# Petersburg Mathematics Cohort

Day 2



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Introduce yourself. Describe your role as an educator. Describe the mathematics you support.





# Schedule for Today

9:00-9:05	- Trajectories in mathematics
9:05-9:45	- Manipulatives: Early Numeracy
9:45-10:30	- Manipulatives: Whole Numbers and Place Value
10:30-10:40	BREAK
10:40-11:45	- Manipulatives: Fractions
11:45-12:00	- Wrap-up

1:00-1:05	- Trajectories in mathematics
1:05-1:40	- Manipulatives: Fraction Concepts
1:40-2:20	- Manipulatives: Fraction Computation
2:20-2:30	BREAK
2:30-2:45	- Manipulatives: Decimals
2:45-3:45	- Manipulatives: Algebra
3:45-4:00	- Wrap-up



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GRADES K-5

GRADES 6-12

# **Trajectories in Mathematics**



#### An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

К	1	2	3	4	5	6	7	8
Know number names and the count sequence Count to tell the number of objects Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11- 19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent & solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply & divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multidigit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and relate volume to multiplication and to addition	<ul> <li>Apply and extend previous understandings of multiplication and division to divide fractions by fractions</li> <li>Apply and extend previous understandings of numbers to the system of rational numbers</li> <li>Understand ratio concepts and use ratio reasoning to solve problems</li> <li>Apply and extend previous understandings of arithmetic to algebraic expressions</li> <li>Reason about and solve one-variable equations and inequalities</li> <li>Represent and analyze quantitative relationships between dependent variables</li> </ul>	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationships and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities

\* Indicates a cluster that is well thought of as a part of a student's progress to algebra, but that is currently not designated as major by the assessment consortia in their draft materials. Apart from the one asterisked exception, the clusters listed here are a subset of those designated as major in the assessment consortia's draft documents.

\*\* Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

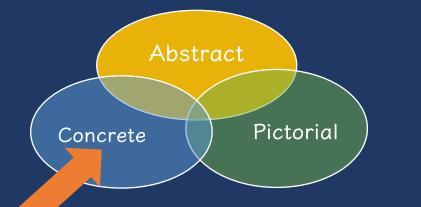
https://achievethecore.org/category/774/mathematics-focus-by-grade-level





# What's the critical math content for your students?





### Three-dimensional objects

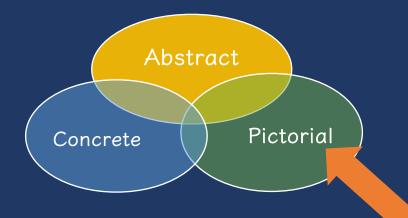




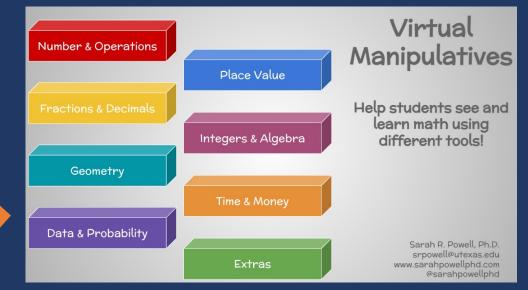


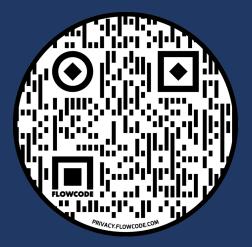




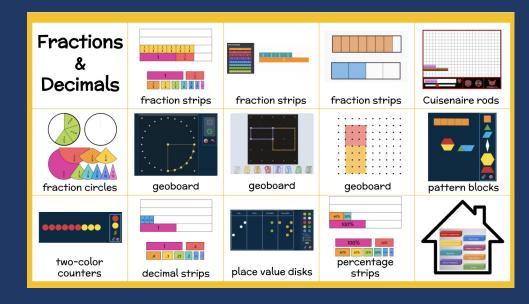


#### Two-dimensional images





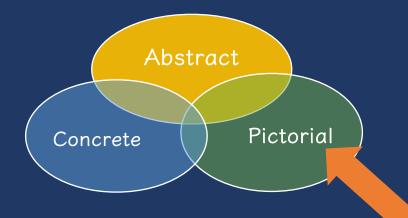
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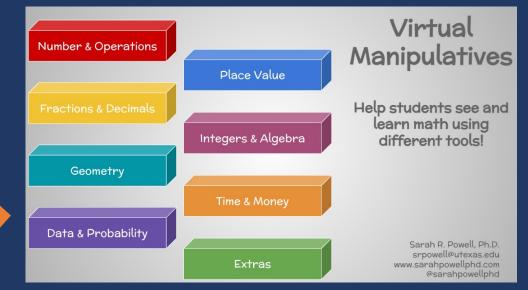


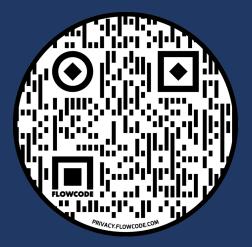
# Manipulatives: Fraction Concepts



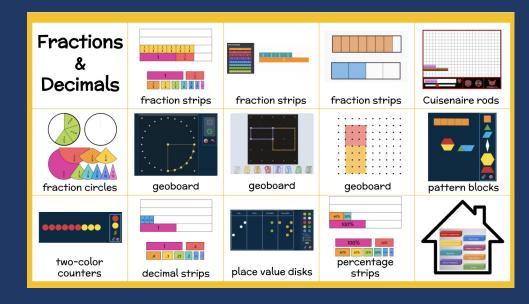


#### Two-dimensional images





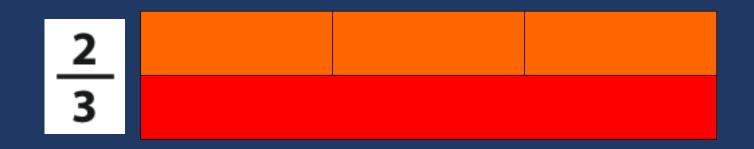
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# Length/Measurement

Fractions are appropriated by length



Fraction tiles/ba



# Length/Measurement

#### Fractions are appropriated by length

2 3



Cuisenaire rods



# Length/Measurement

Fractions are appropriated by length



Number line

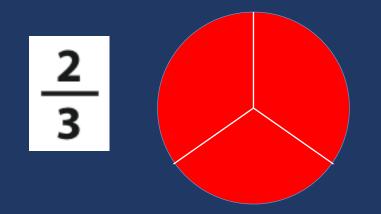




Fraction Concepts			
Fraction	Length	Area	Set
2 3			
<u>1</u> 4			
$1\frac{1}{2}$			
<u>3</u> 7			



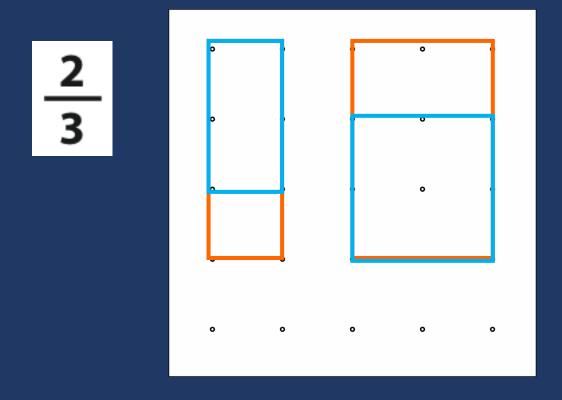












Geoboards





2 3



Pattern blocks









Legos



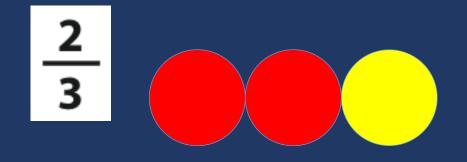


Fraction Concepts			
Fraction	Length	Area	Set
2 3			
<u>1</u> 4			
$1\frac{1}{2}$			
<u>3</u> 7			





Individual shapes match the fraction



Two-color counters





#### Individual shapes match the fraction





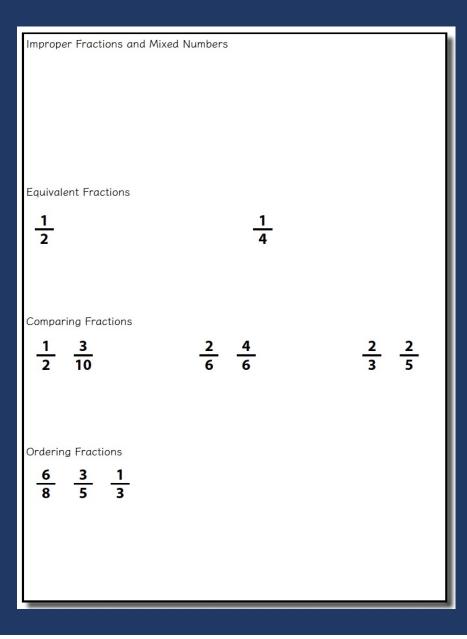




Fraction Concepts			
Fraction	Length	Area	Set
2 3			
<u>1</u> 4			
$1\frac{1}{2}$			
<u>3</u> 7			











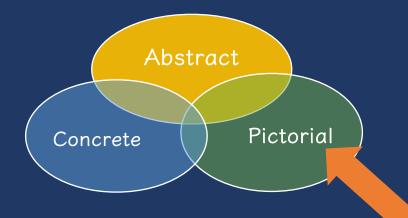
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 Share some of your favorite virtual manipulatives for fractions.

