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Research Article

Applications of Soft pre-open Sets to Soft W-Hausdorff Space in Soft Topological Spaces

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Abstract

In the present article the notion of soft W-Hausdorf or soft W-T_2 structure in soft topological spaces is proclaimed with respect to soft pre-open sets while using the ordinary points of Soft Topology. That's why it is named as soft P-W-Hausdorf or soft P-W-T_2 structure. Some sub-spaces of soft P-W- T_2 structure are also reflected. Product of these spaces is also avail.

Keywords: Soft set; Soft P-open set; Soft P-closed set; Soft topological space; Soft P-W Hausdorf space.

Introduction

General Topology Play attractive role in space time geometry as well as different branches of pure and applied mathematics. In Separation Axioms we discuss points of the space. It shows the points are separated by neighbour-hood. When we are interested to know the distance among the points that are separated from each other, then in that case the concept of separation axioms will come in play. Most of the real life problems have various uncertainties. A number of theory have been proposed for handling with uncertainties in an efficient way in 1999, Molodtsov [1] was first who given birth to a novel concept of soft set theory which is completely a new approach for modelling vagueness and uncertainty in 2011, Shabir and Naz [2] define soft topological spaces and studied separation axioms.

Khattak et al [3] define Soft Separation Axioms in Soft Single Point Spaces and in Soft Ordinary Spaces. [4] Naz et al discussed soft Separation axiom in bi soft topological spaces with great detail. Ittanagi [5] discussed Soft separation axioms bi-topological spaces pairwisely with respect to open sets. El-Sheikh et al [6] studied Characterization of b soft separation axioms in soft topological spaces with respect to b open sets. Khattak et al [7] defined Characterization of Soft b-Separation Axioms in Soft Bi-Topological Spaces with respect to crisp points and soft points and related results in a beautiful way. Khattak et al [8] discussed P-Separation Axioms in Supra Soft Topological Spaces. Khattak et al [9] studied Soft B W-Hausdorff Space in Soft Bi Topological Spaces. Section two of this paper preliminary definition regarding soft sets and soft topological spaces are given in section three of this paper the concept of soft W–Hausdorffness in soft topological spaces is introduced by referring the definition of soft P-open sets. Throughout this paper $\breve{X}$ denotes the father set and E denotes the set of parameter for the father $\breve{X}$.

Preliminaries

Definition 1: [1]. Let $\breve{X}$ be the father set and $\breve{E}$ be a set of parameters. Let $P(\breve{X})$ denotes the power set of and $\breve{A}$ be a nonempty subset of $\breve{E}$. A pair $(F, \breve{A})$ denoted by $F_{\breve{A}}$ Is called a soft set over $\breve{X}$ where F is a mapping given by $F; \breve{A} \rightarrow P(\breve{X})$ .In other words a soft set over $\breve{X}$ is parameterized family of subsets of the universe $\breve{X}$ for a particular $e \epsilon \breve{A}$, $F(e)$ may be considered the set of e-approximate elements of

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the soft set equation $(F,\breve{A})$ if $e \notin \breve{A}$ then $F(e) = \widetilde{\emptyset}$

$F_{\breve{A}}=\{ F(e); e\epsilon\breve{A} \subseteq \breve{E} ; F;\breve{A} \rightarrow P(\breve{X})\}$

The family of all these soft set over $\breve{X}$ w.r.t the parameter set $\breve{E}$ is denoted $SS(\breve{X})_{\breve{E}}$

Definition 2: [10] Let $F_{\breve{A}}, G_{\breve{B}} \in SS(\breve{X})_{\breve{E}}$ .then $F_{\breve{A}}$ is soft sub set of $G_{\breve{B}}$ is denoted by $F_{\breve{A}} \,\widetilde{\sqsubseteq}\, G_{\breve{B}}$ if

