Fractions

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Sarah R. Powell, Ph.D.

Associate Professor
The University of Texas at Austin

www.sarahpowellphd.com

srpowell@utexas.edu

@sarahpowellphd

Say hello.

Describe one thing from our Operations session which you've put into action.

November 2022

Operations

- Addition and subtraction concepts
- Multiplication and division concepts
- Computation with addition, subtraction, multiplication, and division

January 2023

Fractions

- Length, area, and set models
- Comparison of fractions
- Ordering of fractions
- Computation of fractions

March 2023

Word-Problem Solving

- Attack strategies
- Schemas

April 2023

Geometry

- Understanding two-dimensional shapes
- Lines and angles
- Understanding three-dimensional shapes

Model fractions with three models

Compare and order fractions

Add and subtract fractions

Multiply and divide fractions

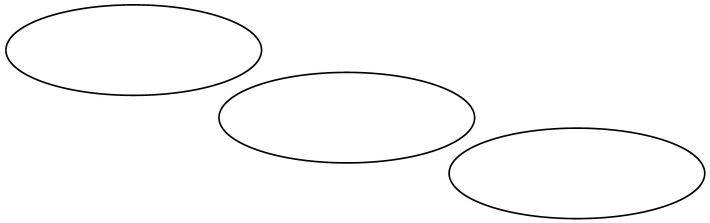
Instructional Platform

Addition and Subtraction

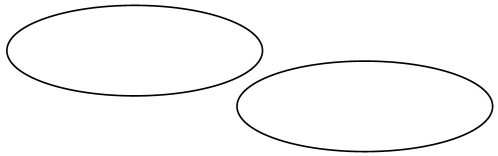
Sarah R. Powell, Ph.D.
srpowell@utexas.edu
@sarahpowellphd
www.sarahpowellphd.com

Instructional Platform

Instructional Delivery



Instructional Strategies



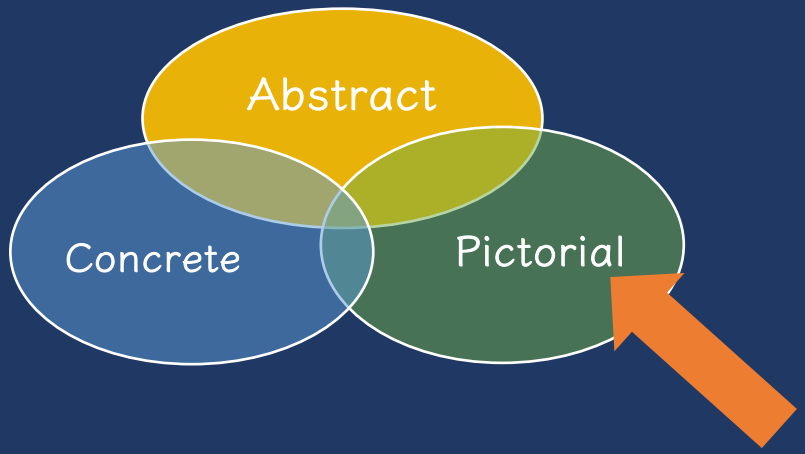
Instructional Platform

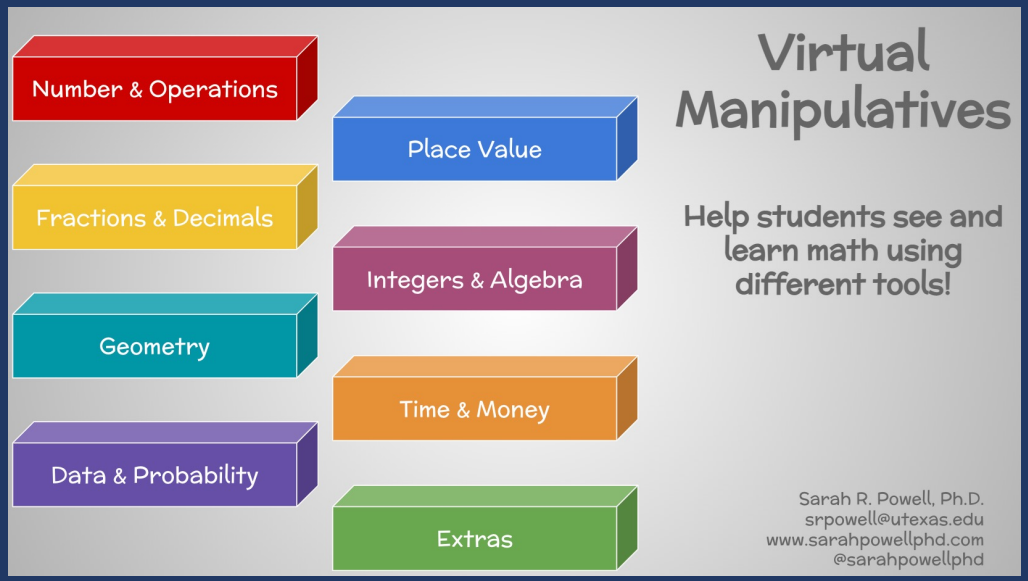
INSTRUCTIONAL DELIVERY

- Explicit instruction
- Precise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

- Fluency building
- Problem solving instruction





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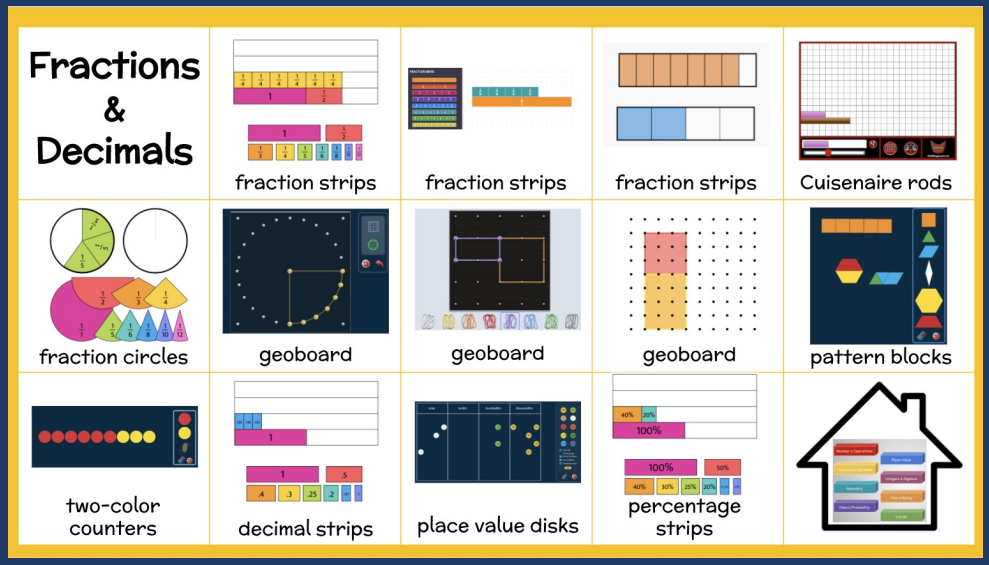


