A Comparitive Study of Sparse Channel Estimation Techniques for OFDM Systems Based on Compressive Sensing

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Abstract: Sparse channels are typically encountered in many communication systems like underwater acoustic channels, communications in a hilly terrain etc. Conventional channel estimation techniques like the Least Squares approach and interpolation based methods do not work well in this case, because these techniques do not exploit the sparse structure of the channel. In this paper, we first present a comparative study of the existing sparse channel estimation techniques based on Matching Pursuit (MP), Orthogonal Matching Pursuit (OMP) and Basis Pursuit (BP) with respect to the mean squared error (MSE) and bit error rate (BER). Then we introduce the novel Compressive Sampling Matching Pursuit (CoSaMP) algorithm and demonstrate its superior performance with respect to the previously mentioned schemes. The channel estimation techniques are compared taking Cramer-Rao lower bound (CRLB) as the reference. Evaluation of the system performance is done in time domain because in frequency domain the system performance becomes dependent on the number of points over which the FFT operation is performed and therefore the interpretation can be misleading.

Keywords: Sparse channels, Compressive Sensing, Matching Pursuit, Basis Pursuit, Cramer Rao Lower bound, MSE and BER.

I. INTRODUCTION

If there is a wireless channel which exhibits a very large delay spread with only a few non-zero channel coefficients, then such a channel is regarded as a sparse channel. There are many communication systems which are regarded as sparse for e.g. terrestrial transmission channel of high definition television (HDTV) [1], a hilly terrain communication channel [2] and underwater acoustic channels [3] to name a few. An example of the sparse channel is shown in fig 1. The reason why these channels are sparse may be attributed as follows: In the underwater acoustic channels, few significant multi paths are relatively close to each other and have a delay spread of almost the same order as in conventional channels. But there may be one signal which is reflected from the sea bed. This signal will have a huge delay compared to the other signals, so overall delay spread of the system is now very high with the inclusion of this far away multipath. Therefore, when the receiver samples the received signal at baseband, all the channel coefficients after the closely spread significant multi paths will be zero, then finally the last channel coefficient (due to reflection from the sea bed) will be non-zero. This will result in a sparse channel.

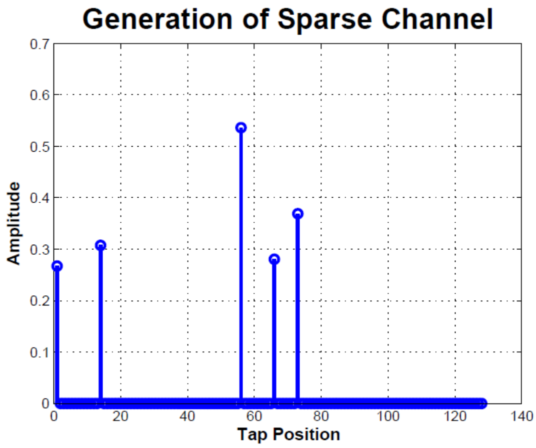

Fig. 1: A typical sparse channel with 5 non-zero multipaths

If the conventional channel estimation techniques are applied for sparse channel estimation, then they do not take into account the sparsity of the channel. These techniques treat the channel as if it has all non-zero coefficients and will try to estimate the channel taps in all the positions. Needless to say, the mean squared error and bit error rate performance will be highly degraded under such a scheme. Moreover, in a sparse channel the multipath delays may not be at the sampling instants. Therefore, ordinary channel estimation schemes cannot capture this delay and thus channel estimation results are more erroneous.