(1) $\breve{A} \subseteq \breve{B}$ and

(2) $F(e) \subseteq G(e)$,$\forall\, e\epsilon\breve{A}$

In this case $F_{\breve{A}}$ is said to be a soft subset of $G_{\breve{B}}$ and $G_{\breve{B}}$ is said to be soft super set of $F_{\breve{A}}$, $G_{\breve{B}} \,\widetilde{\sqsupseteq}\, F_{\breve{A}}$

Definition 3: [11] Two soft subset $F_{\breve{A}}$ and $G_{\breve{B}}$ over a common father set $\breve{X}$ are said to be equal if $F_{\breve{A}}$ is soft subset of $G_{\breve{B}}$ and $G_{\breve{B}}$ is a soft subset of $F_{\breve{A}}$.

Definition 4: [12]. Let τ be the collection of soft sets over X , then τ is said to be a soft topology on X, if

1. $\emptyset, X$ belong to τ
2. The union of any number of soft sets in τ belongs to τ
3. The intersection of any two soft sets in τ belong to τ

The triplet (X,F,E) is called a soft topological space.

Definition 5: [13]. Let (F,A) be any soft set of a soft topological space (X,τ,A) then (F,A) is called

i. 1) (F,A) *soft pre-open set* of X if $(F,A) \subseteq int(cl((F,A))$ and

ii. 2) (F,A) *soft pre closed set* of X if $(F,A) \supseteq cl(int(F,A)))$

The set of all *pre-open soft* sets is denoted by $POS(X)$

Definition 6: [14] Complement of a soft $(F,\tilde{A})$ set denoted by $(F,\tilde{A})^c$ denoted by $(F,\tilde{A})^c$ $(F^c;\breve{A} \rightarrow P(\breve{X}))\}$ is a mapping given by $F^c(e)=X - F(e)$; ,$\forall\, e\epsilon\breve{A}$ and F^c is called the soft complement function of F .clearly $(F^c)^c$ is the same as a $((F,\breve{A})^c)^c = (F,\breve{A})$.

Definition 7: [10] A soft set $(F,\breve{A})$ over $\breve{X}$ is said to be a null soft set denoted by $\breve{\varphi}$ or $\widetilde{\emptyset}_{\breve{A}}$ if $\forall\, e\epsilon\breve{A}$, $F(e)=\varphi$

Definition 8: [14] A soft set $(F,\breve{A})$ over $\breve{X}$ is said in absolute soft set denoted by $\breve{A}$ or $X_{\breve{A}}$ IF $\forall\, e\epsilon\breve{A}$, $F(e)=\breve{X}$ clearly we have $\breve{X}_{\breve{A}}{}^c=\varphi_{\breve{A}}$ and $\varphi_{\breve{A}}{}^c= X_{\breve{A}}$

Definition 9: [10] the union of two soft set $(F,\breve{A})$ and $(G,\breve{B})$ over the common universe $\breve{X}$ is the soft set $(H,\breve{C})$ where $\breve{C} = \breve{A} \cup \breve{B}$ and $\forall\, e\epsilon\breve{C}$

$$H(e)=\begin{cases} F(e), e\epsilon\breve{A} - \breve{B} \\ G(e), e\epsilon\breve{B} - \breve{A} \\ F(e) \cup G(e), e\epsilon\breve{A} \cap \breve{B} \end{cases}$$

Definition 10: [10] The intersection of two soft set $(F,\breve{A})$ and $(G,\breve{B})$ over the common universe $\breve{X}$ Is the soft set $(H,\breve{C})$ where $\breve{C} = \breve{A} \cap \breve{B}$ and $\forall\, e\epsilon\breve{C}$ $\breve{H}$ (e) = $F(e) \cap G(e)$.

Definition 11: [2] Let $(\breve{X},\tau,\breve{E})$ be soft topological space $(F,E)\epsilon\, SS(X)_E$ and Y be a non-null subset of X then the soft subset of (F,E) over $\breve{Y}$ denoted by $(F_{\breve{Y}},\breve{E})$ is defined as follows $F_{\breve{Y}}(e) = \breve{Y} \cap F(e) \forall\, e\epsilon\breve{E}$ In other words $(F_{\breve{Y}},\breve{E}) = \breve{Y}_{\breve{E}} \cap (F,\breve{E})$.