Model fractions with three models

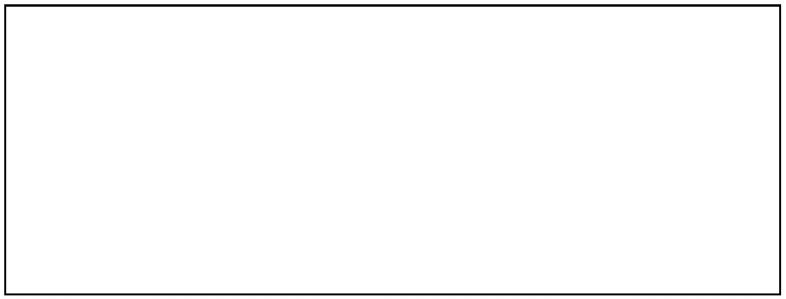
Compare and order fractions

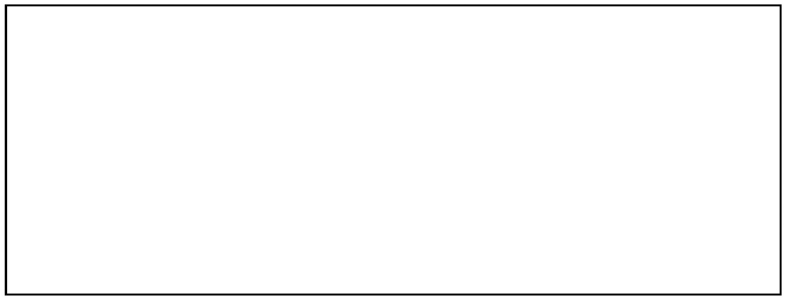
Add and subtract fractions

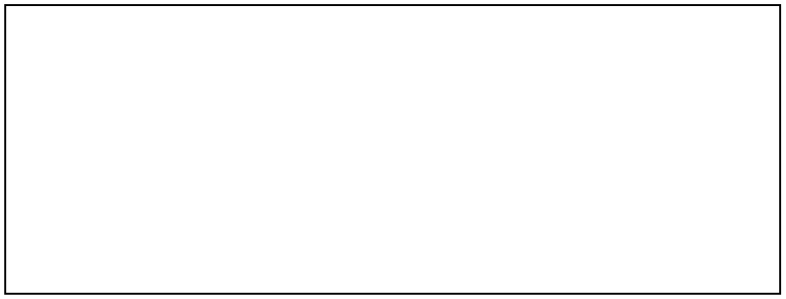
Multiply and divide fractions

Fraction Models

Models of Fractions







LENGTH

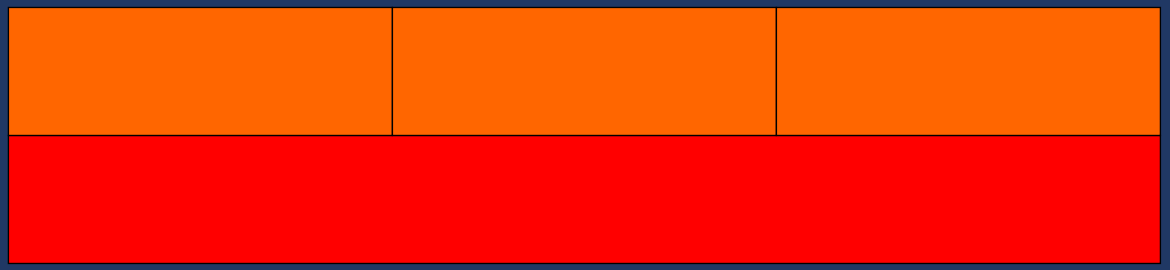
AREA

SET

LENGTH

Fractions are appropriated by length

$\frac{2}{3}$

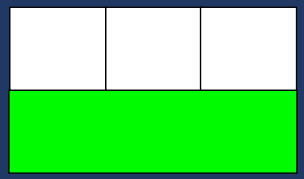


Fraction tiles/bars

LENGTH

Fractions are appropriated by length

$\frac{2}{3}$

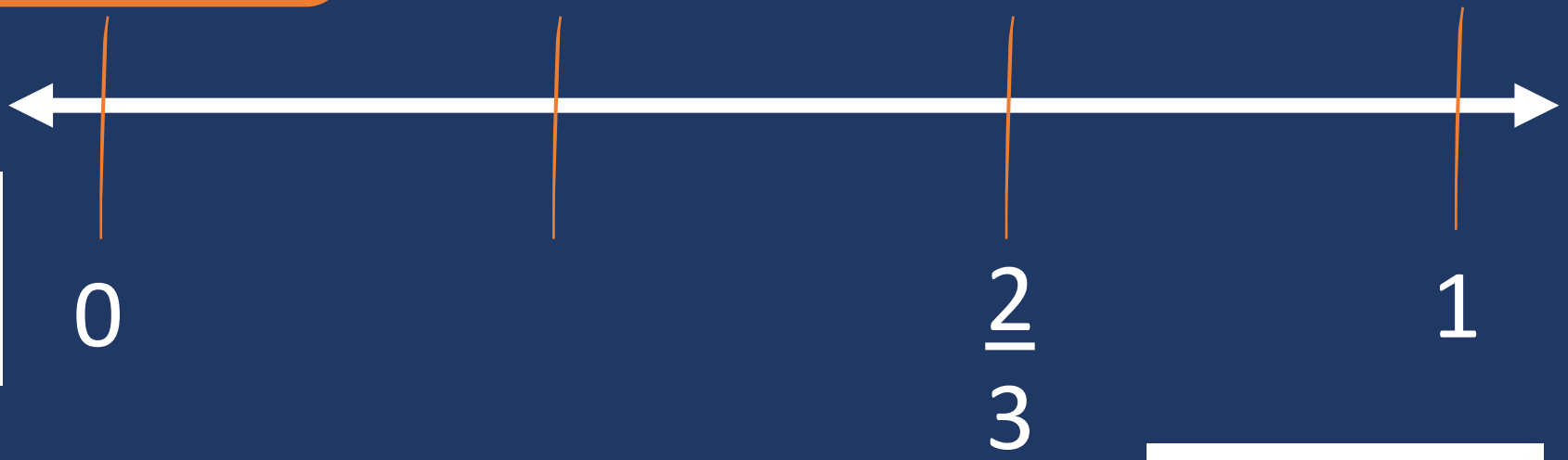


Cuisenaire rods

LENGTH

Fractions are appropriated by length

$\frac{2}{3}$



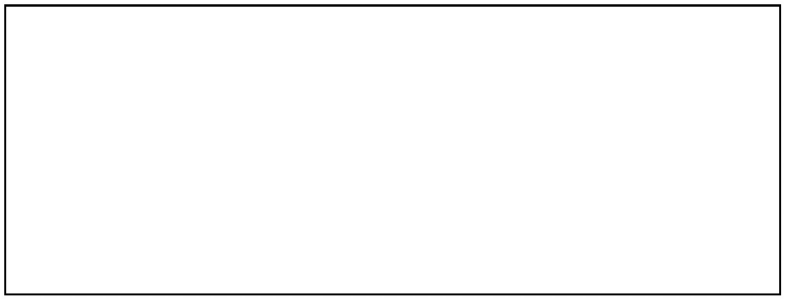
Number line

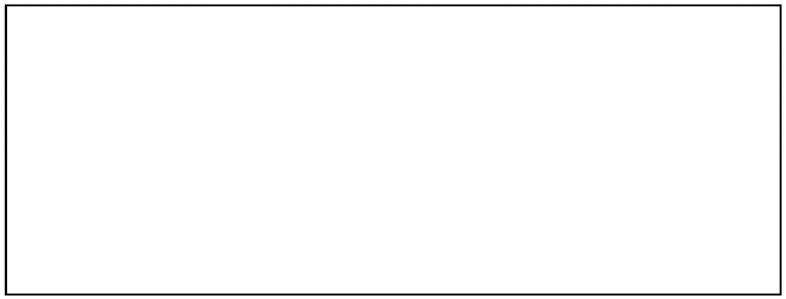
Models of Fractions

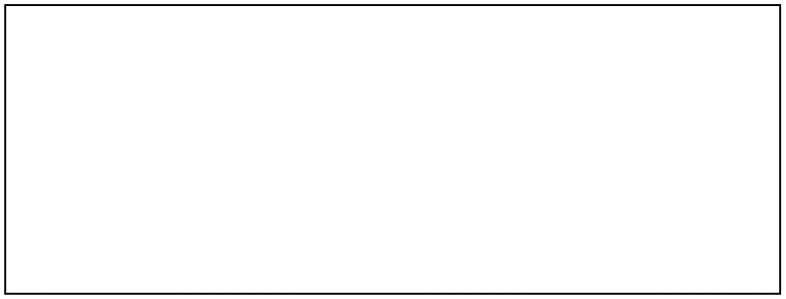
Fraction	Length	Area	Set
$\frac{2}{3}$			
$\frac{1}{4}$			
$1\frac{1}{2}$			
$\frac{3}{7}$			

Show fractions with the length model.