Sparse signal processing is in practice for quite a long time now. It was first reported in the literature for underwater acoustic channel measurements [3, 5]. Terrestrial broadcasting [1] for high definition (HD) television, communications near a hilly terrain [2] are also reported to be sparse. Besides this, sparse signal processing are predominant in many fields like spectral estimation, analysis of noisy images, coding of speech signals etc. A few application areas are listed in [1-5]. The sparse processing techniques does not merely represent a signal as a superposition of several sinusoidal signals, but makes use of available dictionaries. There is no unique way to construct a dictionary. There can be many different ways in which a dictionary can be constructed [6]. The central idea is to provide the sparsest representation of an object by choosing that column of the dictionary which will represent the signal by the least number of non-zero coefficients. Accordingly we have many different types of dictionaries like wavelet dictionaries, Gabor dictionaries, cosine packets etc. Mallat and Zhang [6] have proposed the matching pursuit (MP) algorithm in 1993 which was the first stepping stone in the field of compressive sensing. Although the algorithm gives a much improved performance over the Least Squares, but the algorithm is greedy, and tries to converge to a global maxima from a knowledge of the local maxima. Moreover the chances of reselection of a previously selected vector from the dictionary are also large. In [7], Cotter and Rao compared sparse channel estimation techniques by using Least Squares(LS), variants of LS and Matching Pursuit. The superior performance of MP exploiting the sparseness of the channel is subsequently demonstrated. It is also shown in [7] that MP has both a lower complexity and requires shorter training sequences than LS and all the variants of LS. A slightly improved version of LS which performs channel estimation by taking the channel sparsity into consideration has been proposed in the paper by M.R Raghavendra and K. Giridhar [8]. It uses a Generalized Akaike Information Criterion (GAIC) to estimate the channel taps as well as the channel positions.The performance of this scheme outperforms that of the ordinary LS estimator. Details of Akaike Information Criterion (AIC) has been reported in [9] where it is shown that Generalized Likelihood Ratio Test (GLRT) with a specified threshold is equivalent to AIC. The problem of reselection of basis vectors is avoided by employing the Orthogonal Matching Pursuit algorithm (OMP) [10, 11]. G. Karabulut and A. Yongacoglu[10] presented a comparison of sparse channel estimation with respect to MP and OMP algorithms. It is demonstrated that OMP outperforms the MP algorithm since there is no reselection of basis vectors in the former case. Both MP and OMP come under the category of greedy algorithms. To avoid the shortcomings of greedy algorithm, Chen and Donoho [12] had proposed the Basis Pursuit (BP) algorithm which is the true representation of L_1 norm of a sparse signal. Suppose many solutions are possible by solving the linear set of equations

$$y = W.x \tag{1}$$

Then, by following this method, we should choose that solution whose coefficients have minimum value of L_1-norm [12]. This is done by a convex optimization approach

$$\min_{x} |y - W.x|^2 + \lambda.\|x\|_1 \tag{2}$$

A relaxed basis pursuit is thus a minimization of a mixed norm criterion [13]. Under this family of minimization of a sparse problem which is convex, we have three important algorithms slightly different from each other. These are Dantzig Selector (DS), Basis Pursuit (BP) and LASSO. The performance of Dantzig selector has been categorically verified in [14] by Emmanuel Candes and Terence Tao for the case when the number of parameters p is much larger than the number of equations n as we see in functional MRI and tomographic applications. The Dantzig Selector (DS) has been voted among the top 10 algorithms of this century by SIAM because of its simplicity and powerful computational ability. A comprehensive comparison of LS, OMP and BP with respect to these parameters have been presented in [11] by Joel A. Tropp and Anna C. Gilbert. DeVore and Temlyakov [15] have given a pessimistic reasoning about greedy algorithms. But Joel A. Tropp et el [11] had suggested that results for MP and OMP are quite deceptive and can exhibit a wide variation depending on the number of measurements, the order of sparsity, and the random measurement matrix. The same result has been reestablished in [16]. The work by Kunis and Rauhut [16], have suggested the performance of OMP for signal recovery from random frequency measurements. Kunis and Rauhut also proved that the first iteration in OMP can locate all the dominant taps of the channel by use of pilot sequences. It is shown in their work that OMP produces signal approximations that are superior to BP. Their work also proved that OMP algorithm executes much faster compared to BP. The work by Needell and Vershynin [17] have showed that if Restricted Isometry Property (RIP) is followed, then with a high probability Regularized OMP(ROMP) can recover K-sparse signals from O(K ln d) random observations where d is the number of columns of the DFT matrix. This result is highly esteemed by researchers as a significant advancement of OMP algorithm.

