Algebraic Number Theory: Course Information

Winter and Spring 2019

Professor Ellen Eischen (eeischen@uoregon.edu, 315 Fenton)

Office Hours Tue 3:30-4:30 pm and Thu 11:20-12:20 pm

Target Audience Graduate students in number theory and related fields, including algebraic geometry, representation theory, and topology.

Prerequisites The main prerequisite is abstract algebra at the level of the 500-level algebra sequence (with an emphasis on the Galois theory covered in Math 546), as well as elementary knowledge of modules. Students who have seen some commutative algebra will be better prepared. Students who have not seen commutative algebra will need to do a little extra work or accept a few facts as black boxes.

Learning Goals This sequence will focus on the structure of number fields, including ideal class groups, unit groups, cyclotomic extensions, quadratic reciprocity, special cases of Fermat's Last Theorem, local fields, global fields, adeles, and an introduction to class field theory. Topics and pace may be adjusted according to the backgrounds and interests of enrolled students. Students who complete the problem sets will improve at solving problems and communicating about number theory.

Requirements To accommodate the varied goals of enrolled students, there will be a lot of flexibility in the commitment level expected. You will get an A in this course if you complete Option I or Option II below. (N.B. If you are not a math grad student, you must do only Option I.)

Option I (if you who want to start to understand the details well):

> Submit a total of at least 27 (clear, at least mostly correct) typed solutions to the approximately 50 assigned problems, submitted via Canvas by the due dates for the corresponding assignments. If you'd like, you may replace some written solutions with brief in-class presentations, orally summarizing your solutions instead. (I very strongly encourage you to do such presentations! Let me know, no later than the day the problems are due, if you'd like me to schedule this for you.)

Option II (if you are primarily focusing on other math):

> Contribute typed or neatly written class notes for $\geq d/N$ lectures, with N the number of students choosing this option and d the number of lectures. (To make sure all classes are covered, you will be assigned specific days of note-taking.) The notes will be shared, via Canvas, with the whole class.

Math grad students are also welcome to complete both options, or to mix and match, switching during the term. (e.g. You could submit 9 problems together with 2/3 the amount of notes someone with just Option II would submit.) If you do only 1/2 of the work, you will get a B. If you do 1/4 (resp. none), you will get a C (resp. F).

To prepare to work with me as your advisor, you must (during this course or later on) complete Option I. If you are taking this course to prepare for a career in number theory and are seeing this material for the first time, plan to spend at least 10 hours per week on it outside of class. The more you actively engage with the material, the more you will develop your understanding of it.

Resources:

- There are many useful algebraic number theory textbooks. This quarter, we will primarily follow **Milne's** *Algebraic Number Theory*, **which is available for free at** `https://www.jmilne.org/math/CourseNotes/ANT.pdf.`
  There are also many other useful algebraic number theory textbooks, including:
  (1) *A Classical Introduction to Modern Number Theory*, Ireland and Rosen
  (2) *Algebraic Number Theory*, Fröhlich and Taylor
  (3) *Number Fields*, Marcus
  (4) *Algebraic Number Theory*, Neukirch
  (5) *Algebraic Number Theory*, Lang
  (6) *Algebraic Number Theory*, Osserman
  (7) *Algebraic Number Theory, a Computational Approach*, Stein
- Canvas will be used to inform students of the assignments, course information, and updates.

Policies and Expectations:

- Please respect (and, moreover, encourage and support) your classmates. You will get the most out of the class by working cooperatively with other hard-working classmates.
- I expect you to be actively engaged in class.
- Except in unusual circumstances, cell phones, ipods, computers, etc. are not permitted in this class. They distract other people in the classroom (and, even if you don't realize it, they probably distract you, too).
- Any student needing accommodation for a disability is required to make arrangements through Oregon's Accessible Education Center as early as possible in the semester, preferably within the first two weeks.

Non-negotiable Policy on Honesty:

Students are encouraged to discuss the course material and assignments with each other, as well as to consult additional resources. Unless explicitly instructed otherwise, though, each student must submit their own assignment and acknowledge any resources consulted (names of books, websites, students, etc). Assignments copied from another source will be treated as plagiarism, reported to the university, and result in a course grade of F (and future assignments completed by the student will not be accepted). The one exception is in-class notes for this course, which you may copy and submit *verbatim* from class.

I assume you are in this course to help you prepare to do research in your dissertation area or because you are eager and curious to learn more about number theory. You have absolutely nothing to gain by committing academic dishonesty in this course, since your research - and not your performance in this course - will determine your job prospects if you stay in academia (and, if you leave academia, it is unlikely to matter how you did in this course!). If you are here simply to satisfy your curiosity about number theory, you also will fail at your goal if you decide to cheat your way through the course.

Changes: In the event of unforeseen circumstances (such as a flu epidemic), the professor reserves the right to make changes to the syllabus. Such changes are unlikely, but if they must be made, students will be notified as soon as possible.

On the importance of having already fulfilled the prerequisites: This is a graduate course for students who already have a solid background in algebra, especially Galois theory of finite extensions. If you do not currently know the basics of groups, rings, and fields (at the level of Math 444/544, 445/545, and 446/546) or are unwilling to review concepts you have forgotten, then do not take this class. In addition to traditional courses, there are many excellent introductory abstract algebra textbooks (Dummit and Foote, Fraleigh, Artin, etc.) and online resources. Also, you might find your peers in the graduate program to be excellent people with whom to review prerequisite topics.