Models of Fractions



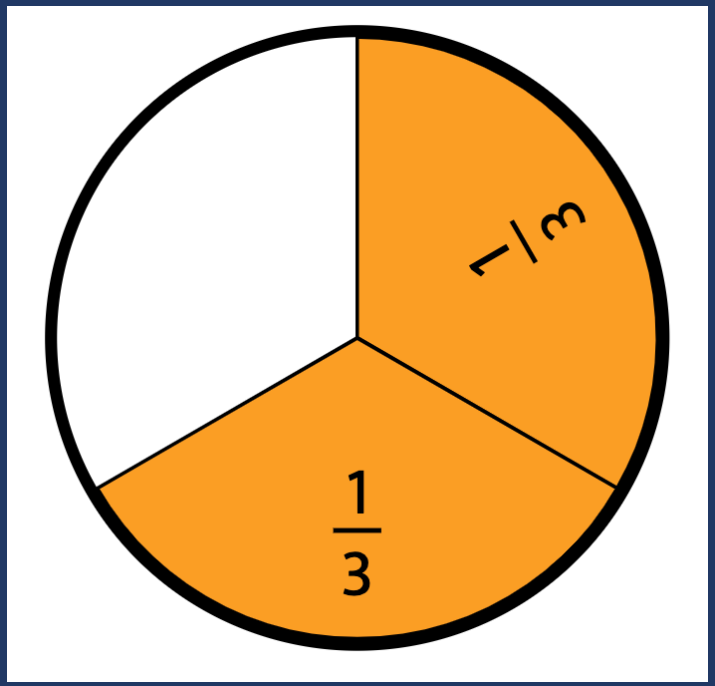




AREA

Shapes divided into equal sections

$\frac{2}{3}$

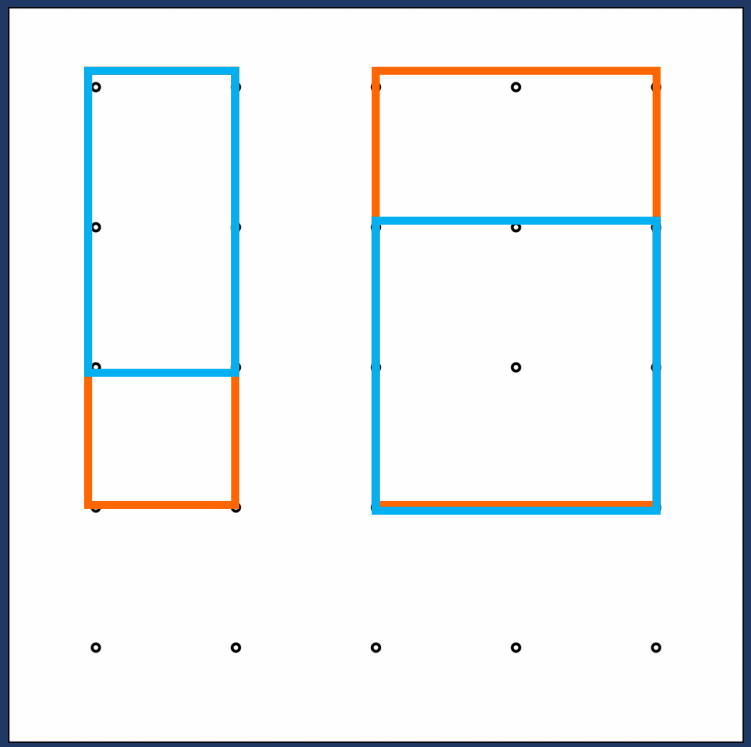


Fraction circles

AREA

Shapes divided into equal sections

$\frac{2}{3}$



Geoboards

AREA

Shapes divided into equal sections

$\frac{2}{3}$

Pattern blocks

AREA

Shapes divided into equal sections

$\frac{2}{3}$

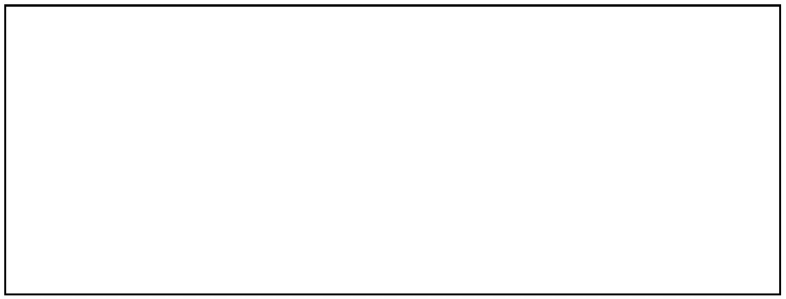
Legos

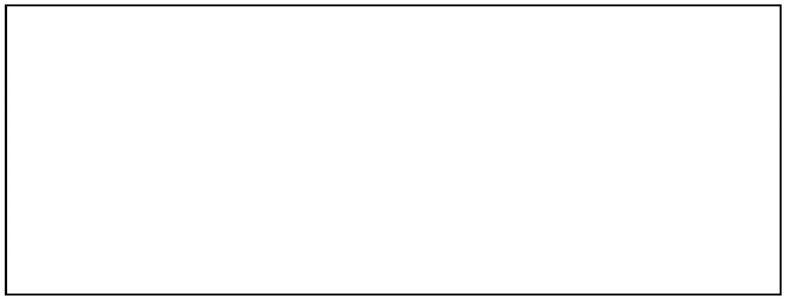
Models of Fractions

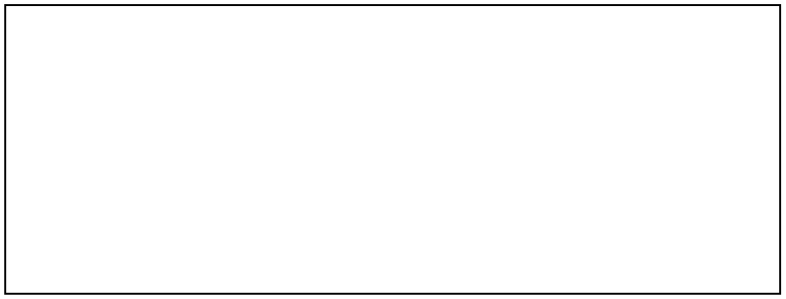
Fraction	Length	Area	Set
$\frac{2}{3}$			
$\frac{1}{4}$			
$1\frac{1}{2}$			
$\frac{3}{7}$			

Show fractions with the area model.

Models of Fractions



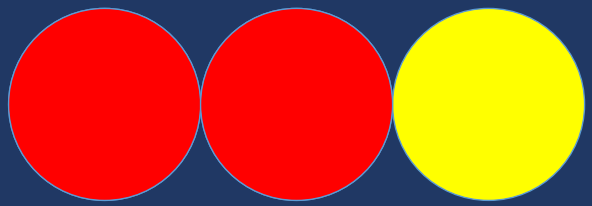




SET

Individual shapes match the fraction

$\frac{2}{3}$

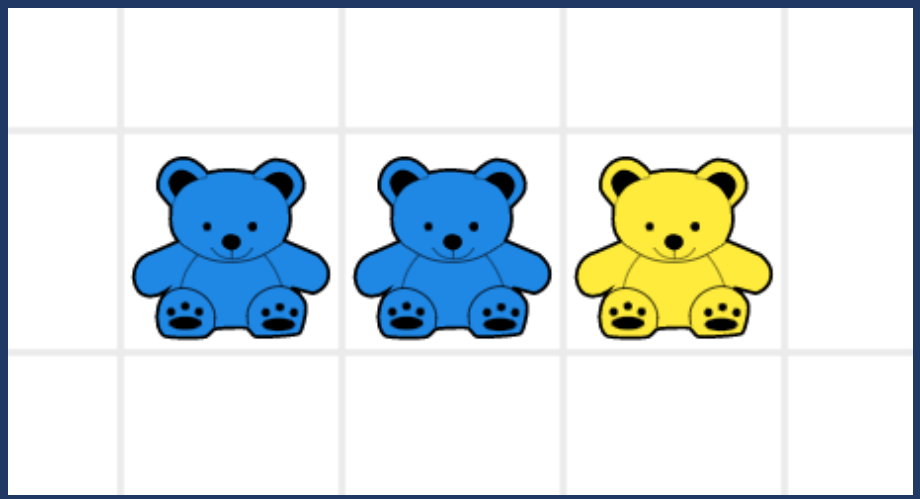


Two-color counters

SET

Individual shapes match the fraction

$\frac{2}{3}$



Models of Fractions

Fraction	Length	Area	Set
$\frac{2}{3}$			
$\frac{1}{4}$			
$1\frac{1}{2}$			
$\frac{3}{7}$			

Show fractions with the set model.

(1) Model a fraction using each of the three models.

(2) Discuss how you would use each of these models in your teaching.

Model fractions with three models

Compare and order fractions

Add and subtract fractions

Multiply and divide fractions

Compare and Order Fractions

Improper Fractions and Mixed Numbers

Equivalent Fractions

$\frac{1}{2}$ $\frac{1}{4}$

Comparing Fractions

$\frac{1}{2}$ $\frac{3}{10}$ $\frac{2}{6}$ $\frac{4}{6}$ $\frac{2}{3}$ $\frac{2}{5}$

Ordering Fractions

$\frac{6}{8}$ $\frac{3}{5}$ $\frac{1}{3}$

(1) Choose one of these activities.

(2) Model with representations.

Model fractions with three models

Compare and order fractions

Add and subtract fractions

Multiply and divide fractions

Addition with Fractions

What does it mean to add?

What are the two ways to interpret addition?

Total
(combine, putting together)

Join
(change increase, add on)

Total
(combine, putting together)

2 + 4

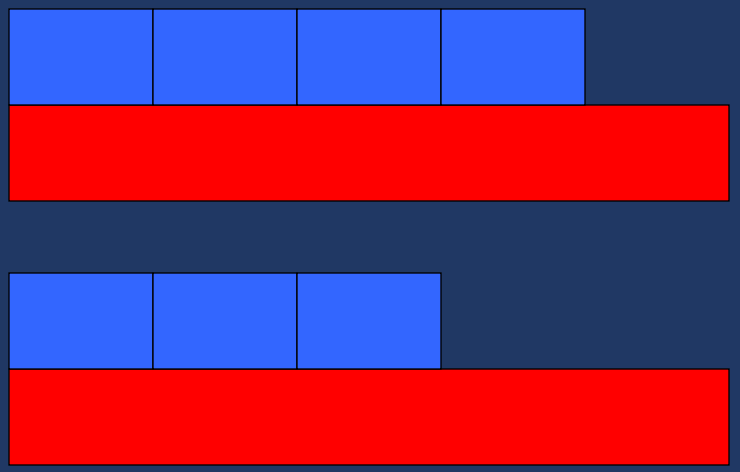
7 + 3

Join
(change increase, add on)

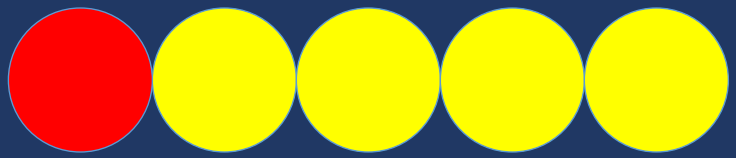
8 + 2

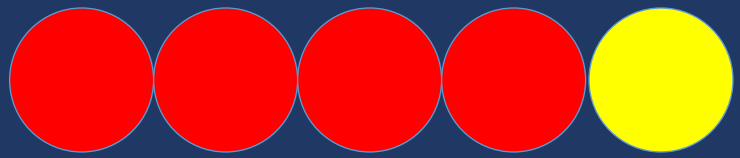
3 + 5

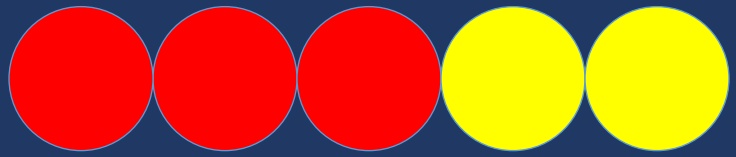
$$\frac{1}{5} + \frac{3}{5}$$

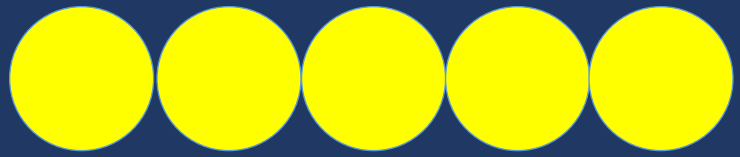


$$\frac{1}{5} + \frac{3}{5}$$









Fraction Computation

Addition

Problem	Representation
$\frac{1}{5} + \frac{3}{5}$	
$\frac{2}{8} + \frac{5}{8}$	
$\frac{2}{3} + \frac{2}{3}$	
$\frac{3}{4} + \frac{2}{4}$	
$\frac{1}{2} + \frac{1}{4}$	
$\frac{4}{6} + \frac{1}{3}$	
$\frac{4}{8} + \frac{3}{4}$	
$\frac{1}{4} + \frac{4}{6}$	

Notes on Addition:

What does it mean to subtract?