3. Considerations for using these tools with students?

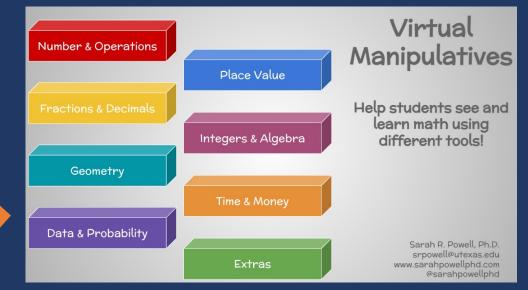


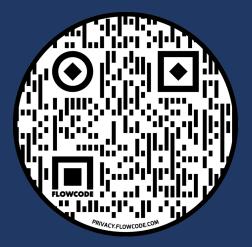
# Manipulatives: Fraction Computation



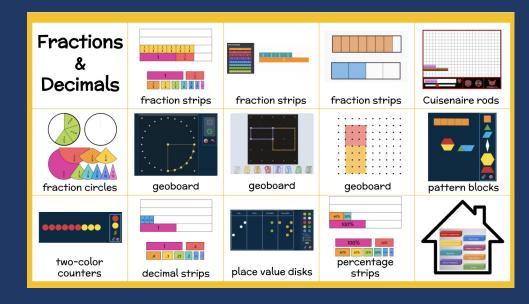


#### Two-dimensional images



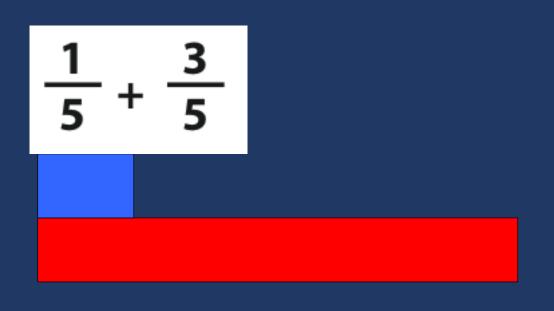


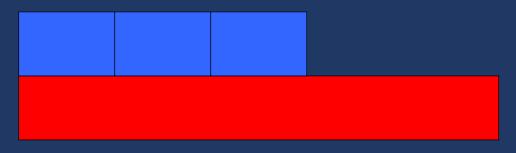
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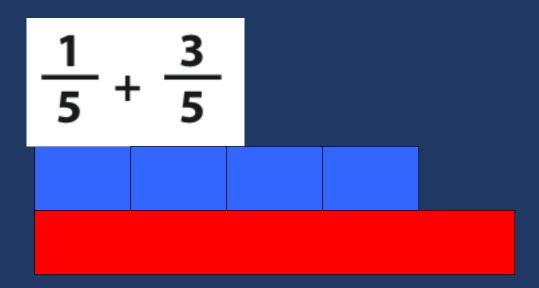


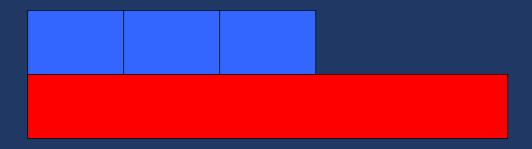
















Fraction Co Addition an	omputation: Id Subtraction
Problem	Representation
$\frac{1}{5} + \frac{3}{5}$	
$\frac{2}{3} + \frac{2}{3}$	
$\frac{1}{2} + \frac{1}{4}$	
$\frac{1}{4} + \frac{4}{6}$	
$\frac{4}{5} - \frac{1}{5}$	
$\frac{6}{5} - \frac{2}{5}$	
$\frac{7}{8} - \frac{2}{4}$	
$\frac{1}{2} - \frac{2}{5}$	



## Addition

 $\frac{1}{2} + \frac{1}{4}$ 

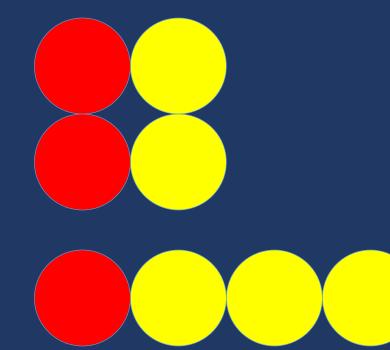








 $\frac{1}{2} + \frac{1}{4}$ 

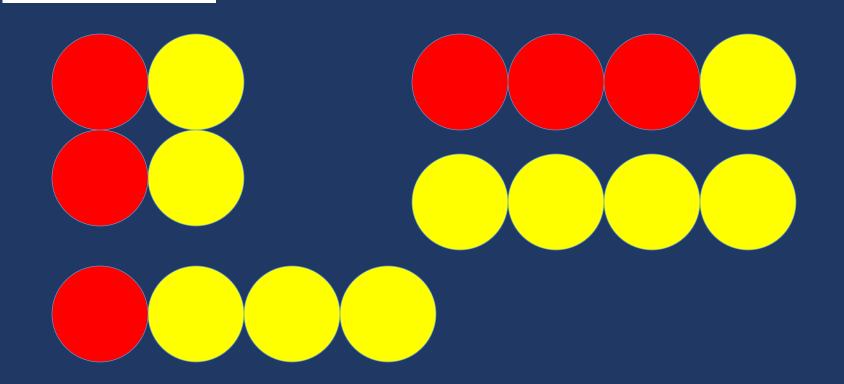






## Addition

 $\frac{1}{2} + \frac{1}{4}$ 



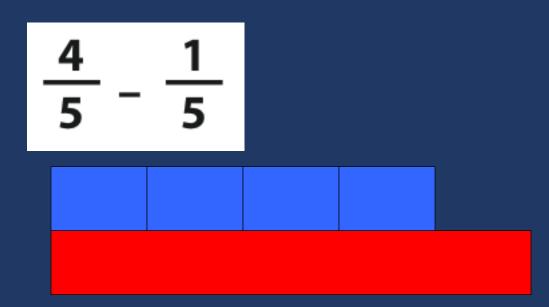




Fraction Co Addition an	omputation: Id Subtraction
Problem	Representation
$\frac{1}{5} + \frac{3}{5}$	
$\frac{2}{3} + \frac{2}{3}$	
$\frac{1}{2} + \frac{1}{4}$	
$\frac{1}{4} + \frac{4}{6}$	
$\frac{4}{5} - \frac{1}{5}$	
$\frac{6}{5} - \frac{2}{5}$	
$\frac{7}{8} - \frac{2}{4}$	
$\frac{1}{2} - \frac{2}{5}$	











Fraction Co Addition an	omputation: Id Subtraction
Problem	Representation
$\frac{1}{5} + \frac{3}{5}$	
$\frac{2}{3} + \frac{2}{3}$	
$\frac{1}{2} + \frac{1}{4}$	
$\frac{1}{4} + \frac{4}{6}$	
$\frac{4}{5} - \frac{1}{5}$	
$\frac{6}{5} - \frac{2}{5}$	
$\frac{7}{8} - \frac{2}{4}$	
$\frac{1}{2} - \frac{2}{5}$	



