Edexcel GCE
Core Mathematics C4
Gold Level G1
(Mark Scheme)

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Question Number	Scheme	Marks
1.	$\int x\sin 2x\,\mathrm{d}x = -\frac{x\cos 2x}{2} + \int \frac{\cos 2x}{2}\,\mathrm{d}x$	M1 A1 A1
	$= \quad \ldots \quad + \frac{\sin 2x}{4}$	M1
	$\left[\ \ldots \ \right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$	M1 A1
		[6]

Question Number	Scheme		Marks
2 (a)	$\text{Area}(R) = \int_0^2 \frac{3}{\sqrt{(1+4x)}}\,\mathrm{d}x = \int_0^2 3(1+4x)^{-\frac{1}{2}}\,\mathrm{d}x$		
	$= \left[\frac{3(1+4x)^{\frac{1}{2}}}{\frac{1}{2}.4}\right]_0^2$	***Integrating*** $3(1+4x)^{-\frac{1}{2}}$ to give $\pm k(1+4x)^{\frac{1}{2}}$. Correct integration. Ignore limits.	M1 A1
	$= \left[\frac{3}{2}(1+4x)^{\frac{1}{2}}\right]_0^2$		
	$= \left(\frac{3}{2}\sqrt{9}\right) - \left(\frac{3}{2}(1)\right)$	Substitutes limits of 2 and 0 into a changed function and subtracts the correct way round.	M1
	$= \frac{9}{2} - \frac{3}{2} = \underline{3}\,(\text{units})^2$	$\underline{3}$	A1 (4)
	(Answer of 3 with no working scores M0A0M0A0.)		
(b)	$\text{Volume} = \underline{\pi \int_0^2 \left(\frac{3}{\sqrt{(1+4x)}}\right)^2 \mathrm{d}x}$	Use of $V = \pi\int y^2\,\mathrm{d}x$. Can be implied. Ignore limits and $\mathrm{d}x$.	B1
	$= (\pi)\int_0^2 \frac{9}{1+4x}\,\mathrm{d}x$		
	$= (\pi)\left[\frac{9}{4}\ln\lvert 1+4x\rvert\right]_0^2$	$\pm k\ln\lvert 1+4x\rvert$ $\frac{9}{4}\ln\lvert 1+4x\rvert$	M1 A1
	$= (\pi)\left[\left(\frac{9}{4}\ln 9\right) - \left(\frac{9}{4}\ln 1\right)\right]$	Substitutes limits of 2 and 0 and subtracts the correct way round.	dM1
	Note that $\ln 1$ can be implied as equal to 0.		
	So Volume $= \underline{\frac{9}{4}\pi\ln 9}$	$\underline{\frac{9}{4}\pi\ln 9}$ or $\underline{\frac{9}{2}\pi\ln 3}$ or $\underline{\frac{18}{4}\pi\ln 3}$	A1 oe isw (5)
	Note the answer must be a one term exact value. Note, also you can ignore subsequent working here.	Note that $= \frac{9}{4}\pi\ln 9 + c$ (oe.) would be awarded the final A0.	**[9]**

Question Number	Scheme		Marks
3.	$\int_0^4 \frac{1}{2+\sqrt{(2x+1)}}\,\mathrm{d}x \ , \ u = 2+\sqrt{(2x+1)}$		
		Either	
	$\frac{\mathrm{d}u}{\mathrm{d}x} = (2x+1)^{-\frac{1}{2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = u-2$	$\frac{\mathrm{d}u}{\mathrm{d}x} = \pm K(2x+1)^{-\frac{1}{2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = \pm\lambda(u-2)$	M1
		Either $\frac{\mathrm{d}u}{\mathrm{d}x} = (2x+1)^{-\frac{1}{2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = (u-2)$	A1
	$\left\{\int \frac{1}{2+\sqrt{(2x+1)}}\,\mathrm{d}x\right\} = \int \frac{1}{u}(u-2)\,\mathrm{d}u$	Correct substitution (Ignore integral sign and $\mathrm{d}u$).	A1
	$= \int\left(1-\frac{2}{u}\right)\mathrm{d}u$	An attempt to divide each term by u.	dM1
	$= u - 2\ln u$	$\pm Au \pm B\ln u$	ddM1
		$u - 2\ln u$	A1 ft
	$\left\{\text{So } \left[u-2\ln u\right]_3^5\right\} = (5-2\ln 5)-(3-2\ln 3)$	Applies limits of 5 and 3 in u or 4 and 0 in x in their integrated function and subtracts the correct way round.	M1
	$= 2+2\ln\left(\frac{3}{5}\right)$	$2+2\ln\left(\frac{3}{5}\right)$	A1 **cao cso**
			[8]