What are the two ways to interpret subtraction?

Separate
(change decrease)

Difference
(compare)

Separate
(change decrease)

$8 - 5$

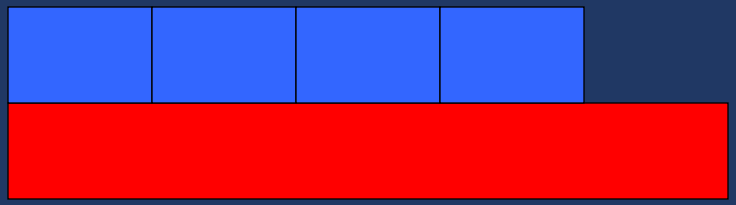
$10 - 7$

Difference
(compare)

$9 - 2$

$14 - 8$

$$\frac{4}{5} - \frac{1}{5}$$



Fraction Computation

Subtraction

Problem	Representation
$\frac{4}{5} - \frac{1}{5}$	
$\frac{6}{8} - \frac{3}{8}$	
$\frac{6}{5} - \frac{2}{5}$	
$\frac{9}{6} - \frac{4}{6}$	
$\frac{7}{8} - \frac{2}{4}$	
$\frac{8}{9} - \frac{1}{3}$	
$\frac{10}{12} - \frac{2}{3}$	
$\frac{1}{2} - \frac{2}{5}$	

Notes on Subtraction:

unlike

uncommon

different

Which vocabulary term do you use?

Multiple: The result when multiplying a number by an integer

What are the first 5 multiples of your favorite number (1-9)?

Multiple: The result when multiplying a number by an integer

Factor: The numbers you multiply together

What are the factors of your favorite number (10-100)?

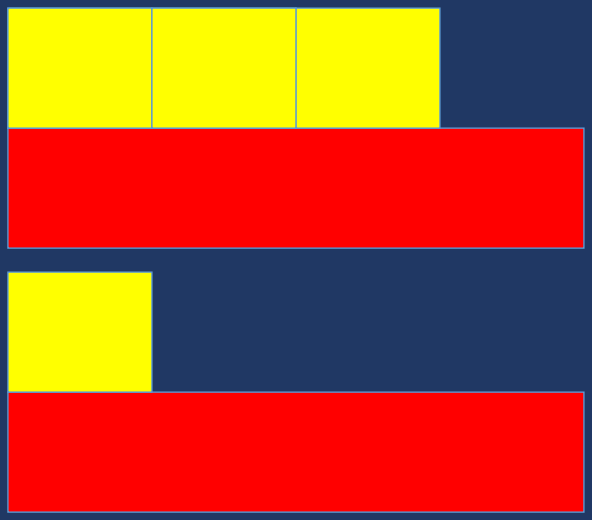
Least Common Multiple (LCM)
Least Common Denominator (LCD)

$$\frac{1}{2}+\frac{1}{3}=\frac{1\times3}{2\times3}+\frac{1\times2}{3\times2}=\frac{5}{6}$$

Greatest Common Factor (GCF)

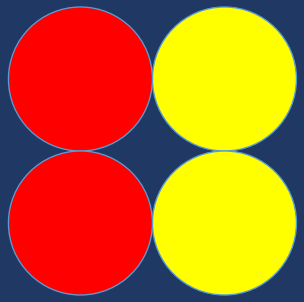
$$\frac{18}{48}=\frac{18\div6}{48\div6}=\frac{3}{8}$$

$$\frac{1}{2} + \frac{1}{4}$$

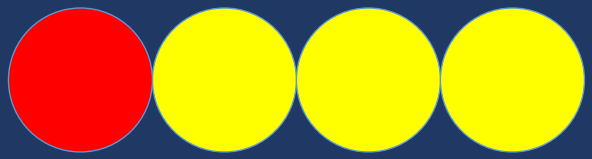


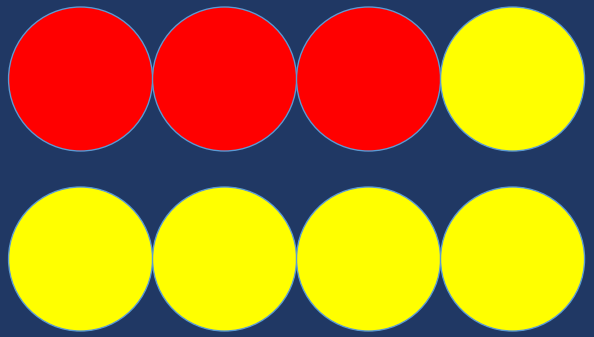
$$\frac{1}{2} + \frac{1}{4}$$

Least/lowest common multiple (LCM)
Least/lowest common denominator (LCD)



2: 2, 4 6, 8, 10
4: 4 8, 12, 16, 20





1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

Multiple Strips

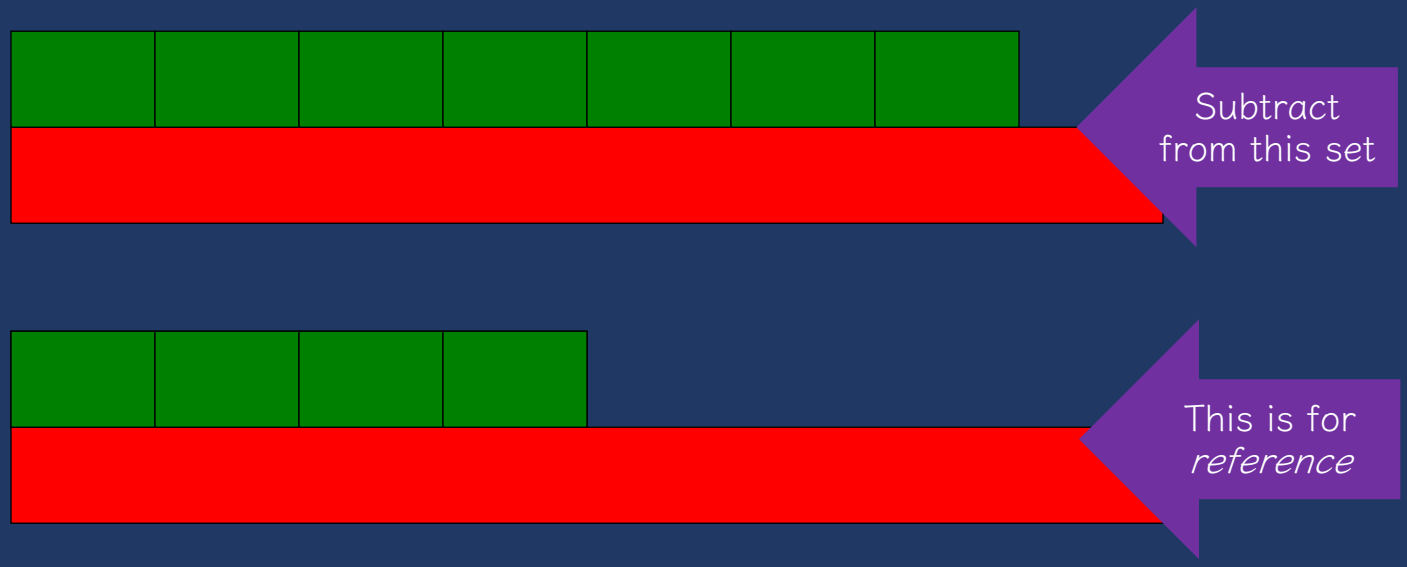
Fraction Computation

Addition

Problem	Representation
$\frac{1}{5} + \frac{3}{5}$	
$\frac{2}{8} + \frac{5}{8}$	
$\frac{2}{3} + \frac{2}{3}$	
$\frac{3}{4} + \frac{2}{4}$	
$\frac{1}{2} + \frac{1}{4}$	
$\frac{4}{6} + \frac{1}{3}$	
$\frac{4}{8} + \frac{3}{4}$	
$\frac{1}{4} + \frac{4}{6}$	

Notes on Addition:

$$\frac{7}{8} - \frac{2}{4}$$

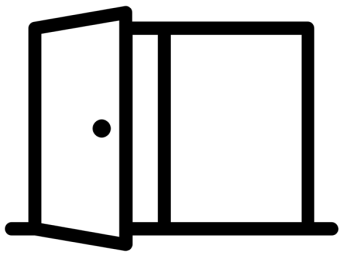


Fraction Computation

Subtraction

Problem	Representation
$\frac{4}{5} - \frac{1}{5}$	
$\frac{6}{8} - \frac{3}{8}$	
$\frac{6}{5} - \frac{2}{5}$	
$\frac{9}{6} - \frac{4}{6}$	
$\frac{7}{8} - \frac{2}{4}$	
$\frac{8}{9} - \frac{1}{3}$	
$\frac{10}{12} - \frac{2}{3}$	
$\frac{1}{2} - \frac{2}{5}$	

Notes on Subtraction:



(1) Teach an addition problem with fractions.