# Subtraction 2 7 8 4 Take away from this set This is for reference





Fraction Computation: Addition and Subtraction		
Problem	Representation	
$\frac{1}{5} + \frac{3}{5}$		
$\frac{2}{3} + \frac{2}{3}$		
$\frac{1}{2} + \frac{1}{4}$		
$\frac{1}{4} + \frac{4}{6}$		
$\frac{4}{5} - \frac{1}{5}$		
$\frac{6}{5} - \frac{2}{5}$		
$\frac{7}{8} - \frac{2}{4}$		
$\frac{1}{2} - \frac{2}{5}$		

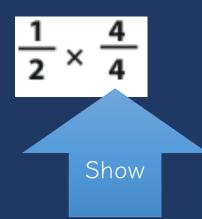


#### Multiplication

- Interpret multiplication sign as "of"
  - 2/3 *of* 3/4

Fraction Computation: Multiplication		
Problem	Representation	
<b>2</b> × $\frac{1}{4}$		
$\frac{1}{2} \times \frac{4}{4}$		
$\frac{1}{2} \times \frac{2}{4}$		
$\frac{1}{2} \times \frac{3}{4}$		
$\frac{2}{3} \times \frac{3}{3}$		
$\frac{2}{5} \times \frac{5}{6}$		
$\frac{1}{4} \times \frac{2}{3}$		
$\frac{3}{4} \times \frac{1}{3}$		

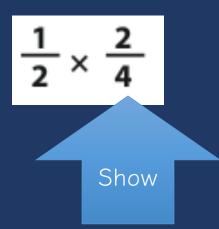




#### One-half *of* four-fourths



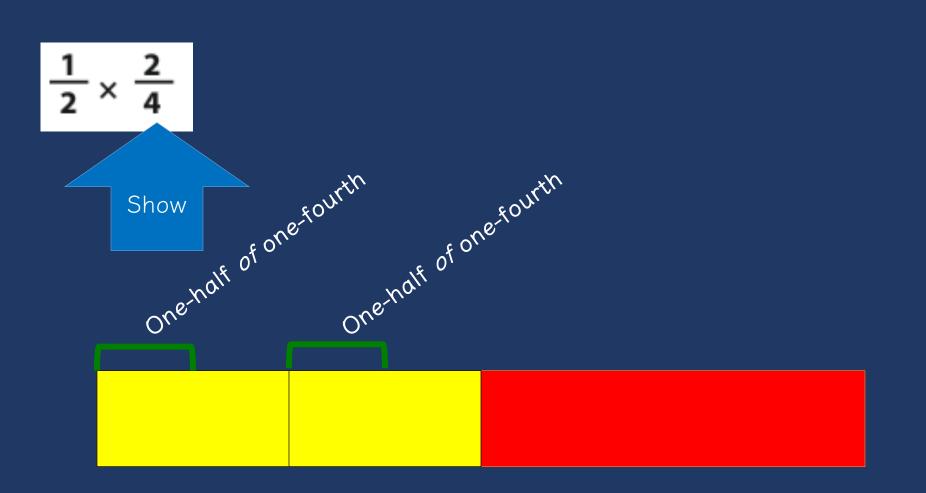




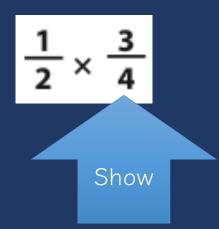
#### One-half of two-fourths



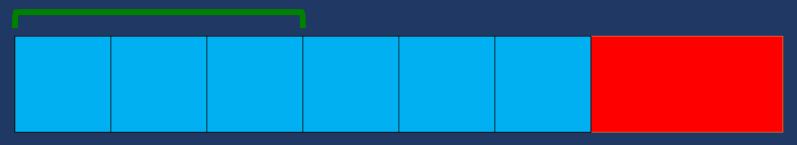




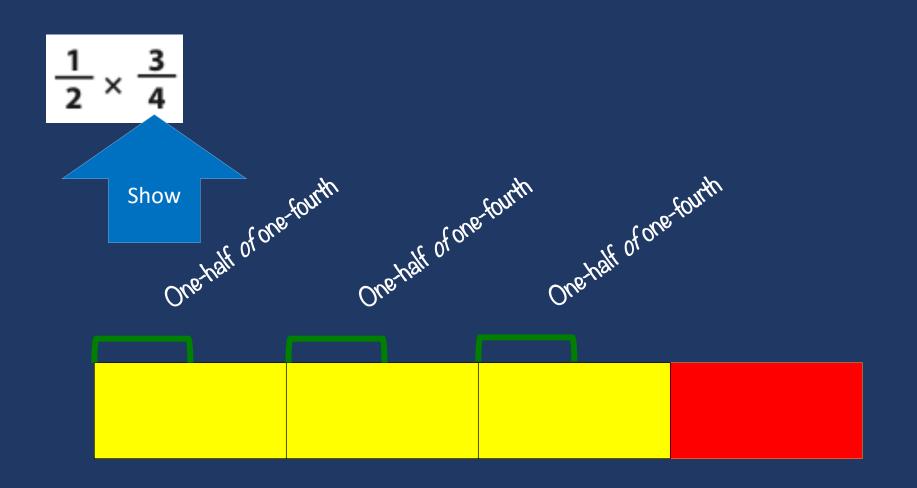




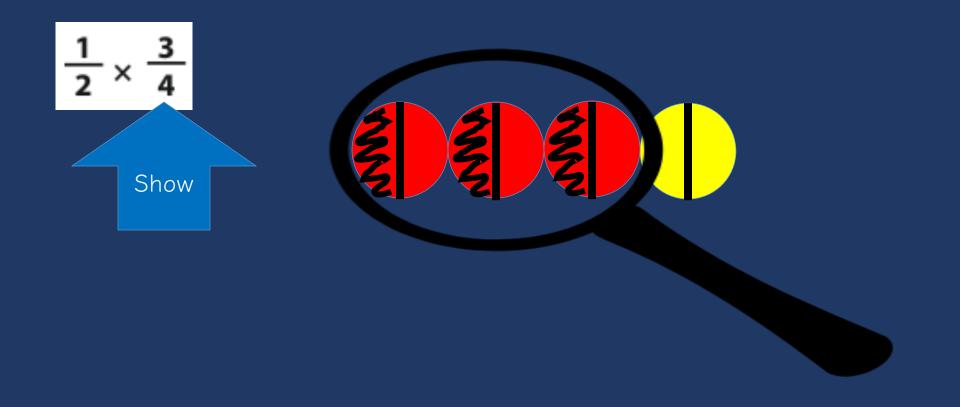
#### One-half of three-fourths







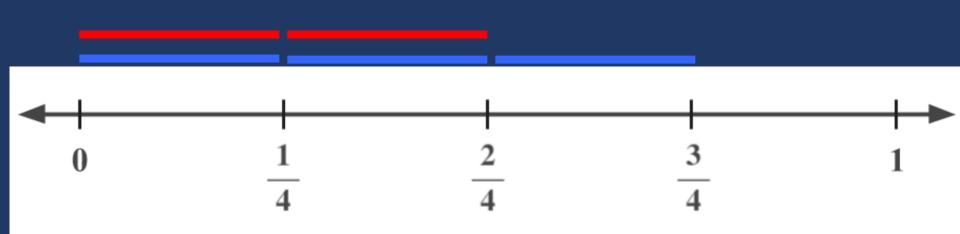






#### Length







Area

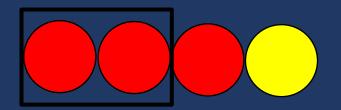
$$\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$$
 "Two-thirds of  
three-fourths"





Set

$$\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$$
 "Two-thirds of  
three-fourths"







Fraction Computation: Multiplication		
Problem	Representation	
<b>2</b> × $\frac{1}{4}$		
$\frac{1}{2} \times \frac{4}{4}$		
$\frac{1}{2} \times \frac{2}{4}$		
$\frac{1}{2} \times \frac{3}{4}$		
$\frac{2}{3} \times \frac{3}{3}$		
$\frac{2}{5} \times \frac{5}{6}$		
$\frac{1}{4} \times \frac{2}{3}$		
$\frac{3}{4} \times \frac{1}{3}$		



#### Division

 "How many sets of the second fraction fit into the first fraction?"