Definition 12: [2] Let $(\breve{X},\tau,\breve{E})$ be soft topological space $(F,\breve{E})$ and $\breve{Y}$ be a non-null subset of $\breve{X}$ .then $\{(F,\breve{E}) : (F,\breve{E})\epsilon\tau\}$ is said to be the relative soft topology on Y and $(\breve{Y},T_{\breve{Y}},\breve{E})$ is called a soft subspace of $(\breve{X},\tau,\breve{E})$.

Definition 13: [15] $F_{\breve{A}}\, \epsilon\, SS(X)_{\breve{E}}$ and $G_{\breve{B}}\, \epsilon\, SS(Y)_k$.The Cartesian product $(F_{\breve{A}} \circledcirc G_{\breve{B}})$ $(e,k)= (F_{\breve{A}}(e) \times G_{\breve{B}}(k) \,\forall\, (e,k)\, \epsilon\breve{A} \times \breve{B}$.

According to this definition $(F_{\breve{A}} \circledcirc G_{\breve{B}})$ is soft set over $\breve{X} \times \breve{Y}$ and its parameter set is $\breve{E} \times \breve{K}$.

Definition 14: [15] $(\breve{X}\, \tau_{\breve{X}}, \breve{E})$ and $(\breve{Y}\, \tau_{\breve{Y}}, \breve{K})$ BE two soft topological spaces the soft product topology $\tau_{\breve{X}} \circledcirc \tau_{\breve{Y}}$ over $\breve{X} \times \breve{Y}$ w.r.t $\breve{E} \times$ $\breve{K}$ *is the soft topology having the collection* $\{F_{\breve{E}} \circledcirc G_{\breve{K}} / F_{\breve{A}} \epsilon\, \tau_{\breve{X}}, G_{\breve{K}}\, \epsilon\, \tau_{\breve{Y}}\}$ is the basis.

Soft P-W-Hausdorff spaces

This section is dedicated to soft Pre-W-T_2Space. Different results are deeply discussed related to this soft topological structure with the application of soft pre-open sets while using ordinary points of Soft Topology.

Definition 15: A soft topological space $(\breve{X},\tau,\breve{E})$ is said to be soft pre W-Hausedorff space of type 1 denoted by $(\mathrm{PSW-H})_1$ if for every $e_1, e_2 \in \breve{E}$, $e_1 \neq e_2$ there exist soft pre-open sets $(F, \breve{A})$, $(G, \breve{B})$ such that $F_{\breve{A}}(e_1) = X$, $G_{\breve{B}}(e_2) = \breve{X}$ and $(F, \breve{A}) \cap (G, \breve{B}) = \breve{\varphi}$.

Proposition 1. Soft subspace of a $(\mathrm{PSW} - \widetilde{H})_1$ space is soft $(\mathrm{PSW} - \mathrm{H})_1$

Proof: let $(\breve{X}, \tau, \breve{E})$ be a $(\mathrm{PSW} - \widetilde{H})_1$ space. Let $\breve{Y}$ be a non-vacuous subset of $\breve{X}$.Let $(\breve{Y}\ \tau_{\breve{Y}}, \breve{E})$ be soft subspace of $(\breve{X}, \tau, \breve{E})$ where $\{\tau_{\breve{Y}} = F_{\breve{Y}}, \breve{E}): (F, \breve{E}) \epsilon \tau\}$ is the relative soft topology on $\breve{Y}$ .considered $e_1, e_2 \in \breve{E}$, $e_1 \neq e_2$, there exist soft Pre-open sets $(F, \breve{A})$, $(G, \breve{B})$ such that $F_{\breve{A}}(e_1) = \breve{X}$, $G_{\breve{B}}(e_2) = \breve{X}$ and $F_{\breve{A}} \cap G_{\breve{B}} = \breve{\varphi}$.Therefore,, $((F_{\breve{A}})_{\breve{Y}}, E), ((G_{\breve{B}})_{\breve{Y}}, \breve{E}) \epsilon \tau_{\breve{Y}}$