(2) Teach a subtraction problem with fractions.

(3) Discuss how you will emphasize addition and subtraction of fractions.

Model fractions with three models

Compare and order fractions

Add and subtract fractions

Multiply and divide fractions

$$\frac{2}{3} \times \frac{3}{4}$$

The procedure of multiplying fractions is *easy*.
Knowing when to multiply fractions is hard.

$$\frac{7}{8} \div \frac{1}{4}$$

The procedure of dividing fractions is *easy*.
Knowing when to divide fractions is hard.

Kate bought 5 and 1/3 feet of ribbon. She plans to make bookmarks, and each bookmark requires 1/8 of a foot of ribbon. How many bookmarks can Kate make?

5 1/3 – 1/8

1/8 ÷ 5 1/3

1/8 × 5 1/3

5 1/3 × 1/8

5 1/3 ÷ 1/8

What does it mean to multiply?

What are the two ways to interpret multiplication?

Equal Groups

Comparison

Equal Groups

3 x 2

2 x 6

Comparison

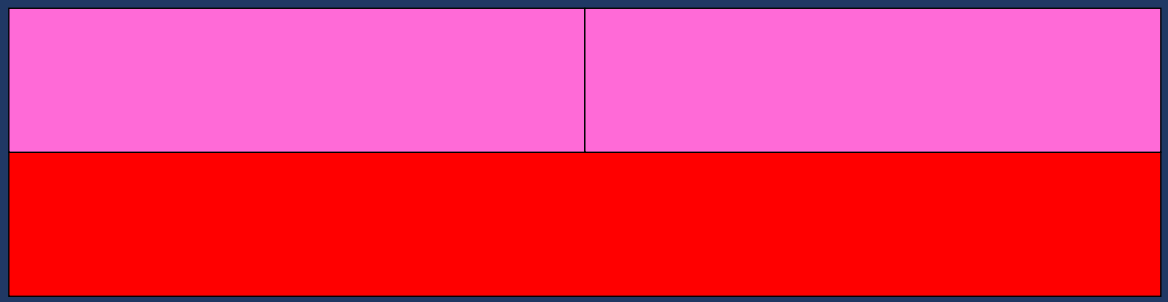
5 x 3

2 x 4

$$2 \times \frac{1}{2}$$

Two groups **of** one-half...

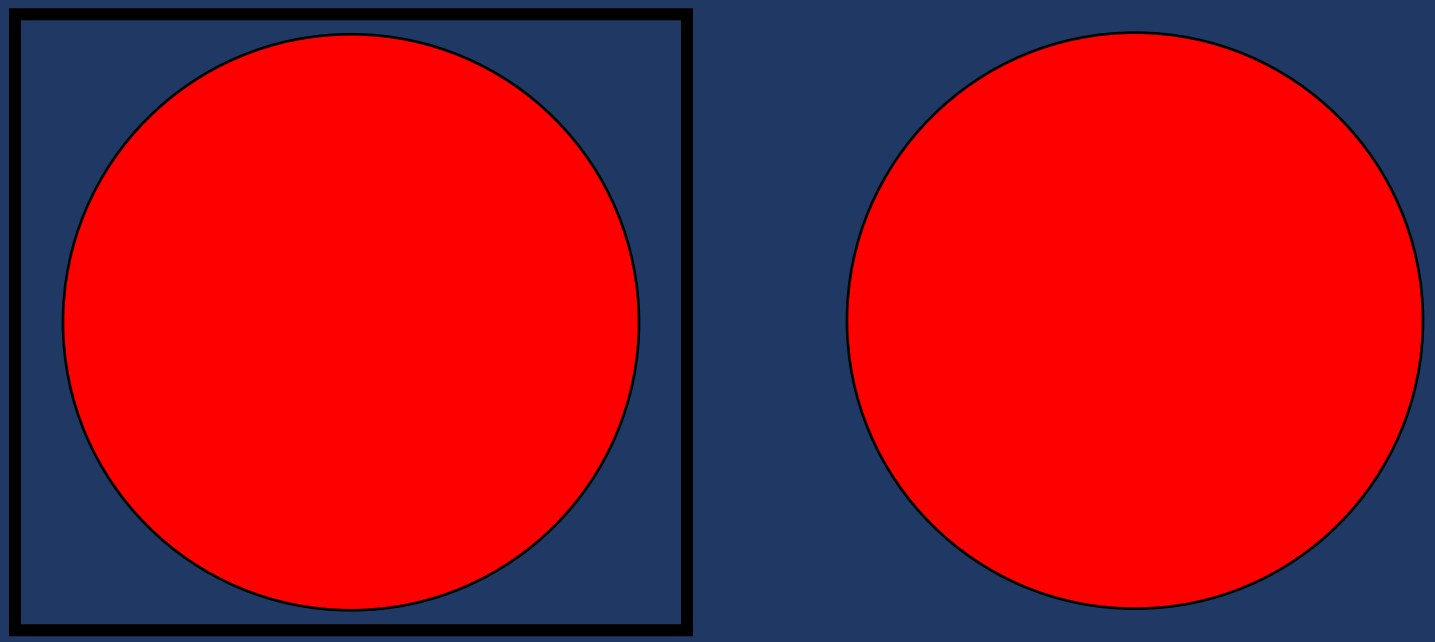
equals one.



$$\frac{1}{2} \times 2$$

One-half **of** two...

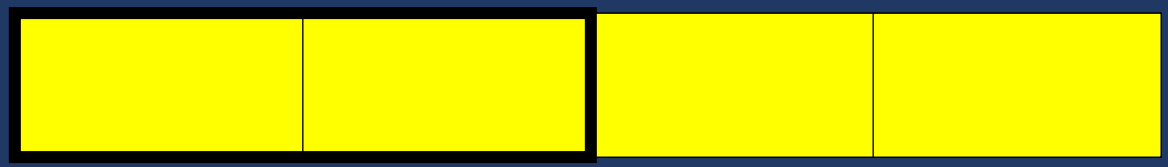
equals one.



$$\frac{1}{2} \times \frac{4}{4}$$

One-half **of** four-fourths...

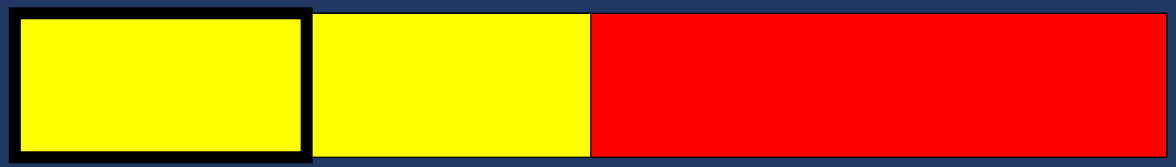
equals one-half.



$$\frac{1}{2} \times \frac{2}{4}$$

One-half **of** two-fourths…

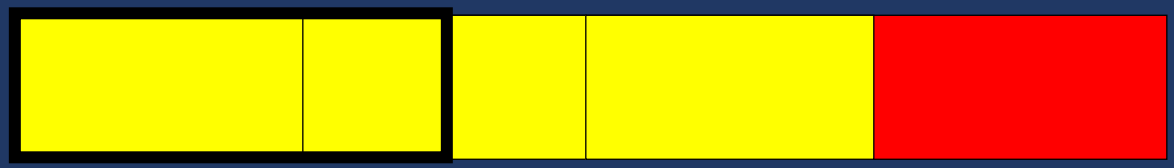
equals one-fourth.



$$\frac{1}{2} \times \frac{3}{4}$$

One-half **of** three-fourths...

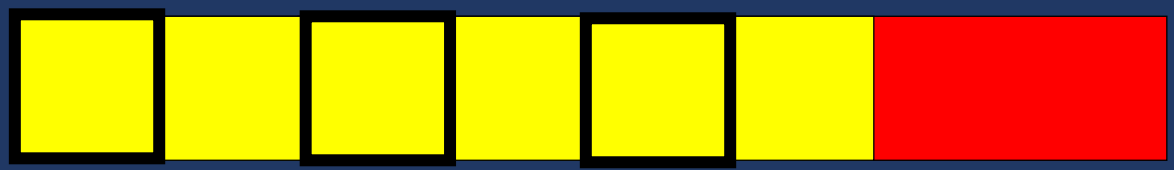
equals three-eighths.



$$\frac{1}{2} \times \frac{3}{4}$$

One-half **of** three-fourths...

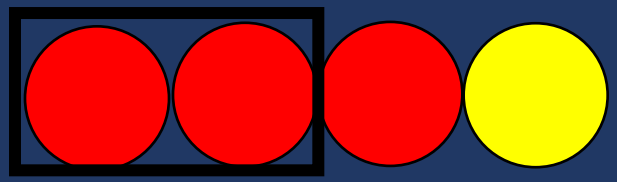
equals three-eighths.



$$\frac{2}{3} \times \frac{3}{4}$$

Two-thirds **of** three-fourths...