Fraction Computation: Division		
Problem	Representation	
$\frac{3}{3} \div 3$		
$\frac{3}{3} \div \frac{1}{3}$		
$\frac{2}{3} \div \frac{1}{3}$		
$\frac{5}{6} \div \frac{1}{2}$		
$\frac{7}{8} \div \frac{3}{4}$		
$\frac{1}{4} \div \frac{1}{2}$		
$\frac{3}{4} \div \frac{2}{3}$		
$\frac{4}{5} \div \frac{1}{2}$		
	·	



Division		
$\frac{3}{3} \div \frac{1}{3}$ Show		
1 set of one-third	2 sets of one-third	3 sets of one-third

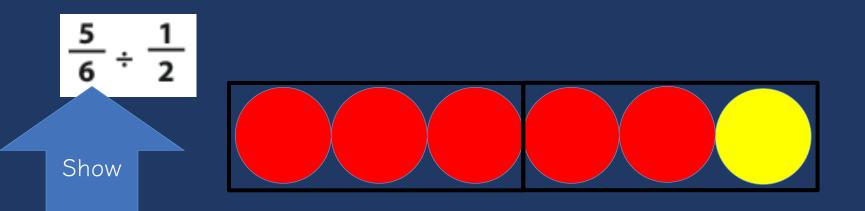


$\frac{2}{3} \div \frac{1}{3}$ Show		
1 set of one-third	2 sets of one-third	



$\frac{5}{6} \div \frac{1}{2}$ Show	
1 set of one-half	Two-thirds set of one- half

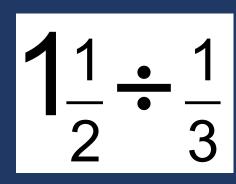




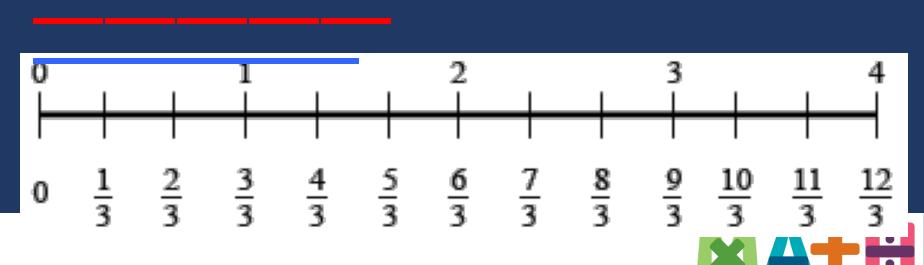




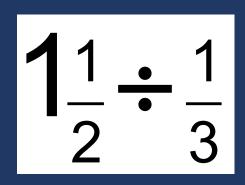
#### Length



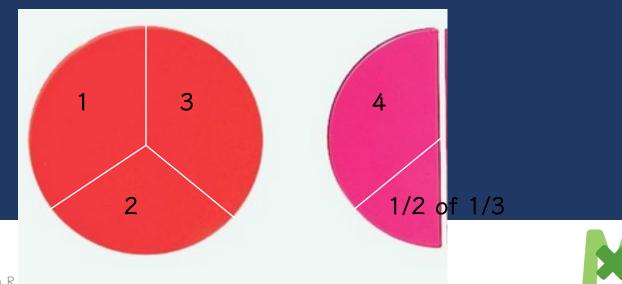
#### "How many sets of 1/3 can be made with 1 and ½?"



#### Area



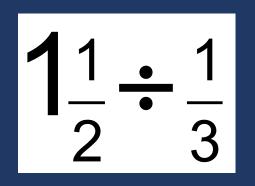
#### "How many sets of 1/3 can be made with 1 and ½?"



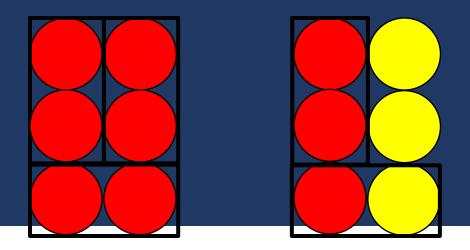
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#### Set



#### "How many sets of 1/3 can be made with 1 and ½?"







Fraction C Division	omputation:
Problem	Representation
$\frac{3}{3}$ ÷ 3	
$\frac{3}{3} \div \frac{1}{3}$	
$\frac{2}{3} \div \frac{1}{3}$	
$\frac{5}{6} \div \frac{1}{2}$	
$\frac{7}{8} \div \frac{3}{4}$	
$\frac{1}{4} \div \frac{1}{2}$	
$\frac{3}{4} \div \frac{2}{3}$	
$\frac{4}{5} \div \frac{1}{2}$	



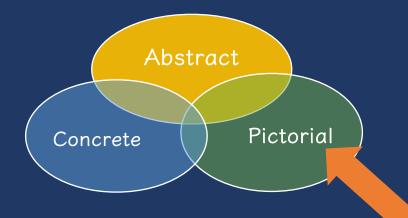


1. Share some of your favorite hands-on materials for fraction computation. 2. Share some of your favorite virtual manipulatives for fraction computation. 3. Considerations for using these tools with students?

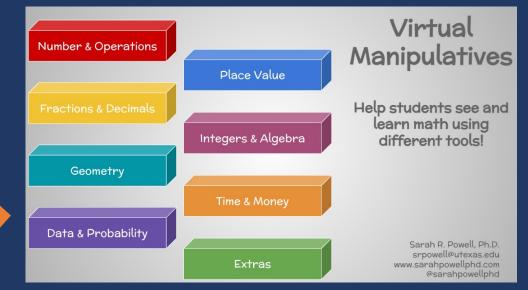


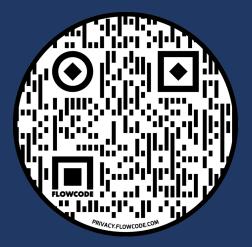
# Manipulatives: Decimals



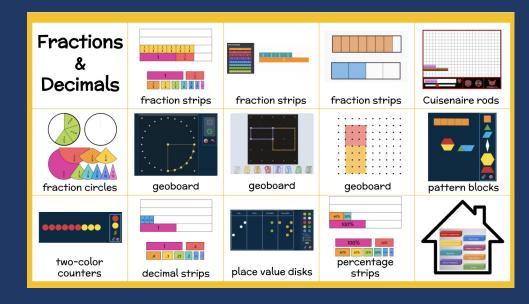


#### Two-dimensional images





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	<u>1</u> 10	
	$\frac{1}{10}$	
	$ \begin{array}{r} \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \\ \end{array} $	
	$\frac{1}{10}$	
1.0	$ \begin{array}{r} \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \end{array} $	
1.0	$\frac{1}{10}$	
	$\frac{1}{10}$	0.1
	$ \begin{array}{r} \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \end{array} $	0.1
	$\frac{1}{10}$	0.1

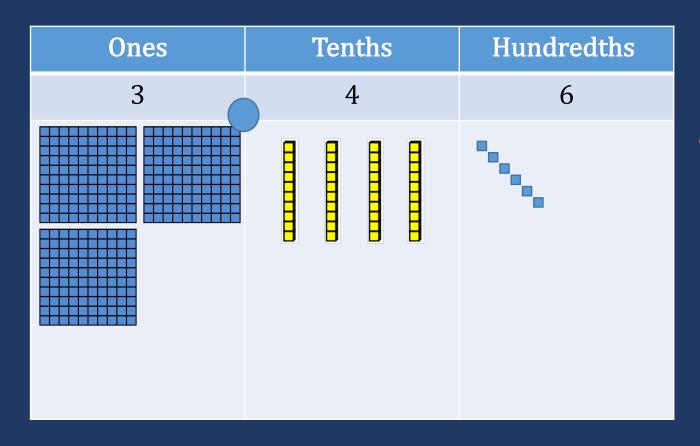
<u>3</u> 10



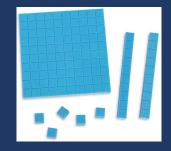




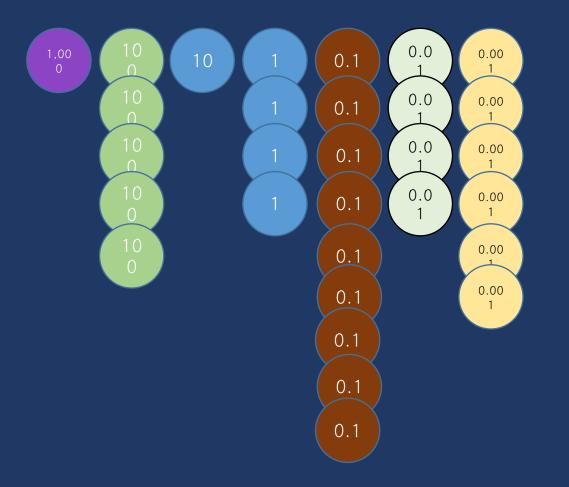












# 1,504.946



## Decimals and Place Value





1.2	0.88	1.034		
2.8	1.04	0.829		
Ones		Tenths	Hundredths	Thousandths





	Decimals	
Computation		
Problem	Representation	
2.34 <u>+ 1.61</u>		
1.98 <u>+ 0.34</u>		
2.34 <u>- 1.61</u>		
3.09 <u>- 1.88</u>		