II. SYSTEM MODEL

The sparse channel is considered to be a frequency selective Rayleigh fading channel. It is described by the equation

$$h(\tau) = \sum_{p=0}^{\alpha-1} c_p \delta(\tau - \tau_{p)}) \tag{3}$$

where α is the number of non-zero channel coefficients (typical value is 5), c_p's are the non-zero channel coefficients and τ_p's are the respective multipath delays associated with the channel coefficients c_p's. The channel is assumed to be wide sense stationary [28] and it is quasi-static in nature i.e. it changes from one OFDM frame to another, but remains constant within the time interval of each OFDM frame.

Considering a standard OFDM transmission and reception scheme [53], the receiver applies an N-point discrete Fourier transform on the received data symbols as

$$r[n] = H[n].d[n] + w[n] \quad n = 0,1 \ldots.(N-1) \tag{4}$$

where r[n] is the received symbol for n^{th} sub-carrier after DFT operation, H[n] is the frequency response of the channel for n^{th} sub-carrier and d[n] is the n^{th} transmitted symbol. w[n] is the AWGN component in the n^{th} sub-carrier . N is the number of active OFDM subcarriers.

The frequency response for n^{th} sub-carrier can be written as

$$H[n] = \sum_{p=0}^{\alpha-1} c_p \, exp\left(\frac{-j.2.\pi.n.\frac{\tau_p}{T_s}}{N}\right), \quad n = 0,1,\ldots(N-1) \tag{5}$$

where τ_p/T_s is an integer, because of the frequency selective nature of the channel and construction of the dictionary. Here T_s is the sampling time period. Writing equation (4) in vector form, we get

$$r = G_s.H + w \tag{6}$$

where r, d, H and w are column vectors containing the r[n], d[n], H[n] and w[n] for all n . G_s is a diagonal matrix with the elements of vector d on the main diagonal. We can rewrite the vector H as

$$H = \begin{bmatrix} H(0) \\ H(1) \\ .. \\ .. \\ H(N-1) \end{bmatrix} \quad (7)$$

$$= \begin{bmatrix} \sum_{p=0}^{\alpha-1} c_p \, exp\left(\frac{-j.2.\pi.0.\frac{\tau_p}{T_s}}{N}\right) \\ .. \\ .. \\ \sum_{p=0}^{\alpha-1} c_p \, exp\left(\frac{-j.2.\pi.(N-1).\frac{\tau_p}{T_s}}{N}\right) \end{bmatrix} \quad (8)$$

$$= \sum_{p=0}^{\alpha-1} c_p W_p(n) \quad (9)$$

where the vector

$$W_p(n) = \left[1 \; e^{-j.2\pi.1.\tau_p/(T_s.N)} \; .. \; .. \; e^{-j.2\pi(N-1)\tau_p/(T_s.N)}\right]^T$$

is nothing but the column vector of the partial discrete Fourier transform matrix. To mathematically express the compressed sensing problem, we should keep a large, but finite dictionary [20]. This is because it is well established that performance of any compressive sensing algorithm depends on the resolution of the dictionary computed beforehand. So we take the maximum possible delay T_{max} and define the delay spread as the interval [0 T_{max}]. We divide this interval into a set of uniformly spaced delays. The interval between any two delay elements are defined by a multiple of the baseband sampling time period T_s. Therefore the delay elements in the delay grid will be uniformly spaced by T_s and the delay grid can be now build as

$$\tau_g = \{0, T_{s,} \; 2.T_{s,} \ldots\ldots\ldots (N_t - 1).T_s\} \quad (10)$$

where $N_t = T_{max}/T_s$

It is a reasonably good assumption that with this fine resolution delay grid at hand, any unknown multipath delay in the received signal will correspond to one of the delay elements from the delay grid. In fact, there will be α such delays for an α sparse channel. Our first job is to precisely estimate the number of multipath delays from the delay dictionary. With the delay grid (of dimension $1xN_t$) at hand, we next construct a partial DFT matrix as

$$W = [W_0(n) \; W_1(n) \ldots\ldots W_{(N_t-1)}(n)],$$
$$n = 0,1,..(N-1) \quad (11)$$

where W is of dimension NxN_t and $W_0(n), W_1(n), \ldots\ldots W_{(Nt-1)}(n)$ each are the column vectors of size Nx1. The general term $W_k(n)$ is defined by as

$$W_k(n) = \begin{bmatrix} exp\left(\frac{-j.2\pi.0.\frac{\tau_k}{T_s}}{N}\right) \\ exp\left(\frac{-j.2\pi.1.\frac{\tau_k}{T_s}}{N}\right) \\ .. \\ .. \\ exp\left(\frac{-j.2\pi.(N-1).\frac{\tau_k}{T_s}}{N}\right) \end{bmatrix} \quad (12)$$