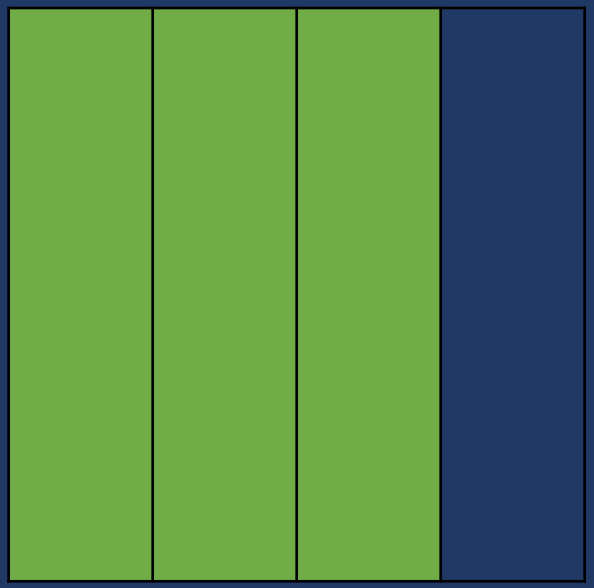
equals one-half.

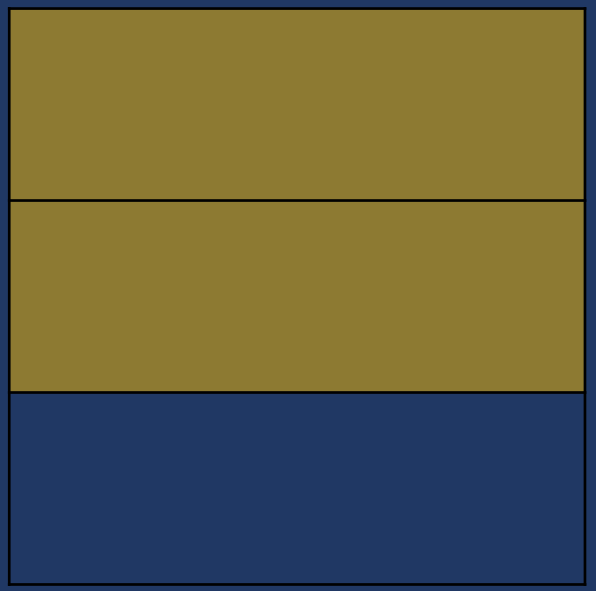


$$\frac{2}{3} \times \frac{3}{4}$$

Two-thirds **of** three-fourths...

equals one-half.

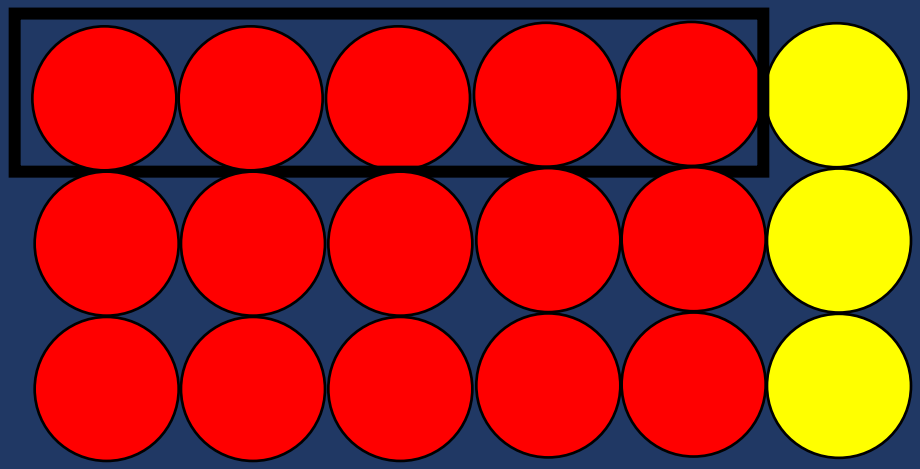




$$\frac{1}{3} \times \frac{5}{6}$$

One-third **of** five-sixths...

equals five-eighteenths.



Fraction Computation

Multiplication

Problem	Representation
2×3	
$2 \times \frac{1}{2}$	
$\frac{1}{2} \times 2$	
$\frac{1}{2} \times \frac{4}{4}$	
$\frac{1}{2} \times \frac{2}{4}$	
$\frac{1}{2} \times \frac{3}{4}$	
$\frac{2}{3} \times \frac{3}{4}$	
$\frac{1}{3} \times \frac{5}{6}$	
$\frac{3}{4} \times \frac{7}{8}$	
$\frac{5}{8} \times \frac{1}{4}$	

What does it mean to divide?

What are the two ways to interpret division?

Partitive Division
(Equal Shares)

Quotative Division

Partitive Division
(Equal Shares)

$10 \div 2$

$15 \div 3$

Quotative Division

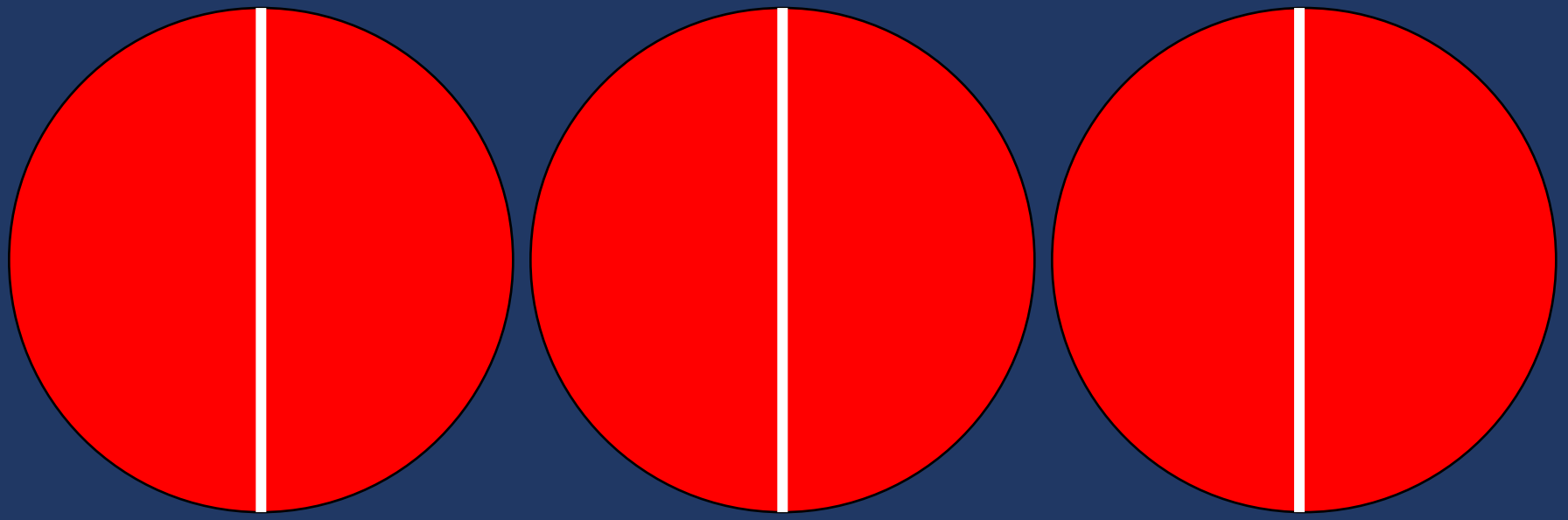
$8 \div 4$

$20 \div 5$

$$3 \div \frac{1}{2}$$

Three divided by groups of one-half...

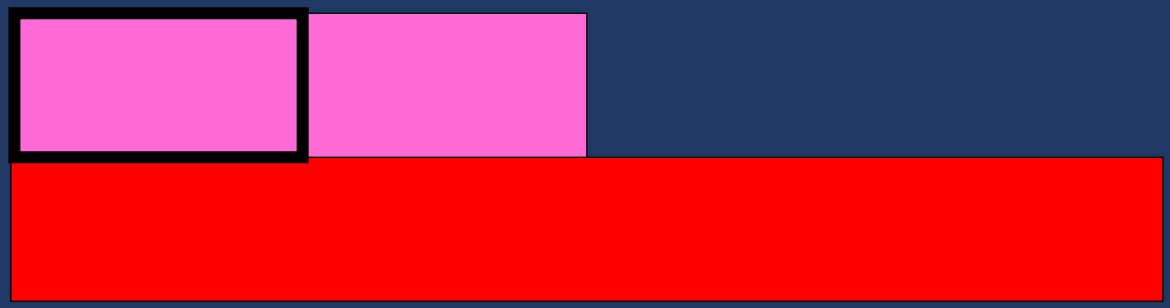
equals six.



$$\frac{1}{2} \div 2$$

A one-half group divided by two...

equals one-fourth.

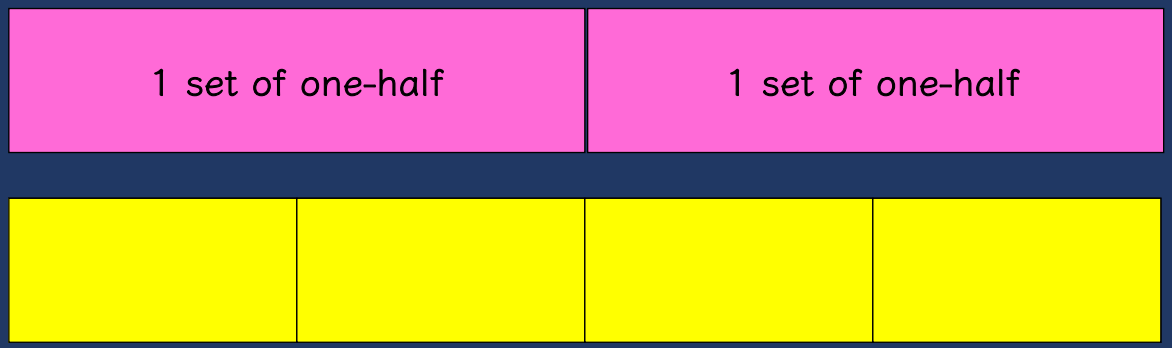


$$\frac{4}{4} \div \frac{1}{2}$$

Four-fourths divided by a group of one-half...

equals two.