Decimals		
Computation		
Problem	Representation	
0.52		
<u>× 3</u>		
0.52		
<u>× 34</u>		
1.24 ÷ 2		
1.24 ÷ 2		
1.27 ÷ 5		





 Share some of your favorite hands-on materials for decimals.
 Share some of your favorite virtual manipulatives for decimals.

3. Considerations for using these tools with students?



# Manipulatives: Algebra



### Integer Concepts

#### <u>Zero pairs</u>

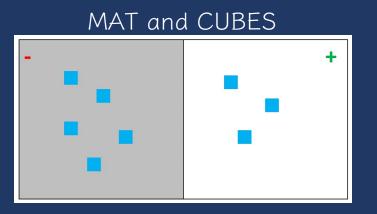
- A <u>positive</u> and a <u>negative</u> cancel one another
  - If you add a positive, you must add a negative
  - If you subtract a positive, you must subtract a negative

A number is assumed positive if a negative sign (-) doesn't proceed it

Be sure to use the negative sign (-) not the minus sign (-)



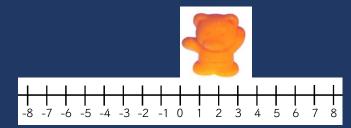
## Ways to Teach Addition and Subtraction



#### TWO-COLOR COUNTERS



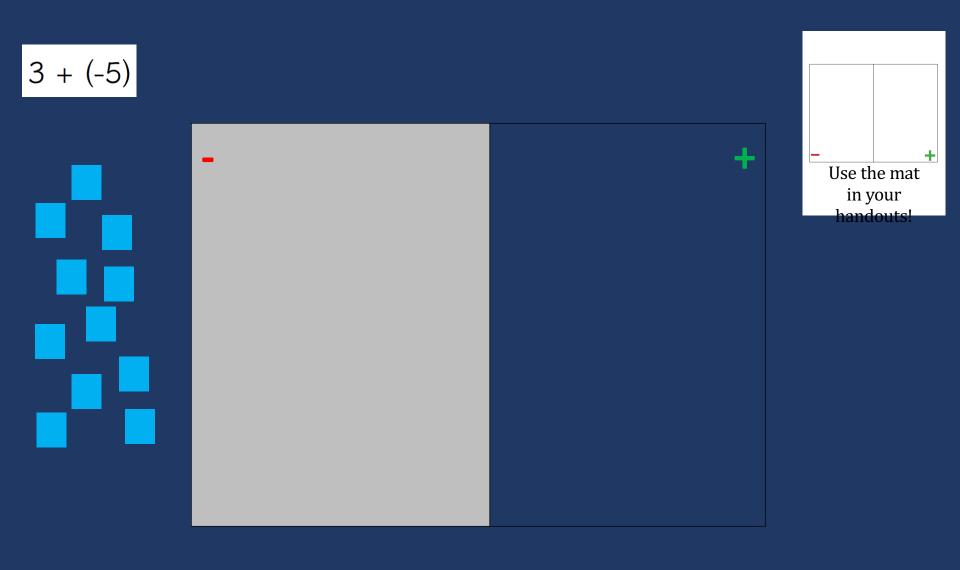




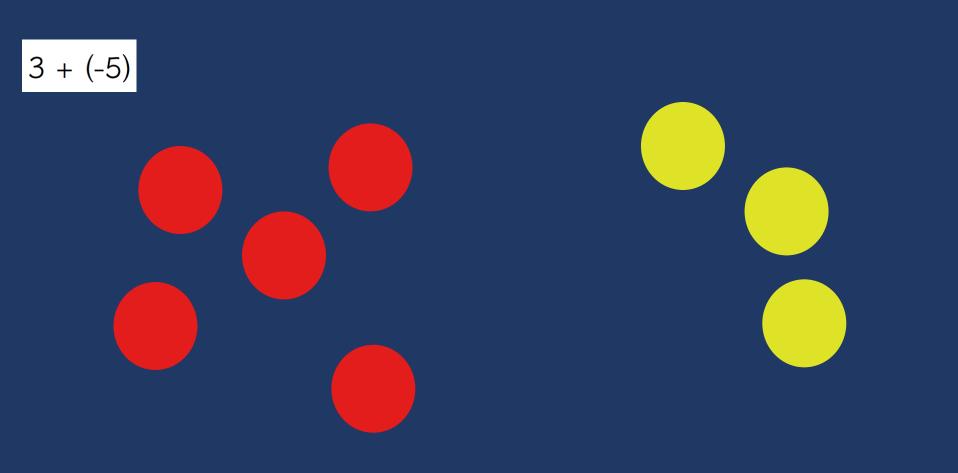


3 + 5

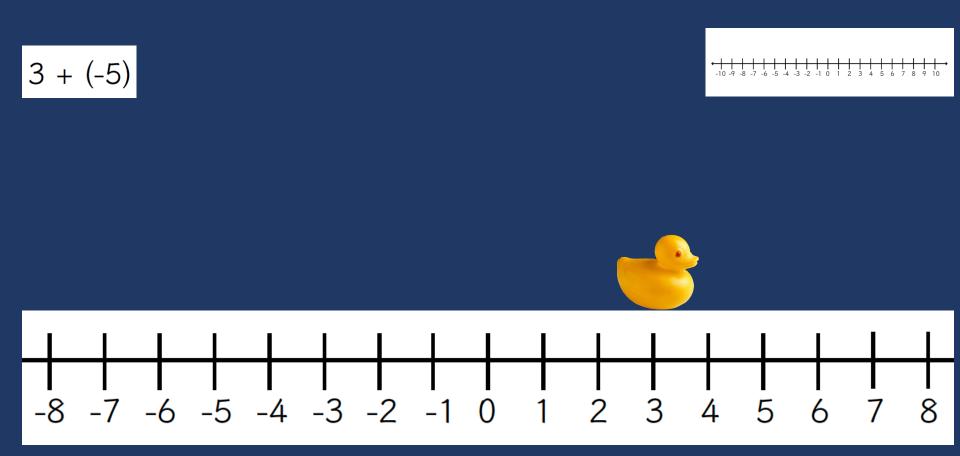










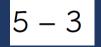






Integers	
Addition	
Problem	Representation
3 + 5	
3 + (-5)	
3 + (-5)	
3 + (-5)	
-2 + 6	
-6 + (-3)	
3 + (-1)	