$$= \breve{Y} \cap \breve{X} = \breve{Y}$$

$$((G_{\breve{B}})_{\breve{Y}}, (e_2) = \breve{Y} \cap G_{\breve{B}}(e_2)$$

$$= \breve{Y} \cap \breve{X} \quad = \breve{Y}$$

$$(F_{\breve{A}})_{\breve{Y}} \cap (G_{\breve{B}})_{\breve{Y}})(e) = (F_{\breve{A}} \cap G_{\breve{B}})_{\breve{Y}})(e)$$

$$= \breve{Y} \cap (F_{\breve{A}} \cap G_{\breve{B}})_{\breve{Y}})(e)$$

$$= \breve{Y} \cap \breve{\varphi}(e)$$

$$= \breve{Y} \cap \breve{\varphi}$$

$$= \breve{\varphi}$$

$$(F_{\breve{A}})_{Y} \cap (G_{\breve{B}})_{\breve{Y}}) = \breve{\varphi}$$

Hence $(Y, \tau_{\breve{Y}}, E)$ is soft pre $(\mathrm{SW} - \widetilde{H})_1$ that is $(\mathrm{PSW} - \widetilde{H})_1$

Definition 16: Let $(\breve{X}, \tau, \breve{E})$ be a soft topological and $\widetilde{H} \subseteq \breve{E}$ then let $(\breve{X}, \tau_{\widetilde{H}}, \breve{E})$ is called soft subspace of $(\breve{X}, \tau, \breve{E})$ relative to the parameter set H where $\tau_{\widetilde{H}} = \{(F_{\breve{A}}/\widetilde{H}: \widetilde{H} \subseteq \breve{A} \subseteq \breve{E}; (F, \breve{A})\ is\ soft\ Pre - open\ set)\}$ and= $\{(F_{\breve{A}})/H\}$ is the restriction map on $\widetilde{H}$

Proposition 2. Soft p –subspace of a $(\mathrm{PSW} - \mathrm{H})_1$ space is $(\mathrm{PSW} - \mathrm{H})_1$

Proof: let $(\breve{X}, \tau, \breve{E})$ be a $(\mathrm{PSW} - \mathrm{H})_1$ space . Let $(Y, \tau_H, \breve{E})$ be soft p- subspace of $(\breve{X}, \tau, \breve{E})$ relative parameter set H where H $\tau_H = \{(F_{\breve{A}}/$
H:
$\widetilde{H} \subseteq \breve{A} \subseteq \breve{E}; where (F, \breve{A}) is\ soft\ Pre -$
$open\ set \in \tau)\}$

Consider $h, h \epsilon \widetilde{H}$ $h_1 \neq h_2$ then $h, h \epsilon \breve{E}$

Therefore, there exists Pre-open sets $(F, \breve{A})$, $(G, \breve{B})$ such that $F_{\breve{A}}(h_1) = X$, $G_{\breve{B}}(h_2) = X$ and $(F, \breve{A}) \cap (G, \breve{B}) = \breve{\varphi}$

Therefore $(F_{\breve{A}})/H$, $(G_{\breve{B}})/\widetilde{H} \in \tau_H$

$$(F_{\breve{A}})/\widetilde{H}(h_1) = X(h_1) = F_{\breve{A}}$$

$$(G_{\breve{B}})/\widetilde{H}(h_2) = G_{\breve{B}}(h_2) = \breve{X}$$

$$(F_{\breve{A}})/\widetilde{H} \cap (G_{\breve{B}})/\widetilde{H} = (F_{\breve{A}} \cap G_{\breve{B}})/\widetilde{H}$$

$$= \breve{\varphi}/H$$

$$= \breve{\varphi}$$

Hence $(\breve{X}, \tau_H, H)$ is $(\mathrm{SSW} - \mathrm{H})_1$

Proposition 3. Product of two $(\mathrm{PSW} - \mathrm{H})_1$ spaces $(\mathrm{PSW} - \mathrm{H})_1$