How many sets of one-half can you make with four-fourths?

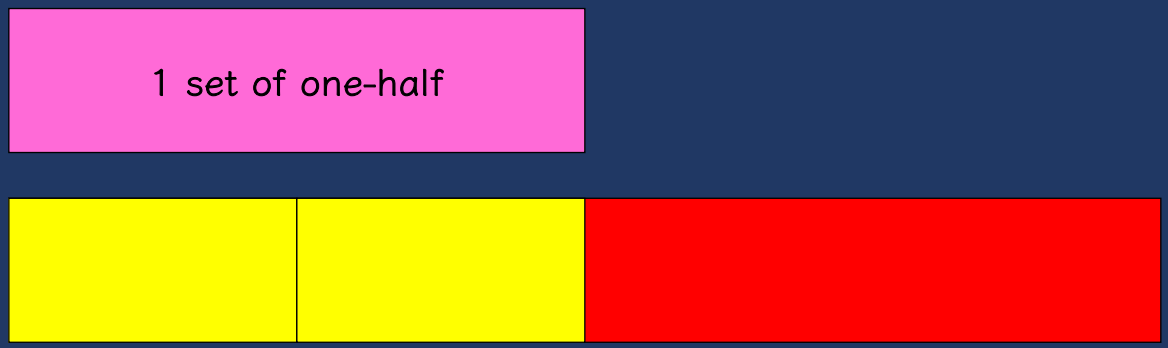


$$\frac{2}{4} \div \frac{1}{2}$$

Two-fourths divided by a group of one-half...

equals one.

How many sets of one-half can you make with two-fourths?

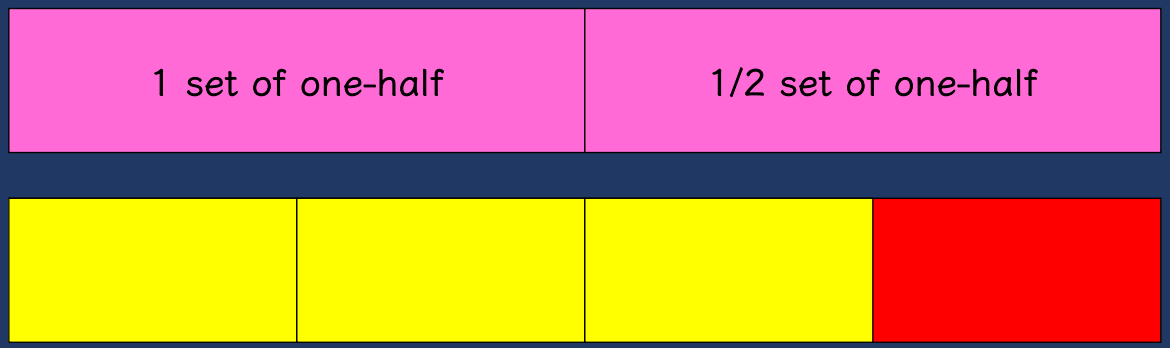


$$\frac{3}{4} \div \frac{1}{2}$$

Three-fourths divided by a group of one-half...

equals one and one-half.

How many sets of one-half can you make with three-fourths?

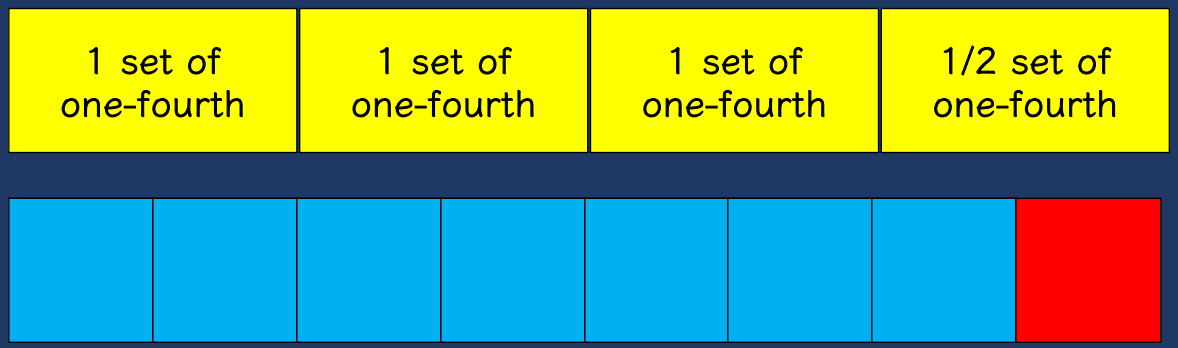


$$\frac{7}{8} \div \frac{1}{4}$$

Seven-eighths divided by a group of one-fourth...

equals three and one-half.

How many sets of one-fourth can you make with seven-eighths?



$$\frac{5}{6} \div \frac{2}{3}$$

Five-sixths divided by a group of two-thirds...

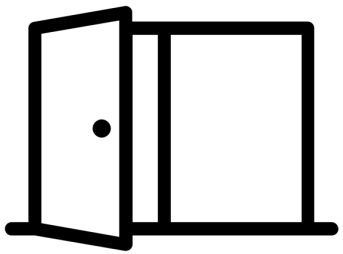
equals one and one-fourth.

How many sets of two-thirds can you make with five-sixths?

Fraction Computation

Division

Problem	Representation
$6 \div 3$	
$3 \div \frac{1}{2}$	
$\frac{1}{2} \div 2$	
$\frac{4}{4} \div \frac{1}{2}$	
$\frac{2}{4} \div \frac{1}{2}$	
$\frac{3}{4} \div \frac{1}{2}$	
$\frac{7}{8} \div \frac{1}{4}$	
$\frac{5}{6} \div \frac{2}{3}$	
$\frac{1}{2} \div \frac{3}{8}$	
$\frac{9}{6} \div \frac{1}{3}$	



(1) Teach a multiplication problem with fractions.

(2) Teach a division problem with fractions.

(3) Discuss how you will emphasize multiplication and division of fractions.

Model fractions with three models

Compare and order fractions

Add and subtract fractions

Multiply and divide fractions

Instructional Platform

INSTRUCTIONAL DELIVERY

- Explicit instruction
- Precise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

- Fluency building
- Problem solving instruction

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

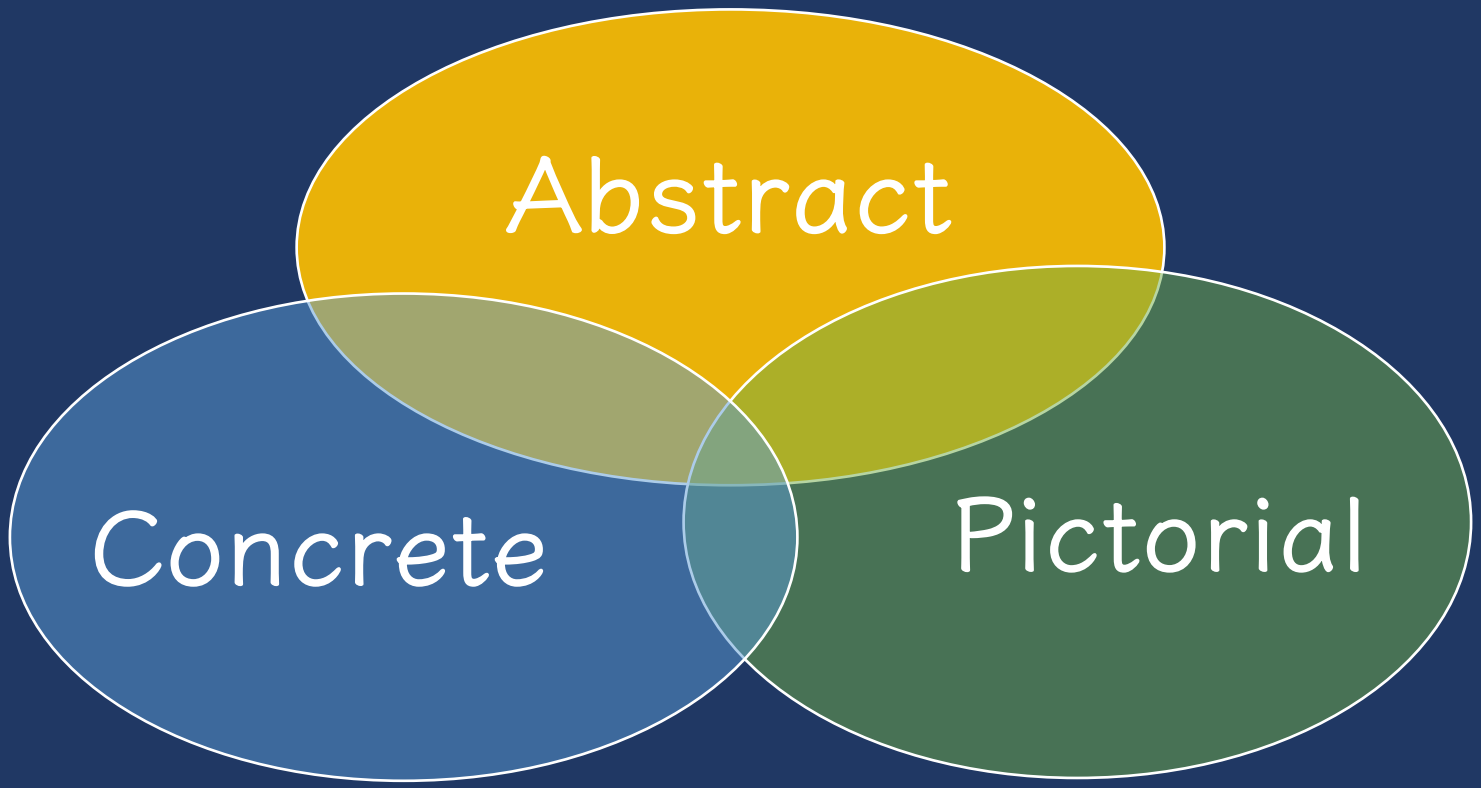
What are your strengths with modeling fractions?