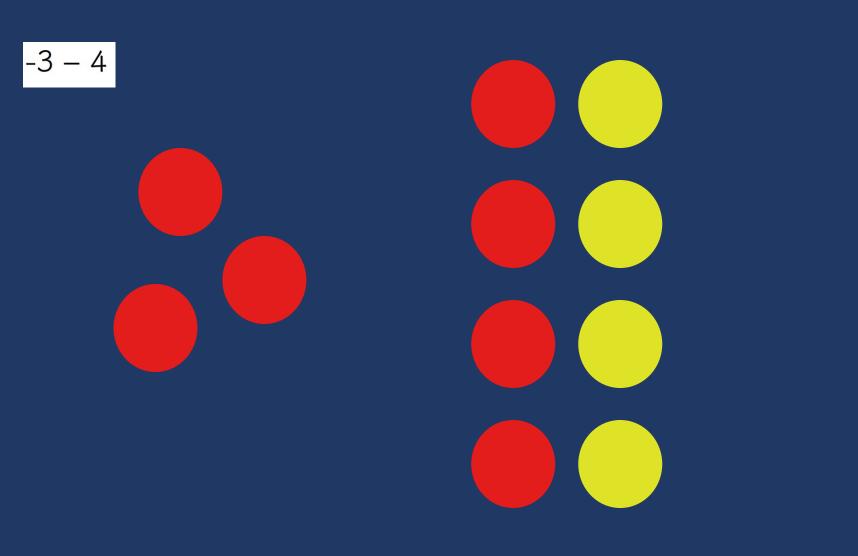




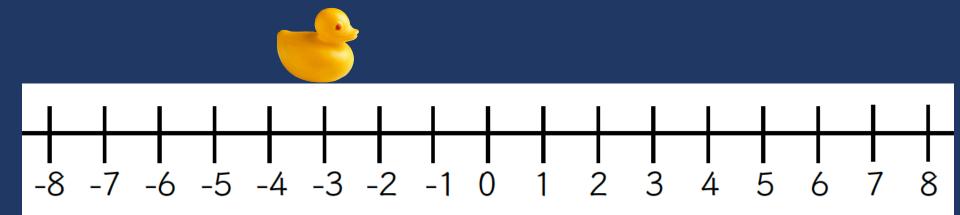




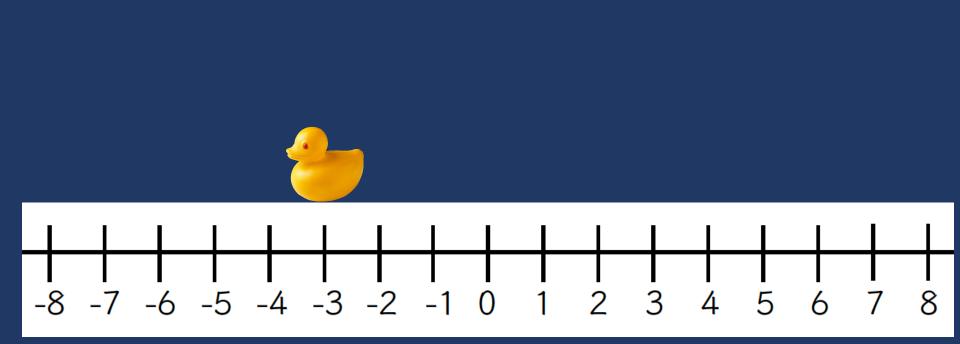














-3 – 4

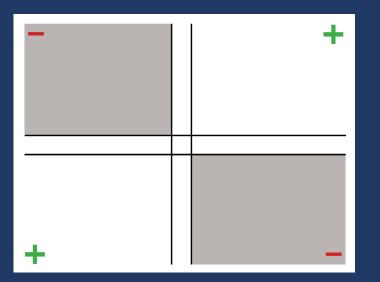


Integers	
Subtraction	
Problem	Representation
5 – 3	
-3 – 4	
-3 – 4	
-3 – 4	
2 – 5	
-2 - (-6)	
-3 – 2	

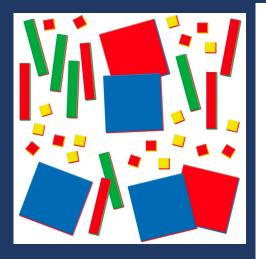


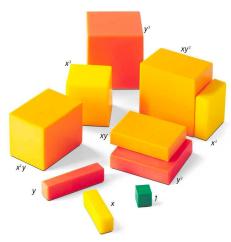
## Ways to Teach Multiplication and Division

#### MAT and CUBES



#### ALGEBRA TILES/ALGEBLOCKS

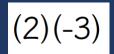


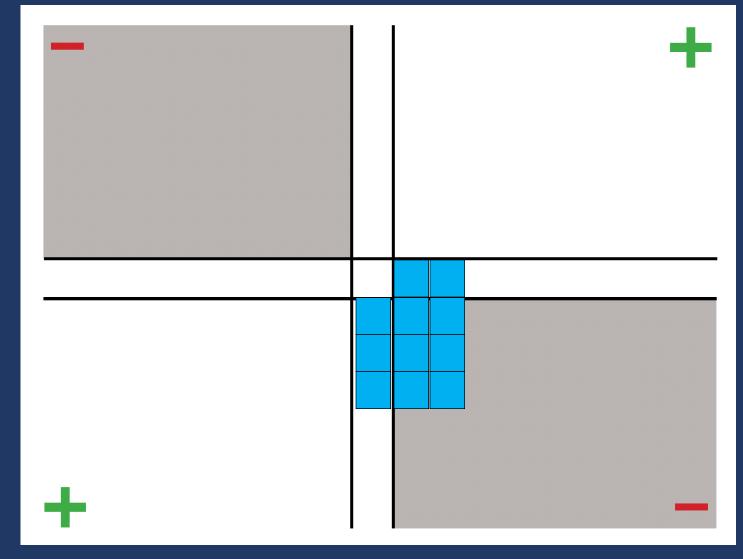










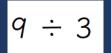




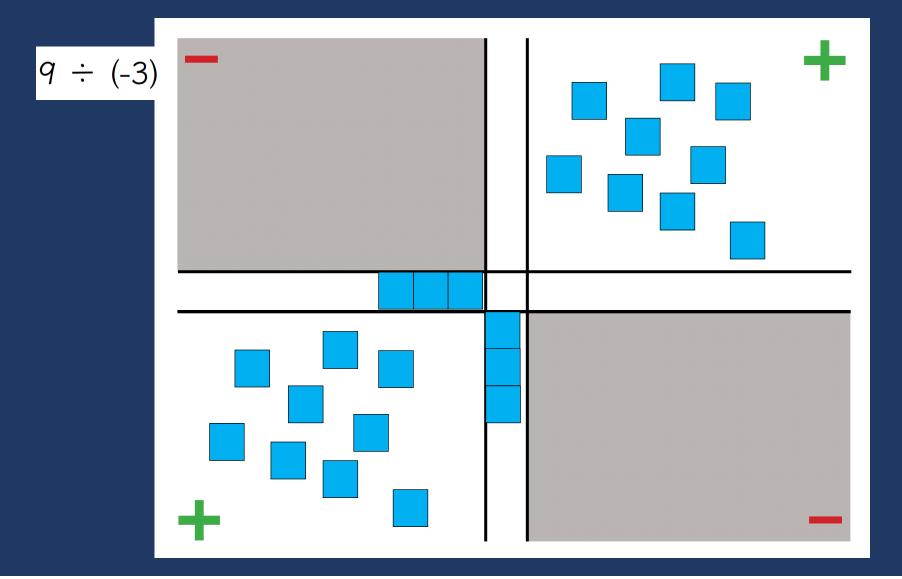


Integers			
	Multiplication and Division		
Problem	Representation		
2(3)			
(2)(-3)			
(-2)(-3)			
9 ÷ 3			
9 ÷ (-3)			
-9 ÷ (-3)			













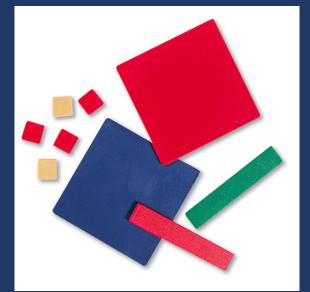
Integers			
	Multiplication and Division		
Problem	Representation		
2(3)			
(2)(-3)			
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9 ÷ 3			
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-9 ÷ (-3)			