Question Number	Scheme		Marks
4.	$x = 2\sin t, \quad y = 1 - \cos 2t \quad \{= 2\sin^2 t\}, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$		
(a)	$\frac{dx}{dt} = 2\cos t, \quad \frac{dy}{dt} = 2\sin 2t$ or $\frac{dy}{dt} = 4\sin t\cos t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	So, $\frac{dy}{dx} = \frac{2\sin 2t}{2\cos t} \left\{= \frac{4\cos t\sin t}{2\cos t} = 2\sin t\right\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and substitutes $t = \frac{\pi}{6}$ into their $\frac{dy}{dx}$.	M1;
	At $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{2\sin\left(\frac{2\pi}{6}\right)}{2\cos\left(\frac{\pi}{6}\right)}; = 1$	Correct value for $\frac{dy}{dx}$ of 1	A1 **cao cso** **(4)**
(b)	$y = 1 - \cos 2t = 1 - (1 - 2\sin^2 t)$ $= 2\sin^2 t$		M1
	So, $y = 2\left(\frac{x}{2}\right)^2$ or $y = \frac{x^2}{2}$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$	$y = \frac{x^2}{2}$ or equivalent.	A1 **cso isw**
	Either $k = 2$ or $-2 \leqslant x \leqslant 2$		B1 **(3)**
(c)	Range: $0 \leqslant \mathrm{f}(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant \mathrm{f} \leqslant 2$		B1 B1 **(2)** **[9]**

Question Number	Scheme		Marks
5. (a)	If l_1 and l_2 intersect then: $\begin{pmatrix}1\\0\\-1\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}1\\3\\6\end{pmatrix}+\mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$		
	Any two of **i**: $1+\lambda=1+2\mu$ (1) **j**: $\lambda=3+\mu$ (2) **k**: $-1=6-\mu$ (3)	Writes down any two of these equations correctly.	M1
	(1) & (2) yields $\lambda=6,\ \mu=3$ (1) & (3) yields $\lambda=14,\ \mu=7$ (2) & (3) yields $\lambda=10,\ \mu=7$	Solves two of the above equations to find … either one of λ or μ correct both λ and μ correct	A1 A1
	Either checking eqn (3), $-1\neq 3$ checking eqn (2), $14\neq 10$ checking eqn (1), $11\neq 15$	Complete method of putting their values of λ and μ into a third equation to show a contradiction.	B1$\surd$
	or for example: checking eqn (3), LHS = -1, RHS = 3 $\Rightarrow$ Lines l_1 and l_2 do not intersect	this type of explanation is also allowed for B1$\surd$.	**[4]**