What are your opportunities for growth?

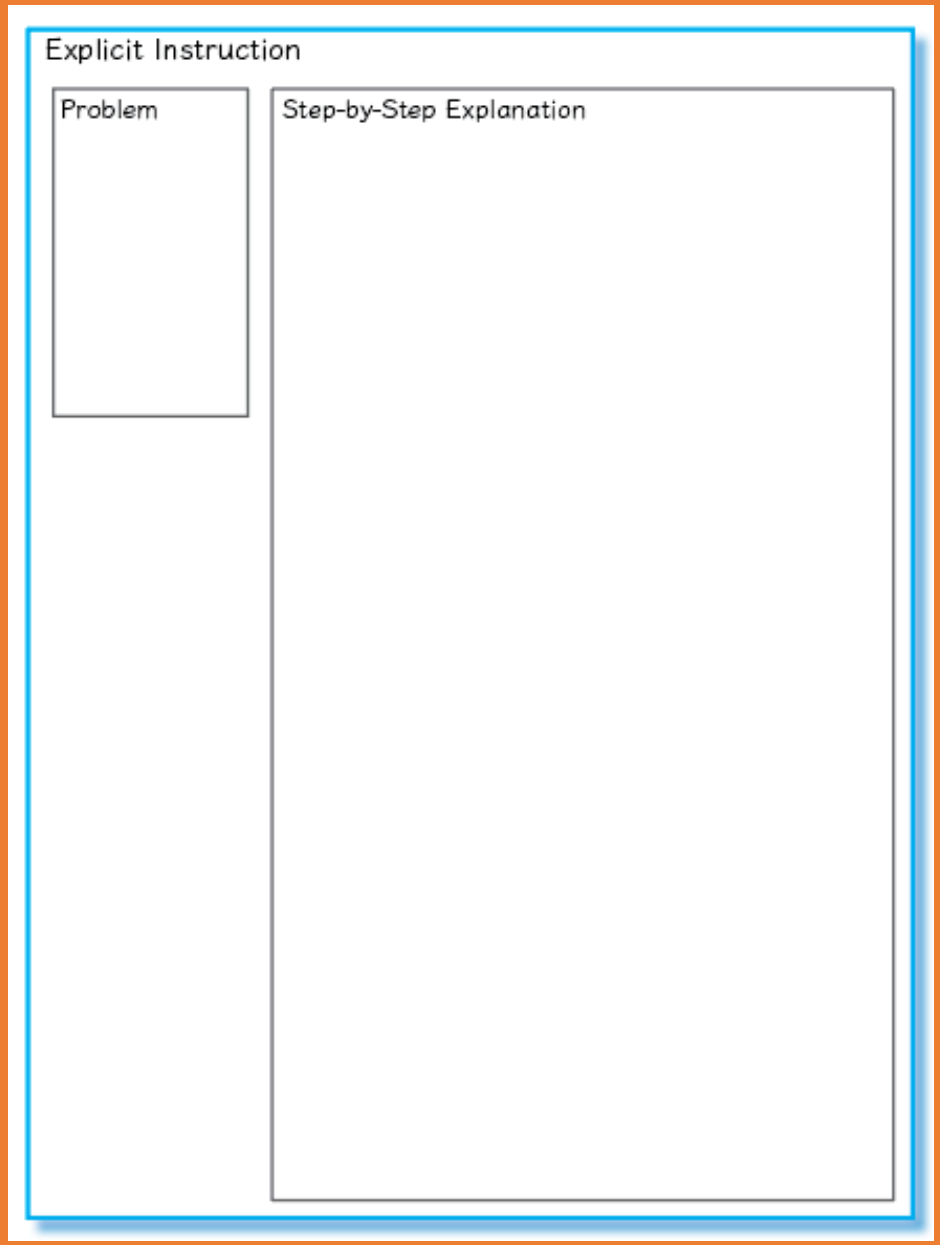
Use formal math language

Use terms precisely

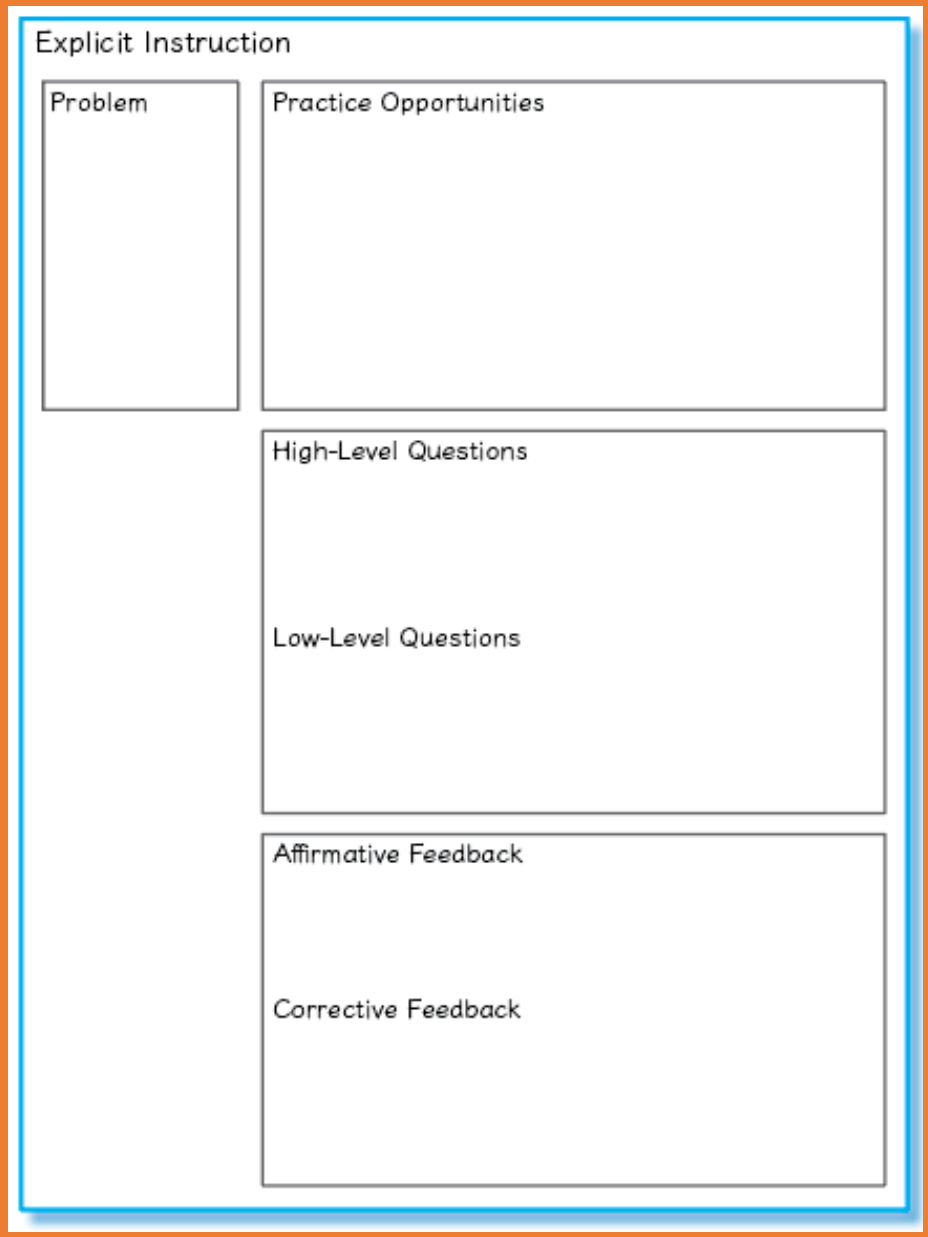
What are five essential math vocabulary for fractions?



What are the representations you'll use to teach fractions?



1. Choose a math problem.
2. Write a step-by-step explanation. Focus on the language of math in your explanation. Consider the representations you will use.



1. Describe the practice opportunities you will use.
2. Write 3 high-level questions.
3. Write 3 low-level questions.
4. Write 2 ways to provide affirmative feedback.
5. Write 2 ways to provide corrective feedback.

November 2022

Operations

- Addition and subtraction concepts
- Multiplication and division concepts
- Computation with addition, subtraction, multiplication, and division

January 2023

Fractions

- Length, area, and set models
- Comparison of fractions
- Ordering of fractions
- Computation of fractions

March 2023

Word-Problem Solving

- Attack strategies
- Schemas

April 2023

Geometry

- Understanding two-dimensional shapes
- Lines and angles
- Understanding three-dimensional shapes

Sarah R. Powell, Ph.D.

Associate Professor
The University of Texas at Austin

www.sarahpowellphd.com

srpowell@utexas.edu

@sarahpowellphd