## Petersburg Mathematics Cohort

## Day 2

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## Introduce yourself.

Describe your role as an educator.
Describe the mathematics you support.

Share your Twitter handle!

## Schedule for Today



$x A+H$

## Trajectories in Mathematics

An important subset of the major work in grades K-8 is the progression that leads toward middle school algebra.

| K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Know number names and the count sequence <br> Count to tell the number of objects <br> Compare numbers <br> Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from <br> Work with numbers 1119 to gain foundations for place value | Represent and solve problems involving addition and subtraction <br> Understand and apply properties of operations and the relationship between addition and subtraction <br> Add and subtract within 20 <br> Work with addition and subtraction equations <br> Extend the counting sequence <br> Understand place value <br> Use place value understanding and properties of operations to add and subtract <br> Measure lengths indirectly and by iterating length units | Represent and solve problems involving addition and subtraction <br> Add and subtract within 20 <br> Understand place value <br> Use place value understanding and properties of operations to add and subtract <br> Measure and estimate lengths in standard units <br> Relate addition and subtraction to length | Represent \& solve problems involving multiplication and division <br> Understand properties of multiplication and the relationship between multiplication and division <br> Multiply \& divide within 100 <br> Solve problems involving the four operations, and identify \& explain patterns in arithmetic <br> Develop understanding of fractions as numbers <br> Solve problems involving measurement and estimation of intervals of time, liquid volumes, \& masses of objects <br> Geometric measurement: understand concepts of area and relate area to multiplication and to addition | Use the four operations with whole numbers to solve problems <br> Generalize place value understanding for multi-digit whole numbers <br> Use place value understanding and properties of operations to perform multidigit arithmetic <br> Extend understanding of fraction equivalence and ordering <br> Build fractions from unit fractions by applying and extending previous understandings of operations <br> Understand decimal notation for fractions, and compare decimal fractions | Understand the place value system <br> Perform operations with multi-digit whole numbers and decimals to hundredths <br> Use equivalent fractions as a strategy to add and subtract fractions <br> Apply and extend previous understandings of multiplication and division to multiply and divide fractions <br> Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition <br> Graph points in the coordinate plane to solve real-world and mathematical problems* | Apply and extend previous understandings of multiplication and division to divide fractions by fractions <br> Apply and extend previous understandings of numbers to the system of rational numbers <br> Understand ratio concepts and use ratio reasoning to solve problems <br> Apply and extend previous <br> understandings of arithmetic to algebraic expressions <br> Reason about and solve one-variable equations and inequalities <br> Represent and analyze quantitative relationships between dependent and independent variables | Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers <br> Analyze proportional relationships and use them to solve real-world and mathematical problems <br> Use properties of operations to generate equivalent expressions <br> Solve real-life and mathematical problems using numerical and algebraic expressions and equations | Work with radical and integer exponents <br> Understand the connections between proportional relationships, lines, and linear equations** <br> Analyze and solve linear equations and pairs of simultaneous linear equations <br> Define, evaluate, and compare functions <br> Use functions to model relationships between quantities |

 listed here are a subset of those designated as major in the assessment consortia's draft documents.
${ }_{* *}$ Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

## $\square$ <br> What's the critical math content for your students?



MA+ $:$


Two-dimensional images


https://bit.ly/srpowell


Manipulatives:
Fraction Concepts


Two-dimensional images


https://bit.ly/srpowell


## Length/Measurement

Fractions are appropriated by length


## Length/Measurement

Fractions are appropriated by length

## $\frac{2}{3}$



Cuisenaire rods

## Length/Measurement

Fractions are appropriated by length



## Area/Region

Shapes divided into equal sections


## Area/Region

Shapes divided into equal sections

## $\frac{2}{3}$



Geoboards

## Area/Region

Shapes divided into equal sections


Pattern blocks

## Area/Region

Shapes divided into equal sections

## $\frac{2}{3}$



Legos


## Set/Discrete

Individual shapes match the fraction
$\frac{2}{3}$

Two-color counters

## Set/Discrete

Individual shapes match the fraction

$$
\frac{2}{3}
$$





1. Share some of your favorite hands-on materials for fractions.

## BREAKOUT

2. Share some of your favorite virtual manipulatives for fractions.
3. Considerations for using these tools with students?