Substituting the equation (12) in equation (9) we get

$$H = W.c \quad (13)$$

where c impulse response of the channel with a span of N_t samples with $N_t << N$. Thus the standard compressive sensing problem may be finally formulated as :

Given $r = (G_sW)c + w$ we are to find c. It is to be noted that G_sW is the dictionary which we will be using for estimating the multipath delays and channel coefficients. It is obtained by multiplying the diagonal matrix G_s with the matrix W which is constructed according to equation (11). Since the channel is sparse, most components of the channel impulse response are zero. Using this helpful knowledge, a sparse solution to $r \approx (G_sW).c$ at high SNR's is achieved by approximating r as a linear combination of a few columns of (G_sW).

III. ALGORITHM DESCRIPTION

A. Matching Pursuit algorithm

For a detailed understanding of MP algorithm the reader is advised to read the famous paper by Mallat and Zhang [6].

The flowchart describing MP algorithm is outlined below:

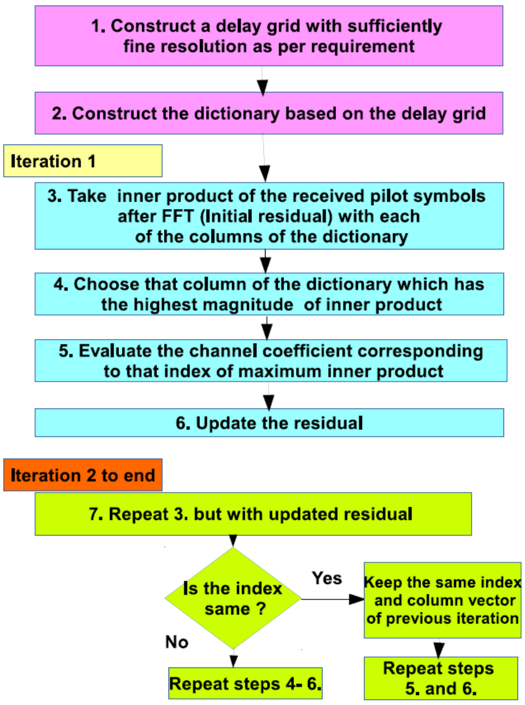

Fig. 2: Flowchart of Matching Pursuit algorithm

The non-zero channel coefficients c_p's are computed as

$$c_p = \max_l \left|G_s W(l)^H b_{p-1}\right| * sign\{G_s W(l)^H b_{p-1}\} \quad p = 1,2 \ldots \alpha \quad l = 0,1,2,\ldots(N_t - 1) \qquad (14)$$

where the sign of a complex number z is defined by

$$sign(z) = \frac{z}{|z|} \qquad (15)$$

The frequency response of the channel is determined by

$$H = \sum_{p=1}^{\alpha} c_p . W(k_p)$$

$$= W.c \qquad (16)$$

B.Orthogonal Matching Pursuit algorithm

In essence, OMP is quite similar in comparison to MP with certain subtle differences. A detailed understanding of OMP algorithm is outlined in [10]. However in OMP, the chance of reselection of a particular basis vector from a stored dictionary is eliminated based on the way orthogonal projection matrix P operates on the extracted columns of the dictionary [10,11]. Otherwise, the computation of the steps of this algorithm and matching pursuit algorithm are exactly identical as far as sparse channel estimation is concerned. The frequency response of the channel may be computed as in equation (16)