Proof: let $(\breve{X}, \tau_{\breve{X}}, \breve{E})$ and let $(\breve{Y}, \tau_{\breve{Y}}, \breve{K})$ be two $(\mathrm{PSW} - \mathrm{H})_1$ spaces Consider two distinct points $(e_1, \breve{K}_1)$ and $(e_2, \breve{K}_2) \epsilon\ \breve{E} \times \breve{K}$

Either $e_1 \neq e_2$ or $\breve{K}_1 \neq \breve{K}_2$ Assume $e_1 \neq e_2$ $(\breve{X}, \tau_{\breve{X}}, \breve{E})$ is $(\mathrm{PSW} - \mathrm{H})_1$ there exist soft Pre-open sets $(F, \breve{A}), (G, \breve{B})$ such that $F_{\breve{A}}(e_1) = \breve{X}$, $G_{\breve{B}}(e_2) = \breve{X}$

and $(F, \breve{A}) \cap (G, \breve{B}) = \breve{\varphi}$

Therefore $F_{\breve{A}} \circledcirc Y_K$, $G_{\breve{B}} \circledcirc \breve{Y}_K \in \tau_{\breve{X}} \circledcirc \tau_{\breve{Y}}$

$$(F_{\breve{A}} \circledcirc Y_{\breve{K}})(e_1, \breve{K}_1) = F_{\breve{A}}(e_1) \times \breve{Y}_K(\breve{K}_1)$$

$$= \breve{X} \times \breve{Y}$$

$$(G_{\breve{B}} \circledcirc \breve{Y}_{\breve{K}})(e_2, \breve{K}_2) = G_{\breve{B}}(e_2) \times \breve{A}_{\breve{K}}(\breve{K}_2)$$

$$= \breve{X} \times \breve{Y}$$

If for any

$(e, k) \in \breve{E} \times \breve{K}$, $(F_{\breve{A}} \circledcirc Y_{\breve{K}}), (e, k) \neq \widetilde{\emptyset}$

$$\Rightarrow F_{\breve{A}}(e) \times \breve{Y}_{\breve{K}}(k) \neq \widetilde{\emptyset}$$

$$\Rightarrow F_{\breve{A}}(e) \times \breve{Y}_{\breve{K}} \neq \widetilde{\emptyset}$$

$$\Rightarrow F_{\breve{A}}(e) \neq \widetilde{\emptyset}$$

$$\Rightarrow G_{\breve{B}}(e) = \widetilde{\emptyset}$$

Since $F_{\breve{A}} \cap G_{\breve{B}} = \breve{\varphi}$

$$\Rightarrow F_{\breve{A}}(e) \cap G_{\breve{B}}(e) = \widetilde{\emptyset}$$

$$\Rightarrow F_{\breve{A}}(e) \times \breve{Y}_{\breve{K}}(k) = \widetilde{\emptyset}$$

$$(G_{\breve{B}} \circledcirc \breve{Y}_{\breve{K}}), (e, k) = \widetilde{\emptyset}$$

$$(F_{\breve{A}} \circledcirc \breve{Y}_{\breve{K}}) \cap (G_{\breve{B}} \circledcirc \breve{Y}_{\breve{K}}) = \widetilde{\emptyset}$$

Assume $\breve{K}_1 \neq \breve{K}_2$ $(\breve{Y}, \tau_{\breve{Y}}, \breve{K})$ is $(\mathrm{PSW} - \mathrm{H})_1$ there exist Pre-open sets $(F, \breve{A}), (G, \breve{B})$