### algebra tiles

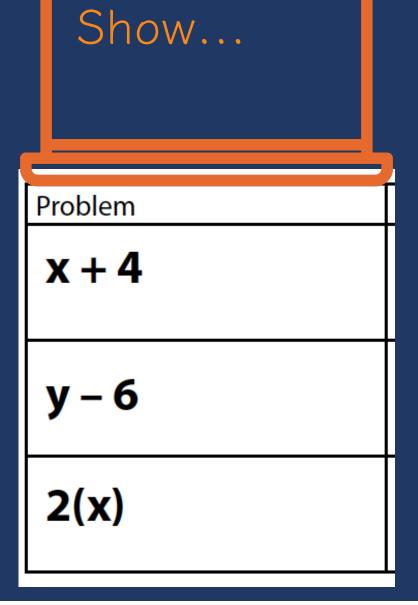
#### Algeblocks





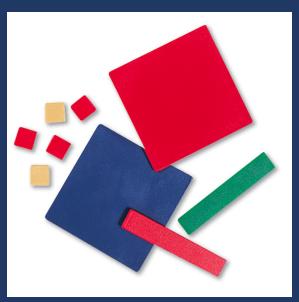


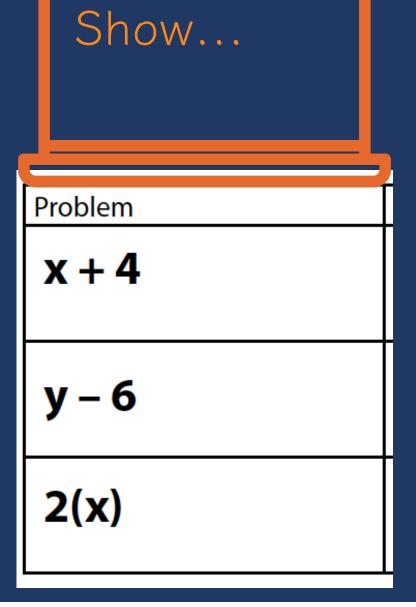






#### algebra tiles

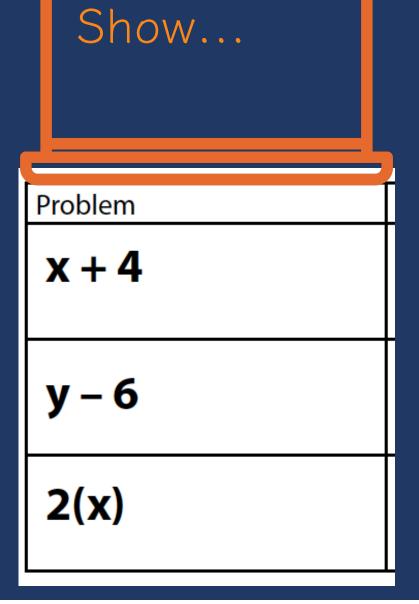






#### Algeblocks







# properties of equality

zero pairs

> If you do something to one side of the equal sign, you do the same thing to the other side.

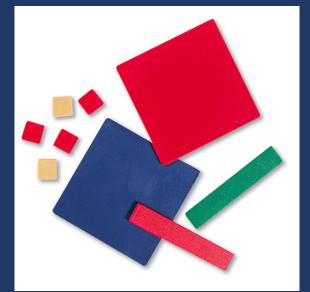


### -1 and 1 equal 0



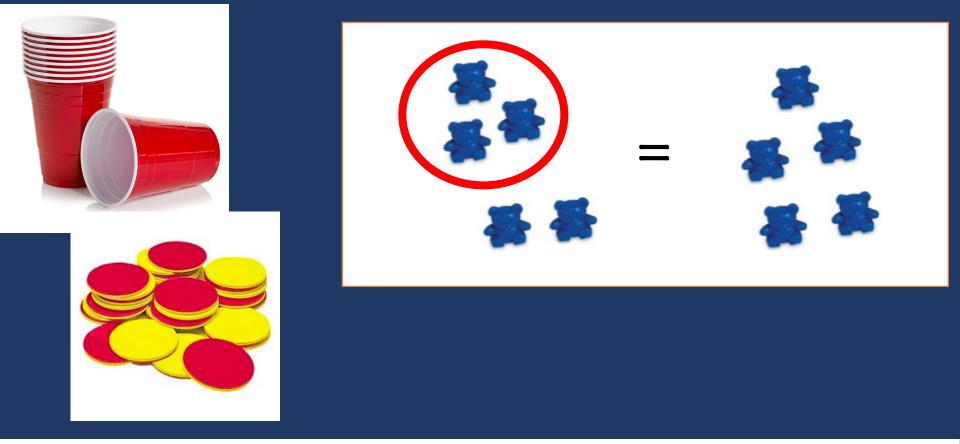
### algebra tiles

#### Algeblocks

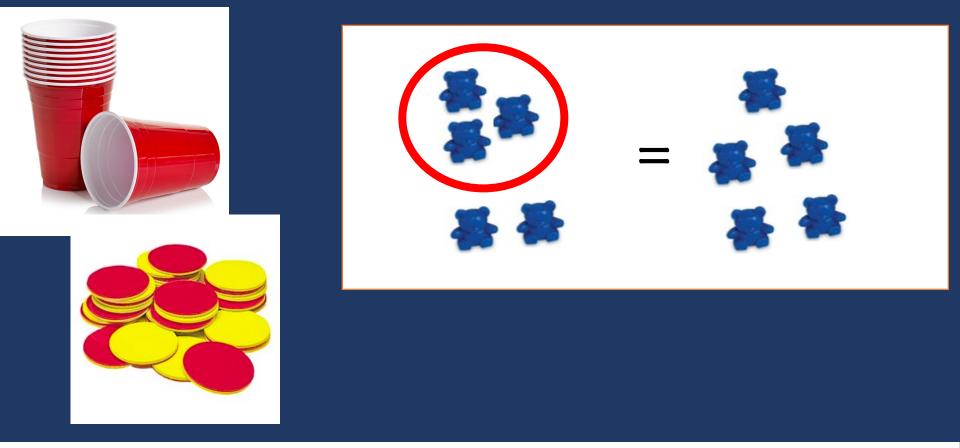












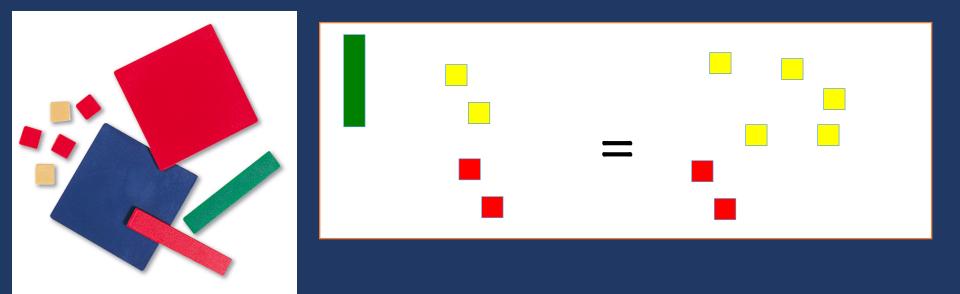




Solving Equations	
With Cups/Plates and Counters	
Problem	Representations
x + 2 = 5	
x + 2 = 5	
4 + x = 6	
5 = x - 3	
-2 = x + 3	
With Algebra Tile	25
Problem	Representations
x + 2 = 5	
4 + x = 6	
x + 3 = 7	
y – 4 = 2	
5 = x - 3 y - 2 = 5	

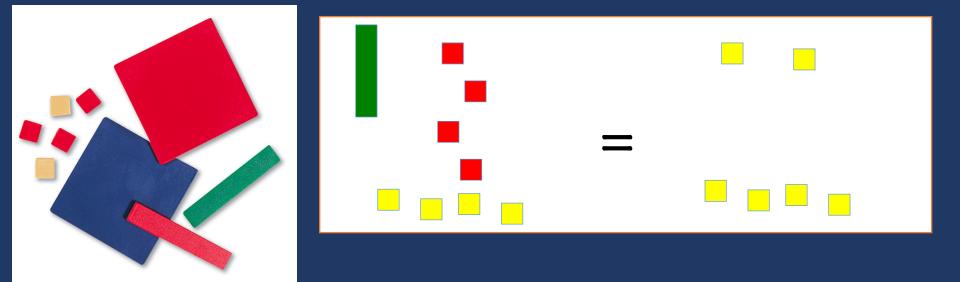


# algebra tiles x+2=5





algebra tiles 
$$y - 4 = 2$$





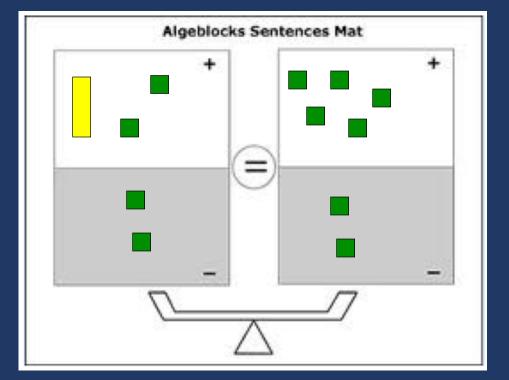


Solving Equations	
With Cups/Plates and Counters	
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x + 2 = 5	
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With Algebra Tile	25
Problem	Representations
x + 2 = 5	
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y – 4 = 2	
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#### Algeblocks