C. Basis Pursuit algorithm

The standard compressive sensing equation is restated as :

$$r = (G_s W).c + w \qquad (17)$$

The Basis Pursuit formulated [12,14] based on the above equation is given by

$$\min_c |r - (G_s.W).c|^2 + \lambda.\|c\|_1 \qquad (18)$$

Here, the L_1 norm of a vector x is defined by the equation

$$\|x\|_1 = \sum_{i=1}^{n} |x_i| \qquad (19)$$

Since, in our case the channel coefficients are complex, so the L_1 norm of c is defined as

$$\|c\|_1 = \sum_{p=0}^{N_t-1} \sqrt{\left|R(c_p)^2\right| + \left|I(c_p)^2\right|} \qquad (20)$$

In the above equation the parameter λ depends on the noise variance. Typical value of λ for signal to noise ratio of 5 dB is 0.3162. we have used the CVX routine of MATLAB to solve this optimization problem.

D. Compressive Sensing Matching Pursuit algorithm

CoSaMP algorithm is the latest development in the field of compressive sensing. It offers the advantages of both the greedy algorithm and the convex program [18,19]. The flowchart describing the sequence of operations in the CoSaMP algorithm is described below.

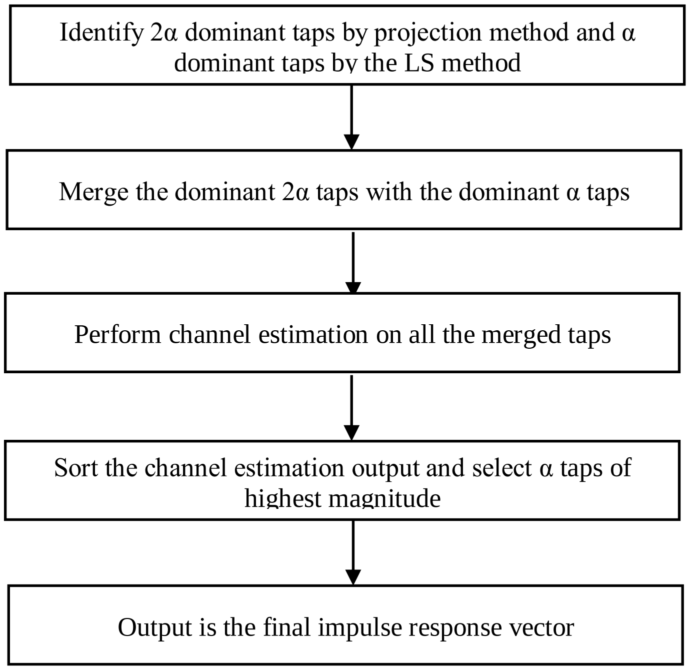

Fig. 3: Flowchart of Compressive Sampling Matching Pursuit algorithm

IV. SIMULATION RESULTS

The following parameters have been fixed for Simulations:

Simulation parameters of OFDM	Values with respective units
Number of OFDM Subcarriers	1024
Sampling time period	0.1 μs
Length of Cyclic Prefix	128
Modulation Type	4 QAM
Physical Channel Length	128
Range of SNR	5 to 25 dB in steps of 4
PDF of the Channel	Uniform