$F_{\breve{A}}(K_1) = Y$, $G_{\breve{B}}(K_2) = \breve{Y}$

and $(F, \breve{A}) \cap (G, \breve{B}) = \breve{\varphi}$

$$(\breve{X}_{\breve{E}} \circledcirc F_{\breve{A}})(e_1, K_1) = \breve{X}_{\breve{E}}(e_1) \times F_{\breve{A}}(K_1)$$

$$= \breve{X} \times \breve{Y}$$

$$(\breve{X}_{\breve{E}} \circledcirc G_{\breve{B}})(e_2, \breve{K}_2) = (\breve{X}_{\breve{E}}(e_2) \times G_{\breve{B}}(K_2)$$

$= \breve{X} \times \breve{Y}$

If for any $(e,k) \in \breve{E} \times \breve{K}$

$\Rightarrow (\breve{X}_{\breve{E}}(e) \times F_{\breve{A}}(k) \neq \tilde{\emptyset}$

$\Rightarrow \breve{X} \times F_{\breve{A}}(k) \neq \tilde{\emptyset}$

$\Rightarrow F_{\breve{A}}(k) \neq \tilde{\emptyset}$

$G_{\breve{B}}(k) = \phi$ Since $(F,\breve{A}),(G,\breve{B}) = \breve{\varphi}$

that is $F_{\breve{A}} \cap G_{\breve{B}} = \breve{\varphi}$

$\Rightarrow F_{\breve{A}}(k) \cap G_{\breve{B}}(K) = \tilde{\emptyset}$

$\Rightarrow \breve{X}_{\breve{E}}(e) \times G_{\breve{B}}(e) = \tilde{\emptyset}$

$\Rightarrow (\breve{X}_{\breve{E}} \circledcirc G_{\breve{B}})$(e,k) $= \tilde{\emptyset}$

$(\breve{X}_{\breve{E}} \circledcirc F_{\breve{A}}) \cap (\breve{X}_{\breve{E}} \circledcirc G_{\breve{B}}) = \breve{\varphi}$

Hence $\breve{X} \times \breve{Y}$ $(\tau_{\breve{X}} \circledcirc \tau_{\breve{Y}}, \breve{E} \times \breve{K}$ }is $(\mathrm{PSW}-\mathrm{H})_1$

Definition 17: A soft topological space (X,τ,E) is said to be soft Semi W-hausdorff that is P-W-Hausdorrf space of type 2 denoted by $(PSW-H)_2$

If for every $e_1, e_2 \in \breve{E}, e_1 \neq e_2$ there exists soft Per-open sets $(F,\breve{E}),(G,\breve{E})$ such that $F_e(e_1) = \breve{X}$, $G_E(e_2) = \breve{X}$and $F_{\breve{E}} \cap G_{\breve{E}} = \tilde{\emptyset}$

Proposition 4. Soft subspace of a $(PSW-\tilde{H})_2$ space is $(PSW-\tilde{H})_2$

Proof: Let $(\tilde{X},\tau,\tilde{E})$ be a $(PSW-\tilde{H})_2$space. Let $\breve{Y}$ be a Non-null subset of$\breve{X}$. Let $(\breve{Y},\tau_{\breve{Y}},\breve{Y})$ be a soft subspace of $((\breve{X},\tau,\tilde{E})$ where

$\tau_{\breve{Y}} = \{(F_{\breve{Y}},\breve{E}): where (F,\breve{E}) is\ soft\ Pre-open\ sets \in \tau\}$

is the relative soft topology on$\breve{Y}$. Consider$e_1, e_2 \in \tau$, $e_1, e_2 \neq \tau$ There exist soft Pre-open sets $(F,\breve{E}),(G,\breve{E})$ such that $F_{\breve{E}}(e_1) = \breve{X}, G_E(e_2) = \breve{X}\ and\ (F,\breve{E}),(G,\breve{E}) = \tilde{\emptyset}\ that\ is F_{\breve{E}} \cap G_{\breve{E}} = \tilde{\emptyset}$.