Manipulatives:
Fraction Computation


Two-dimensional images


https://bit.ly/srpowell


Addition

## $\frac{1}{5}+\frac{3}{5}$



## Addition



| Problem | Representation |
| :--- | :--- |
| $\frac{1}{5}+\frac{3}{5}$ |  |
| $\frac{2}{3}+\frac{2}{3}$ |  |
| $\frac{1}{2}+\frac{1}{4}$ |  |
| $\frac{1}{4}+\frac{4}{6}$ |  |
| $\frac{4}{5}-\frac{1}{5}$ |  |
| $\frac{6}{5}-\frac{2}{5}$ |  |
| $\frac{7}{8}-\frac{2}{4}$ |  |
| $\frac{1}{2}-\frac{2}{5}$ |  |

Addition


## Addition

## $\frac{1}{2}+\frac{1}{4}$



$$
\begin{aligned}
& 2: 2 \sqrt{4} 6,8,10 \\
& 4:(4,8,12,16,20
\end{aligned}
$$



## Addition

## $\frac{1}{2}+\frac{1}{4}$



| Problem | Representation |
| :--- | :--- |
| $\frac{1}{5}+\frac{3}{5}$ |  |
| $\frac{2}{3}+\frac{2}{3}$ |  |
| $\frac{1}{2}+\frac{1}{4}$ |  |
| $\frac{1}{4}+\frac{4}{6}$ |  |
| $\frac{4}{5}-\frac{1}{5}$ |  |
| $\frac{6}{5}-\frac{2}{5}$ |  |
| $\frac{7}{8}-\frac{2}{4}$ |  |
| $\frac{1}{2}-\frac{2}{5}$ |  |

## Subtraction



| Problem | Representation |
| :--- | :--- |
| $\frac{1}{5}+\frac{3}{5}$ |  |
| $\frac{2}{3}+\frac{2}{3}$ |  |
| $\frac{1}{2}+\frac{1}{4}$ |  |
| $\frac{1}{4}+\frac{4}{6}$ |  |
| $\frac{4}{5}-\frac{1}{5}$ |  |
| $\frac{6}{5}-\frac{2}{5}$ |  |
| $\frac{7}{8}-\frac{2}{4}$ |  |
| $\frac{1}{2}-\frac{2}{5}$ |  |

## Subtraction



Take away from this set


| Problem | Representation |
| :--- | :--- |
| $\frac{1}{5}+\frac{3}{5}$ |  |
| $\frac{2}{3}+\frac{2}{3}$ |  |
| $\frac{1}{2}+\frac{1}{4}$ |  |
| $\frac{1}{4}+\frac{4}{6}$ |  |
| $\frac{4}{5}-\frac{1}{5}$ |  |
| $\frac{6}{5}-\frac{2}{5}$ |  |
| $\frac{7}{8}-\frac{2}{4}$ |  |
| $\frac{1}{2}-\frac{2}{5}$ |  |

## Multiplication

## Multiplication

- Interpret multiplication sign as "of"
- $2 / 3$ of $3 / 4$

$$
\frac{1}{2} \times \frac{3}{4}
$$

$$
\frac{2}{3} \times \frac{3}{3}
$$

$$
\frac{2}{5} \times \frac{5}{6}
$$

$$
\frac{1}{4} \times \frac{2}{3}
$$

$$
\frac{3}{4} \times \frac{1}{3}
$$

## Multiplication



One-half of four-fourths


## Multiplication



One-half of two-fourths


## Multiplication



## Multiplication



One-half of three-fourths


## Multiplication



## Multiplication



## Multiplication

Length

$$
\frac{2}{3} \times \frac{3}{4}=\frac{1}{2} \quad \text { "Two-thirds of } \begin{gathered}
\text { three-fourths" }
\end{gathered}
$$



## Multiplication

Area

$$
\frac{2}{3} \times \frac{3}{4}=\frac{1}{2}
$$

"Two-thirds of three-fourths"


## Multiplication

Set

$$
\frac{2}{3} \times \frac{3}{4}=\frac{1}{2} \quad \begin{gathered}
\text { "Two-thirds of } \\
\text { three-fourths" }
\end{gathered}
$$


$2 \times \frac{1}{4}$
$\frac{1}{2} \times \frac{4}{4}$
$\frac{1}{2} \times \frac{2}{4}$
$\frac{1}{2} \times \frac{3}{4}$
$\frac{2}{3} \times \frac{3}{3}$
$\frac{2}{5} \times \frac{5}{6}$
$\frac{1}{4} \times \frac{2}{3}$
$\frac{3}{4} \times \frac{1}{3}$

## Division

Division

- "How many sets of the second fraction fit into the first fraction?"