Question Number	Scheme		Marks
5. (b)	$\lambda = 1 \Rightarrow \overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ & $\mu = 2 \Rightarrow \overrightarrow{OB} = \begin{pmatrix} 5 \\ 5 \\ 4 \end{pmatrix}$	**Only one of either** $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ or $\overrightarrow{OB} = \begin{pmatrix} 5 \\ 5 \\ 4 \end{pmatrix}$ or $A(2,1,-1)$ or $B(5,5,4)$. (can be implied)	B1
	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 5 \\ 5 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$ or $\overrightarrow{BA} = \begin{pmatrix} -3 \\ -4 \\ -5 \end{pmatrix}$	Finding the difference between their $\overrightarrow{OB}$ and $\overrightarrow{OA}$. (can be implied)	M1$\surd$
	$\overrightarrow{AB} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$, $\mathbf{d}_1 = \mathbf{i} + \mathbf{j} + 0\mathbf{k}$ & θ is angle		M1
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \mathbf{d}_1}{\left\lvert\overrightarrow{AB}\right\rvert . \left\lvert\mathbf{d}_1\right\rvert} = \pm\left(\dfrac{3+4+0}{\sqrt{50}.\sqrt{2}}\right)$	Applies dot product formula between $\mathbf{d}_1$ and their $\pm\overrightarrow{AB}$. Correct expression.	M1$\surd$ A1
	$\cos\theta = \frac{7}{10}$	$\frac{7}{10}$ or 0.7 or $\frac{7}{\sqrt{100}}$ but not $\frac{7}{\sqrt{50}\sqrt{2}}$	A1 **cao** **[6]**
			10 marks

Question Number	Scheme	Marks
6. (a)	$\left\{x = u^2 \Rightarrow\right\} \dfrac{\mathrm{d}x}{\mathrm{d}u} = 2u$ or $\dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{1}{2}x^{-\frac{1}{2}}$ or $\dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{1}{2\sqrt{x}}$	B1
	$\left\{\int \dfrac{1}{x(2\sqrt{x}-1)}\,\mathrm{d}x\right\} = \int \dfrac{1}{u^2(2u-1)}\,2u\,\mathrm{d}u$	M1
	$= \int \dfrac{2}{u(2u-1)}\,\mathrm{d}u$	A1 * **cso** **(3)**
(b)	$\dfrac{2}{u(2u-1)} \equiv \dfrac{A}{u} + \dfrac{B}{(2u-1)} \Rightarrow 2 \equiv A(2u-1) + Bu$	
	$u = 0 \Rightarrow 2 = -A \Rightarrow A = -2$ $u = \frac{1}{2} \Rightarrow 2 = \frac{1}{2}B \Rightarrow B = 4$	M1 A1
	So $\int \dfrac{2}{u(2u-1)}\,\mathrm{d}u = \int \dfrac{-2}{u} + \dfrac{4}{(2u-1)}\,\mathrm{d}u$	Integrates $\dfrac{M}{u} + \dfrac{N}{(2u-1)}$, $M \neq 0$, $N \neq 0$ to obtain any one of $\pm\lambda \ln u$ or $\pm\mu \ln(2u-1)$ M1
	$= -2\ln u + 2\ln(2u-1)$	At least one term correctly followed through $-2\ln u + 2\ln(2u-1)$. A1 ft
		A1 **cao**
	So, $\left[-2\ln u + 2\ln(2u-1)\right]_1^3$	
	$= \left(-2\ln 3 + 2\ln(2(3)-1)\right) - \left(-2\ln 1 + 2\ln(2(1)-1)\right)$	Applies limits of 3 and 1 in u or 9 and 1 in x in their integrated function and subtracts the correct way round. M1
	$= -2\ln 3 + 2\ln 5 - (0)$	
	$= 2\ln\left(\dfrac{5}{3}\right)$ $2\ln\left(\dfrac{5}{3}\right)$	A1 **cso** **cao** **(7)** **[10]**