Therefore $((F_{\breve{E}})_{\breve{Y}},\breve{E}),((G_{\breve{E}})_Y,\breve{E}) \in \tau_{\breve{Y}}$

Also $(F_E)_Y(e_1) = \breve{Y} \cap F_{\breve{E}}(e_1)$

$= \breve{Y} \cap \breve{X}$

$= \breve{Y}$

$(G_E)_Y(e_2) = \breve{Y} \cap G_{\breve{E}}(e_2)$

$= \breve{Y} \cap \breve{X}$

$= \breve{Y}$

$((F_E)_{\breve{Y}} \cap (G_E)_{\breve{Y}})(e) = ((F_{\breve{E}} \cap G_{\breve{E}})_{\breve{Y}}(e)$

$= \breve{Y} \cap (F_{\breve{E}} \cap G_{\breve{E}})_{\breve{Y}})(e)$

$= \breve{Y} \cap \tilde{\emptyset}$

$= \breve{Y} \cap \tilde{\emptyset}$

$= \tilde{\emptyset}$

$(F_{\breve{A}})_{\breve{Y}} \cap (G_{\breve{B}})_{\breve{Y}} = \tilde{\emptyset}$

Hence $(\breve{Y},\tau_{\breve{Y}},\breve{E})\ is\ (SSW-\tilde{H})_2$

Proposition 5. Soft p-subspace of a $(PSW-\tilde{H})_2$ is $(PSW-\tilde{H})_2$

Proof: Let $(\tilde{X},\tau,\tilde{E})$ be a $(PSW-\tilde{H})_2$ space. Let $\tilde{H} \subseteq \tilde{E}$

Let $(\tilde{X},\tau_{\tilde{H}},\tilde{H})$ be a soft p-subspace of $(\tilde{X},\tau,\tilde{E})$, relative to the parameter set $\tilde{H}$ where $\{(F_{\tilde{A}})/\tilde{H}: \tilde{H} \subseteq \tilde{A} \subseteq \tilde{E},$
$(F,\tilde{A}) \in \tau\}\ where\ (F,\tilde{A}) is\ soft\ Pre-open\ set$

Consider $h_1, h_2 \in \tilde{H}, h_1 \neq h_2$.

Then $h_1, h_2 \in \tilde{H}$ there exists soft pre-open set $(F,\tilde{E})$,$(G,\tilde{E})$

Such that

$F_{\tilde{E}}(h_1) = \tilde{X}, G_{\tilde{E}}(h_2) = \tilde{X}\ and (F,\tilde{E})$,$(G,\tilde{E}) = \tilde{\emptyset}\ that\ is\ F_{\tilde{E}} \cap G_{\tilde{E}} = \tilde{\emptyset}$

Therefore $(F_{\tilde{A}})/\tilde{H},((G_{\tilde{E}})/H \in \tau_{\tilde{H}}$

Also $\left((F_{\tilde{E}})\ /\ \tilde{H}\right)(h_1) = F_{\tilde{E}}(h_1) = \tilde{X}$

$\left((G_{\tilde{E}})\ /\ \tilde{H}\right)(h_2) = G_{\tilde{E}}(h_2) = \tilde{X}$ &

$((F_{\tilde{E}})/\ \tilde{H}) \cap ((G_{\tilde{E}})/\tilde{H}) = (F_{\tilde{E}} \cap G_{\tilde{E}})/\tilde{H}$

$= \tilde{\emptyset}/H$

$= \tilde{\emptyset}$

Hence (X,τ,E) is $(SSW-H)_2$

Proposition 6. Product of two $(PSW-H)_2$ spaces is$(PSW-H)_2$

Proof: Let $(\tilde{X},\tau_{\tilde{X}},\tilde{E})$ & $(\tilde{Y},\tau_{\tilde{Y}},\tilde{K})$ be two $SSW-H)_2$ Soft spaces. Consider two distinct points (e_1,k_1),
(e_2,k_2). $\in \tilde{E} \times \tilde{K}$

