

Elmwood Press
Core Mathematics C3
Paper F
(Mark Scheme)

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Worked Solutions

Edexcel C3 Paper F

1. (a) $\frac{2}{x+3} - \frac{1}{(x+3)(x+4)}$

$$\frac{2(x+4) - 1}{(x+3)(x+4)}$$

$$\frac{2x+7}{(x+3)(x+4)} \quad (4)$$

(b) $2x+7=0 \Rightarrow x = -3\frac{1}{2}$ (2)

2. (a) $f^{-1}: x \mapsto \frac{2}{x} + 3, \quad x \in \mathbb{R}, \quad x \neq 0$

$$\text{let } y = \frac{2}{x-3}$$

$$yx - 3y = 2$$

$$yx = 2 + 3y$$

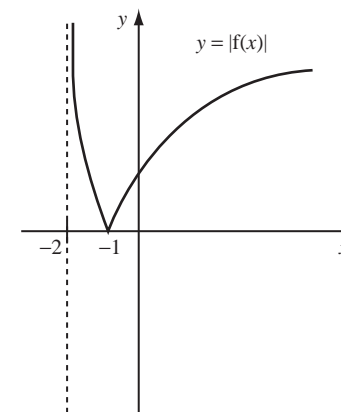
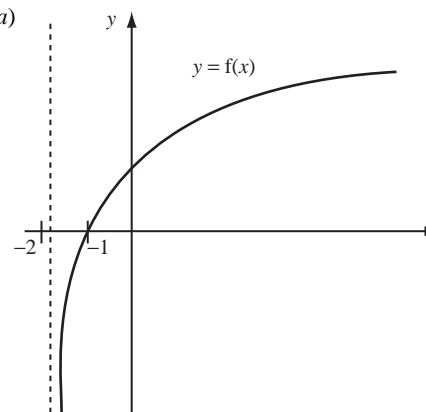
$$x = \frac{2}{y} + 3$$

$$\therefore f^{-1}: x \rightarrow \frac{2}{x} + 3, \quad x \in \mathbb{R}, \quad x \neq 0 \quad (4)$$

(b) $f(4) = \frac{2}{4-3} = 2$

$$ff^{-1}(7) = 7 \quad (2)$$

3. (a)



(3)

(b) The line $y = x$ only cuts the graph once. (1)

$$\left. \begin{array}{l} \ln(2+2) - 2 = -0.6137 \\ \ln(1+2) - 1 = 0.098 \end{array} \right\} \text{change of sign } \therefore \text{root lies in interval } [1, 2]$$

(2)

(d) $x_1 = 1.0986, x_2 = 1.1309, x_3 = 1.1413, x_4 = 1.1446, x_5 = 1.1457$

solution 1.146 (3 d.p.). (3)

4. (a) $x^2 \cdot \frac{1}{x} + 2x \ln x = x(1 + 2 \ln x)$ (4)

(b) $2 \cos 3x \cdot (-\sin 3x) \cdot 3 = -6 \sin 3x \cos 3x$ (3)

(c) $\frac{x \cdot \cos x - \sin x}{x^2}$ (3)

$$5. (a) \text{ R.H.S} = \frac{1 - \tan^2 \theta}{2 \tan \theta} \quad (\times \tan^2 \theta)$$

$$\equiv \frac{1}{\tan 2\theta}$$

$$\equiv \cot 2\theta \quad (5)$$

$$(b) \cot^2 \theta - 1 = 2 \cot \theta$$

$$\therefore \cot 2\theta = 1 \Rightarrow \tan 2\theta = 1$$

$$2\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$$

$$\therefore \theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8} \quad (4)$$

$$6. (a) e^{2x} + 6 = 5e^x$$

$$e^{2x} - 5e^x + 6 = 0$$

$$(e^x - 3)(e^x - 2) = 0 \quad (3)$$

$$(b) e^x = 3 \Rightarrow x = \ln 3 \text{ (or 1.0986 4 d.p.)}$$

$$\text{and } e^x = 2$$

$$\Rightarrow x = \ln 2 \text{ (or 0.6931 to 4 d.p.)} \quad (4)$$

$$(c) e^{2(x+1)} - 5e^{x+1} + 6 = 0$$

$$\left. \begin{array}{l} e^{x+1} = 3 \\ e^{x+1} = 2 \end{array} \right\} \Rightarrow \begin{array}{l} x = \ln 3 - 1 \text{ (or 0.0986 4 d.p.)} \\ x = \ln 2 - 1 \text{ (or -0.3069 4 d.p.)} \end{array} \quad (4)$$

$$7. (a) 25 \left(\frac{7}{25} \sin x + \frac{24}{25} \cos x \right) \equiv R(\cos \alpha \sin x + \sin \alpha \cos x)$$

$$\Rightarrow R = 25 \text{ and } \tan \alpha = \frac{24}{7} \Rightarrow \alpha = 73.7^\circ \text{ (1 d.p.)} \quad (4)$$

$$(b) 25 \sin(x + 73.7) = 15$$

$$\sin(x + 73.7) = \frac{3}{5}$$

$$x + 73.7 = 36.9, 143.1, 360 + 36.9$$

$$x = 69.4^\circ, 323.2^\circ \quad (4)$$

$$(c) 15 \sec x - 7 \tan x = 24$$

$$\Rightarrow 15 - 7 \sin x = 24 \cos x. \quad (2)$$

$$(d) 25, x = 16.3 \quad (2)$$

$$8. (a) \frac{dx}{dy} = \cos y \quad (2)$$

$$(b) y = \frac{\pi}{4} \quad \cos y = \frac{1}{\sqrt{2}} \therefore \text{grad. of tgt} = \sqrt{2} \quad (3)$$

$$(c) y - \frac{\pi}{4} = \sqrt{2} \left(x - \frac{1}{\sqrt{2}} \right) \quad (2)$$

$$(d) y = 0, -\frac{\pi}{4} = \sqrt{2}x - 1$$

$$\sqrt{2}x = 1 - \frac{\pi}{4}$$

$$x = \frac{1 - \frac{\pi}{4}}{\sqrt{2}} = \frac{4 - \pi}{4\sqrt{2}} \quad (3)$$

$$(e) \text{ area} = \left(\frac{4 - \pi}{4\sqrt{2}} \right) \cdot \frac{\pi}{8} \quad (2)$$