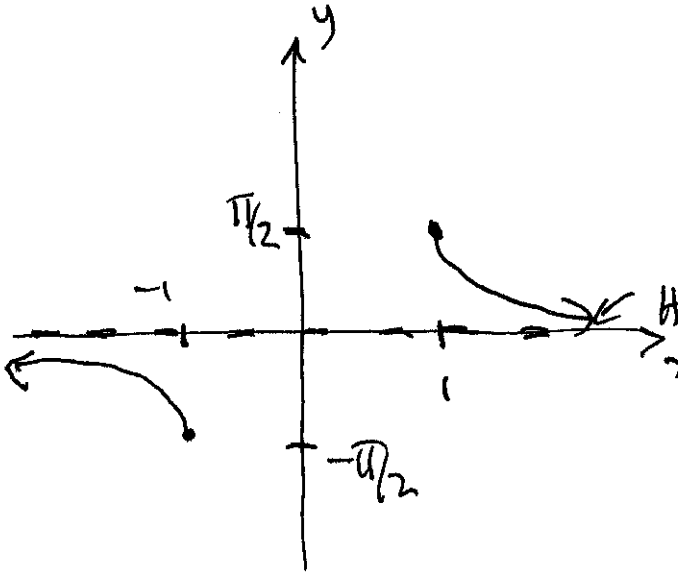
We consider only the part of the graph $[-\pi/2, \pi/2]$

the VA $x = 0$ becomes the HA $y = 0$.

The pts $(-\pi/2, -1)$ & $(\pi/2, 1)$ become

$(-1, -\pi/2)$ $(1, \pi/2)$

as $x \to 0$ $\csc x \to \pm\infty$



and the graph is ← here