Either $e_1 \neq e_2\ or\ k_1 \neq k_2$

Assume $e_1 \neq e_2$ Since $(\tilde{X},\tau_{\tilde{X}},\tilde{E})$ is $(PSW-H)_2$, there

Soft Pre-open sets$(F,\tilde{E})$,$(G,\tilde{E}) \in \tau_{\tilde{X}}$ such that $F_{\tilde{E}}(e_1) = \tilde{X}, G_E(e_2) = \tilde{X}$ and

$(F,\tilde{E})$,$(G,\tilde{E}) = \tilde{\emptyset}$ that is $F_{\tilde{E}} \cap G_{\tilde{E}} = \tilde{\emptyset}$

Therefore $F_{\tilde{E}} \circledcirc \tilde{Y}_{\tilde{K}}, G_{\tilde{E}} \circledcirc \tilde{Y}_{\tilde{K}} \in \tau_{\tilde{X}} \circledcirc \tau_{\tilde{Y}}$

$(F_{\tilde{E}} \circledcirc \tilde{Y}_{\tilde{K}})(e_1,k_1) = F_{\tilde{E}}(e_1) \times \tilde{Y}_{\tilde{K}}(k_1)$

$= \tilde{X} \times \tilde{Y}$

$(G_{\tilde{E}} \circledcirc \tilde{Y}_{\tilde{K}})(e_2,k_2) = G_{\tilde{E}}(e_2) \times \tilde{Y}_{\tilde{K}}(k_2)$

$= \tilde{X} \times \tilde{Y}$

If for any
$(e,k) \in (\tilde{E} \times \tilde{K}), (F_{\tilde{E}} \circledcirc \tilde{Y}_{\tilde{K}})(e,k) \neq \tilde{\emptyset}$

$\Rightarrow F_{\tilde{E}}(e) \times \tilde{Y}_{\tilde{K}}(\tilde{k}) \neq \tilde{\emptyset}$

$\Rightarrow F_{\tilde{E}}(e) \times \tilde{Y} \neq \tilde{\emptyset}$

$\Rightarrow F_{\tilde{E}}(e) \neq \tilde{\emptyset}$

$\Rightarrow G_{\tilde{E}}(e) = \tilde{\emptyset}$ $(since\ F_{\tilde{E}} \cap G_{\tilde{E}} = \tilde{\emptyset} \Rightarrow F_{\tilde{E}}(e) \cap G_{\tilde{E}}(e) = \tilde{\emptyset})$

$\Rightarrow G_{\tilde{E}}(e) \times \tilde{Y}_K(k) = \tilde{\emptyset}$

$\Rightarrow G_{\tilde{E}} \circledcirc \tilde{Y}_K(e,k) = \tilde{\emptyset}$

$\Rightarrow (F_{\tilde{E}} \circledcirc \tilde{Y}_K) \cap (G_{\tilde{E}} \circledcirc \tilde{Y}_{\tilde{K}}) = \tilde{\emptyset}$

In a similar fashion, one can prove the case when $k_1 \neq k_2$

Hence $(\tilde{X} \times \tilde{Y}, \tau_{\tilde{X}} \circledcirc \tau_{\tilde{Y}}, E \times K)$ is$(PSW - H)_2$.

Conclusions

In the present paper the notion of Soft Pre-W-Hausdorff spaces is presented with respect to ordinary points of the soft topological spaces and some basic properties regarding this concept are established in a better way. I have fastidiously studied numerous homes on the behalf of Soft Topology. And lastly I determined that soft Topology is totally linked or in other sense we can correctly say that Soft Topology (Separation Axioms) are connected with structure. Provided if it is related with structures then it gives the idea of non-linearity beautifully. In other ways we can rightly say Soft Topology is somewhat directly proportional to non-linearity, although we use non-linearity in Applied Math. So it is not wrong to say that Soft Topology is applied Math in itself. It means that Soft Topology has the taste of both of pure and applied math. In future I will discuss Separation Axioms in Soft Topology With respect to soft points.

Conflicts of Interest

The authors declare no conflict of interest.

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