Question Number	Scheme		Marks
7.	$x = 27\sec^3 t, \quad y = 3\tan t, \quad 0 \leqslant t \leqslant \frac{\pi}{3}$		
(a)	$\frac{dx}{dt} = 81\sec^2 t \sec t \tan t, \quad \frac{dy}{dt} = 3\sec^2 t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	$\frac{dy}{dx} = \frac{3\sec^2 t}{81\sec^3 t \tan t} \left\{= \frac{1}{27\sec t \tan t} = \frac{\cos t}{27\tan t} = \frac{\cos^2 t}{27\sin t}\right\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$	M1;
	At $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{3\sec^2\left(\frac{\pi}{6}\right)}{81\sec^3\left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{6}\right)} = \frac{4}{72} \left\{= \frac{3}{54} = \frac{1}{18}\right\}$	$\frac{4}{72}$	A1 **cao cso** **(4)**
(b)	$\left\{1 + \tan^2 t = \sec^2 t\right\} \Rightarrow 1 + \left(\frac{y}{3}\right)^2 = \left(\sqrt[3]{\left(\frac{x}{27}\right)}\right)^2 = \left(\frac{x}{27}\right)^{\frac{2}{3}}$		M1
	$\Rightarrow 1 + \frac{y^2}{9} = \frac{x^{\frac{2}{3}}}{9} \Rightarrow 9 + y^2 = x^{\frac{2}{3}} \Rightarrow y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$ *		A1 * **cso**
	$a = 27$ and $b = 216$ or $27 \leqslant x \leqslant 216$	$a = 27$ and $b = 216$	B1 **(3)**
(c)	$V = \pi\int_{27}^{125}\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2 dx$ or $\pi\int_{27}^{125}\left(x^{\frac{2}{3}} - 9\right)dx$	For $\pi\int\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2$ or $\pi\int\left(x^{\frac{2}{3}} - 9\right)$	B1
	$= \{\pi\}\left[\frac{3}{5}x^{\frac{5}{3}} - 9x\right]_{27}^{125}$	Either $\pm Ax^{\frac{5}{3}} \pm Bx$ or $\frac{3}{5}x^{\frac{5}{3}}$ oe	M1
		$\frac{3}{5}x^{\frac{5}{3}} - 9x$ oe	A1
	$= \{\pi\}\left(\left(\frac{3}{5}(125)^{\frac{5}{3}} - 9(125)\right) - \left(\frac{3}{5}(27)^{\frac{5}{3}} - 9(27)\right)\right)$		dM1
	$= \{\pi\}\left((1875 - 1125) - (145.8 - 243)\right)$		
	$= \frac{4236\pi}{5}$ or 847.2π	$\frac{4236\pi}{5}$ or 847.2π	A1 **(5)** **[12]**

Question Number	Scheme	Marks
8.	(a) $\frac{dV}{dt} = 0.48\pi - 0.6\pi h$	M1 A1
	$V = 9\pi h \quad \Rightarrow \frac{dV}{dt} = 9\pi \frac{dh}{dt}$	B1
	$9\pi \frac{dh}{dt} = 0.48\pi - 0.6\pi h$	M1
	Leading to $75\frac{dh}{dt} = 4 - 5h$ ∗ cso	A1 **(5)**
	(b) $\int \frac{75}{4-5h} dh = \int 1 dt$ separating variables	M1
	$-15\ln(4-5h) = t \ (+C)$	M1 A1
	$-15\ln(4-5h) = t + C$	
	When $t = 0$, $h = 0.2$	
	$-15\ln 3 = C$	M1
	$t = 15\ln 3 - 15\ln(4-5h)$	
	When $h = 0.5$	
	$t = 15\ln 3 - 15\ln 1.5 = 15\ln\left(\frac{3}{1.5}\right) = 15\ln 2$ awrt 10.4	M1 A1 **(6)**
		[11]

				Mean score for students achieving grade:							
Qu	**Max score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	6		76	4.55	5.92	5.34	4.47	3.21	2.22	1.41	0.64
2	9		70	6.27		7.98	6.00	4.33	3.11	2.49	0.91
3	8		69	5.54	7.67	6.55	4.78	3.02	2.21	1.16	0.34
4	9	9	63	5.63	8.28	7.06	5.89	4.72	3.59	2.44	1.17
5	10		60	5.95		8.06	6.01	4.55	3.23	2.10	1.10
6	10	10	54	5.43	9.51	7.34	5.44	3.86	2.63	1.58	0.65
7	12		55	6.63	10.35	7.67	5.03	3.19	2.83	1.78	0.36
8	11		40	4.36	10.25	6.65	3.33	1.38	0.48	0.18	0.06
	75		**59**	**44.36**		**56.65**	**40.95**	**28.26**	**20.30**	**13.14**	**5.23**