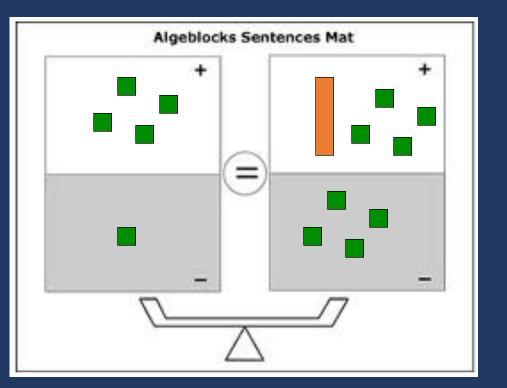






Algeblocks 
$$-1 = y - 4$$









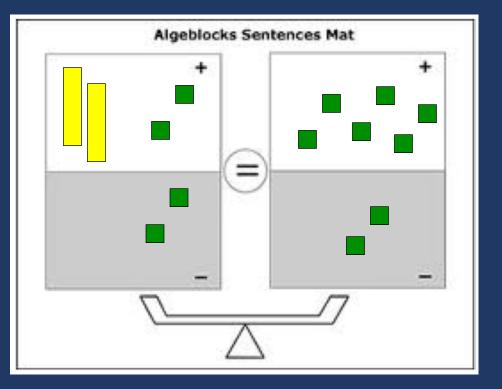
	Solving Equations	
With Algeblocks		
Problem	Representations	
x + 2 = 5		
4 + x = 6		
-2 = x + 3		
-1 = y - 4		
2x + 2 = 6		
x + 4 = 3x		
2(x+3) = x+4		
2x-4=1+3x		
3y - 5 = -y - 1		
1 - x = x + 1		



#### Algeblocks

$$\mathbf{2X} + \mathbf{2} = \mathbf{6}$$



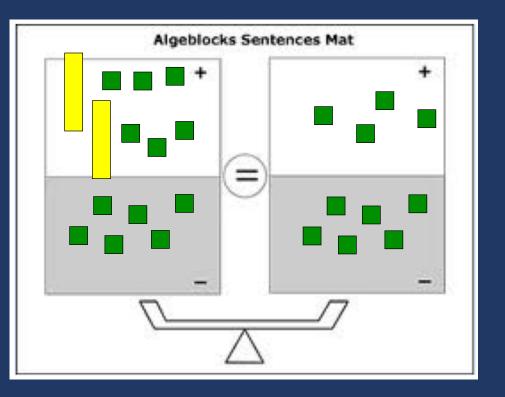




#### Algeblocks

$$2(x + 3) = 4$$







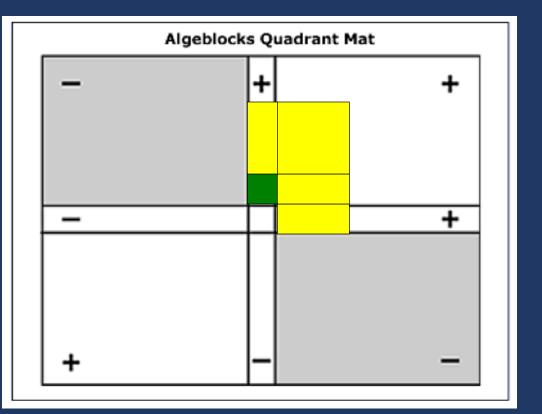


	Solving Equations	
With Algeblocks		
Problem	Representations	
x + 2 = 5		
4 + x = 6		
-2 = x + 3		
-1 = y - 4		
2x + 2 = 6		
x + 4 = 3x		
2(x+3) = x+4		
2x-4=1+3x		
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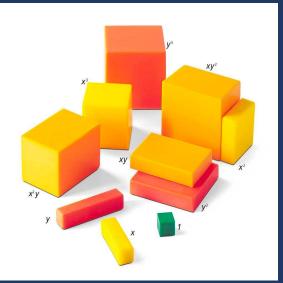
## Algeblocks **x(1 + x)**

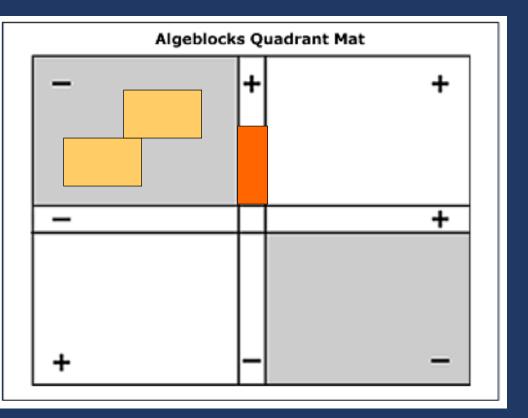














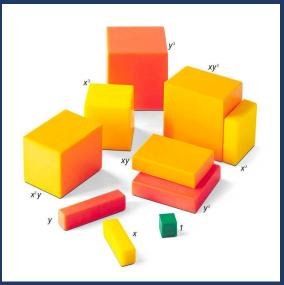


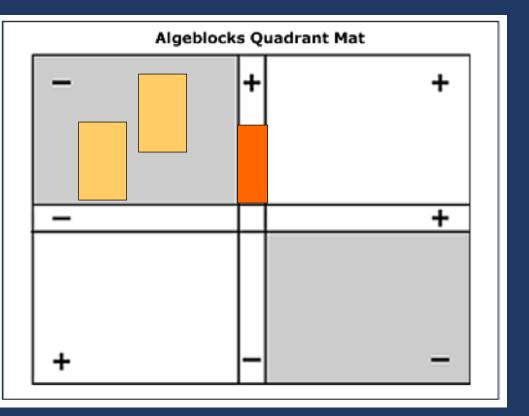
Solving Equations	
With Algeblocks	
Problem	Representation
x(3)	
-2(y)	
x(1 + x)	
-y(y + 2)	
(x – 2)(-2x)	
(y – 1)(y + 2)	
3y ÷ 3	
-2xy ÷ y	
-3x ÷ 3x	
4x <sup>2</sup> ÷-x	



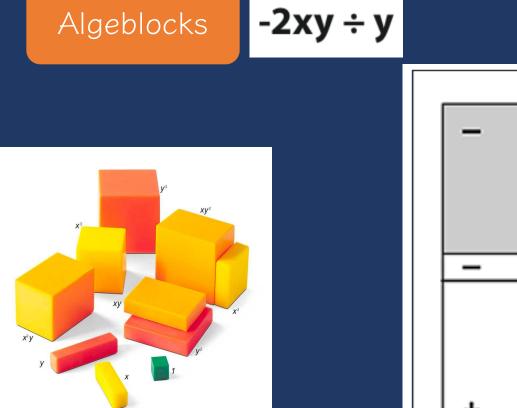


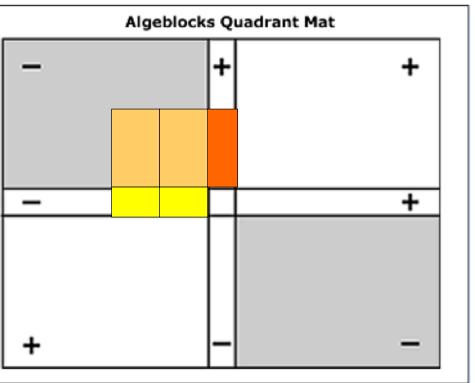
















Solving Equations	
With Algeblocks	
Problem	Representation
x(3)	
-2(y)	
x(1 + x)	
-y(y + 2)	
(x – 2)(-2x)	
(y – 1)(y + 2)	
3y ÷ 3	
-2xy ÷ y	
-3x ÷ 3x	
4x <sup>2</sup> ÷-x	





1. Share some of your favorite hands-on materials for algebra. 2. Share some of your favorite virtual manipulatives for algebra. 3. Considerations for using

these tools with students?



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