Representation
$\frac{3}{3} \div 3$
$\frac{3}{3} \div \frac{1}{3}$
$\frac{2}{3} \div \frac{1}{3}$
$\frac{5}{6} \div \frac{1}{2}$
$\frac{7}{8} \div \frac{3}{4}$
$\frac{1}{4} \div \frac{1}{2}$
$\frac{3}{4} \div \frac{2}{3}$
$\frac{4}{5} \div \frac{1}{2}$

## Division

## $\frac{3}{3} \div \frac{1}{3}$

Show

1 set of one-third
2 sets of one-third
3 sets of one-third

## Division



1 set of one-third
2 sets of one-third

## Division



Show

1 set of one-half

Two-thirds set of onehalf

## Division



Show


## Division

Length

"How many sets of $1 / 3$ can be made with 1 and $1 / 2$ ?"


## Division

Area

"How many sets of $1 / 3$ can be made with 1 and $1 / 2$ ?"


## Division

Set

"How many sets of $1 / 3$ can be made with 1 and $1 / 2$ ?"



1. Share some of your favorite hands-on materials for fraction computation.
2. Share some of your favorite virtual manipulatives for fraction computation.
3. Considerations for using these tools with students?

## Manipulatives: Decimals



Two-dimensional images


https://bit.ly/srpowell


## Decimal



$\left.$| $\frac{1}{10}$ |
| :---: |
| $\frac{1}{10}$ |
| $\frac{1}{10}$ |
| $\frac{1}{10}$ |
| $\frac{1}{10}$ |
| $\frac{1}{10}$ |\(\left|\begin{array}{|c|}\hline \frac{1}{10} <br>


\hline\end{array}\right|\)| 10.1 |
| :---: | \right\rvert\,


$1 x A+H$

1.0

$$
\begin{array}{l|l}
\frac{1}{8} & 0.125
\end{array}
$$




## $1,504.946$

## Decimals and Place Value





1. Share some of your favorite hands-on materials for decimals.

## BREAKOUT

2. Share some of your favorite virtual manipulatives for decimals.
3. Considerations for using these tools with students?

Manipulatives:
Algebra

## Integer Concepts

Zero pairs

- A positive and a negative cancel one another
- If you add a positive, you must add a negative
- If you subtract a positive, you must subtract a negative

A number is assumed positive if a negative sign (-) doesn't proceed it


## Ways to Teach Addition and Subtraction

MAT and CUBES


TWO-COLOR COUNTERS


NUMBER LINE

xA+ $1 \cdot$

## $3+(-5)$



Use the mat in your
handouts!
$3+(-5)$

$5-3$
$x A+|\cdot|$

## $-3-4$


$-3-4$


## $-3-4$

\section*{|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| -8 | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 8 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |}

$x A+1 \dot{1}$

## $-3-4$

\section*{|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |}

$x A+1 \cdot 1$


## Ways to Teach Multiplication and Division

MAT and CUBES


ALGEBRA TILES/ALGEBLOCKS


## 2(3)

$x A+1 \cdot 1$

$x A+1 \cdot 1$

Sh〇W...
(2)(-3)
$(-2)(-3)$

$$
9 \div 3
$$

$$
9 \div(-3)
$$

$-9 \div(-3)$

## $9 \div 3$



Sh〇W...
(2)(-3)
$(-2)(-3)$

$$
9 \div 3
$$

$$
9 \div(-3)
$$

$-9 \div(-3)$

## cups/plates and counters

## algebra tiles

Algeblocks


## cups/plates

 and counters

Problem
$x+4$

$$
y-6
$$

2(x)

## Show...

## algebra tiles



Problem
$x+4$
$y-6$

2(x)
$\times A+H$

## Show...

## Algeblocks



Problem
$x+4$
$y-6$
2(x)

## zero pairs <br> properties of equality

-1 and 1 equal 0
If you do something to one side of the equal sign, you do the same thing to the other side.

## cups/plates and counters

## algebra tiles

Algeblocks


MA+: $:$

## cups/plates and counters <br> $x+2=5$



## cups／plates and counters <br> $x+2=5$


（in的是是 름


## algebra tiles $\mathbf{x + 2 = 5}$


$x A+1 \dot{1}$

## algebra tiles

$$
y-4=2
$$




## Algeblocks <br> $x+2=5$



Algeblocks
$-1=y-4$


Algeblocks Sentences Mat



## Algeblocks <br> $2 x+2=6$



## Algeblocks

$2(x+3)=4$


Algeblocks Sentences Mat



## Algeblocks $\mathbf{x}(\mathbf{1}+\mathbf{x})$



Algeblocks Quadrant Mat


Algeblocks
$-2 x y \div y$


Algeblocks Quadrant Mat



Algeblocks
$-2 x y \div y$


Algeblocks
$-2 x y \div y$


Algeblocks Quadrant Mat



1. Share some of your favorite hands-on materials for algebra.

BREAKOUT
2. Share some of your favorite virtual manipulatives for algebra.
3. Considerations for using these tools with students?

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