Table 1: Parameters used for Sparse Channel Simulation

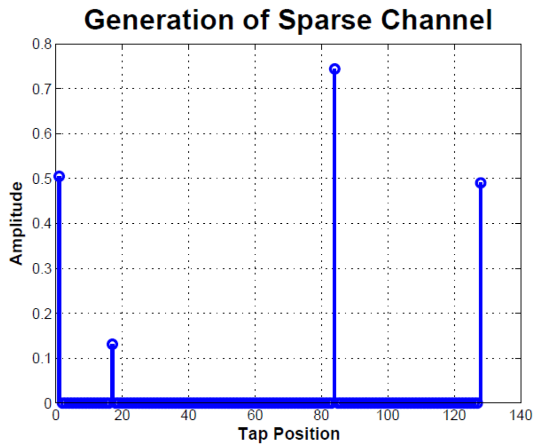

Fig. 4: A sparse channel with 4 Multi paths

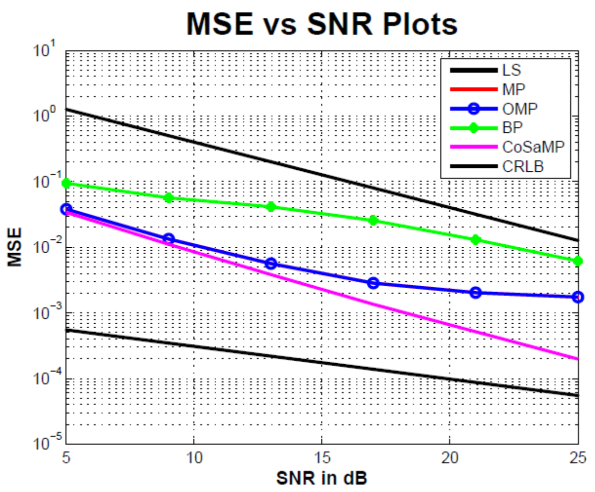

Fig. 5: MSE vs SNR for pilot interval = 4

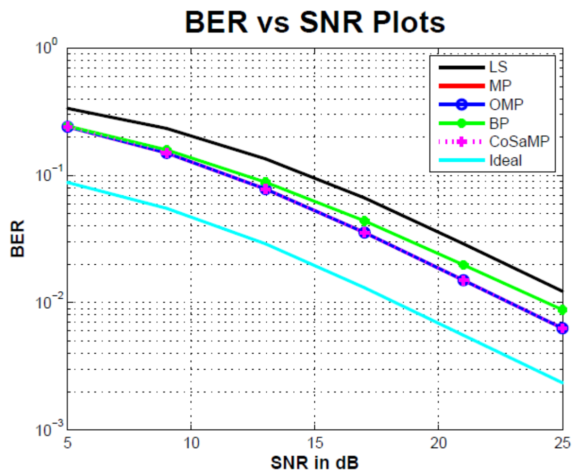

Fig. 6: BER vs SNR for pilot interval = 4

We have the following observations:

- Performance of all the estimators are uniformly better than the Least Square channel estimate, which does not take into account the sparsity of the channel.
- The novel CoSaMP algorithm outperforms all other algorithms in terms of time domain MSE and is closest to the Cramer-Rao lower bound which is the minimum lower bound on variance of these estimators.
- The basis pursuit (BP) based on minimization of the L_1 norm is found to exhibit an inferior performance compared to MP or OMP. The reason is that convex program is based on an optimized solution and an optimized solution is not necessarily the best solution.
- The performance of MP and OMP are exactly same in this case, since there is no question of reselection of the previously selected basis vector from dictionary. This is because the sparse channel coefficients generated are unique and the dictionary is constructed with fine resolution so that even small changes in multipath delays can be adequately captured by the dictionary.

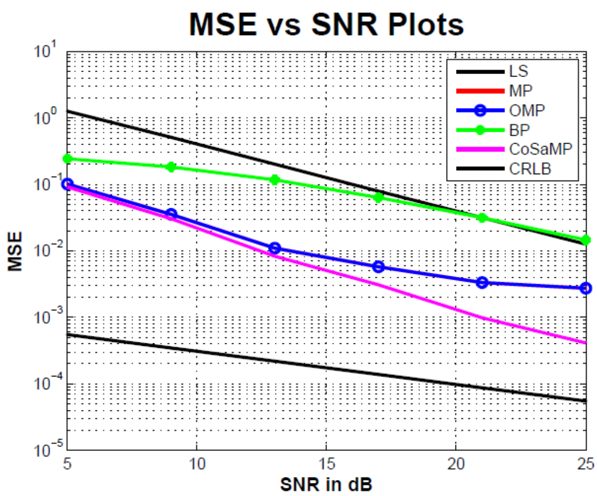

Fig.7: MSE vs SNR for pilot interval = 8

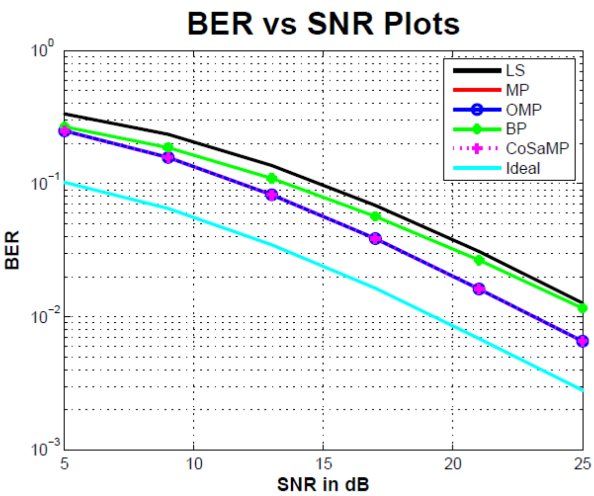

Fig.8: BER vs SNR for pilot interval = 8

We have the following observations:

- When the number of pilots reduces by a factor of 2, then also the compressive sensing schemes give an acceptable level of MSE and BER performance.
- This scheme of pilot arrangement can therefore be regarded as optimal. With a slightly wider pilot separation, the system performance becomes unacceptable w.r.t MSE and BER

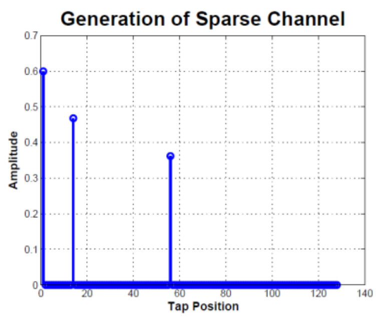

Fig. 9: Generation of a 3 tap sparse channel

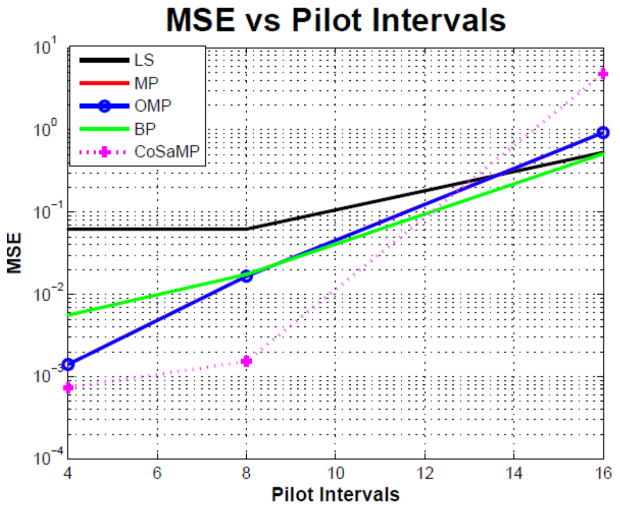

Fig. 10: MSE vs SNR for variable number of pilots

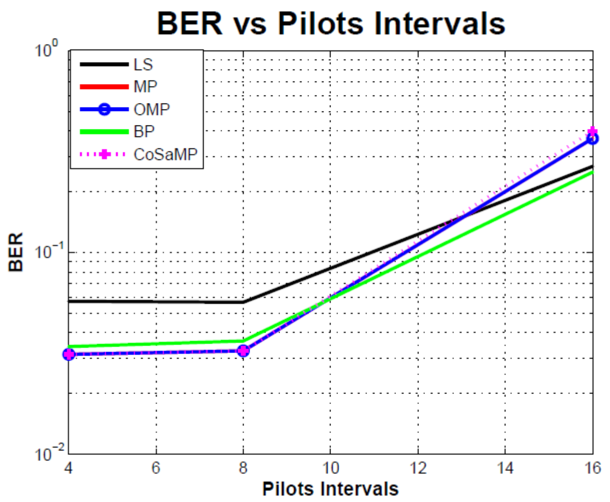

Fig. 11: BER vs SNR for variable number of pilots

The plot of mean squared error(MSE) and bit error rate(BER) with respect to variable number of pilots shows a sharp performance degradation when the number of pilots fall below a certain number (pilot interval 8 in this case). This number is fixed for a particular set of active OFDM subcarriers. This is in conformity with the restricted isometric property (RIP) [21] which is the fundamental backbone of compressive sensing. For our special case where 1024 OFDM subcarriers are employed in a 10 MHz bandwidth, the optimal number of pilots is found to be 128 which calls for a comb type pilot fashion separated at an interval of 1024/128 = 8 locations and interleaved with the data symbols.

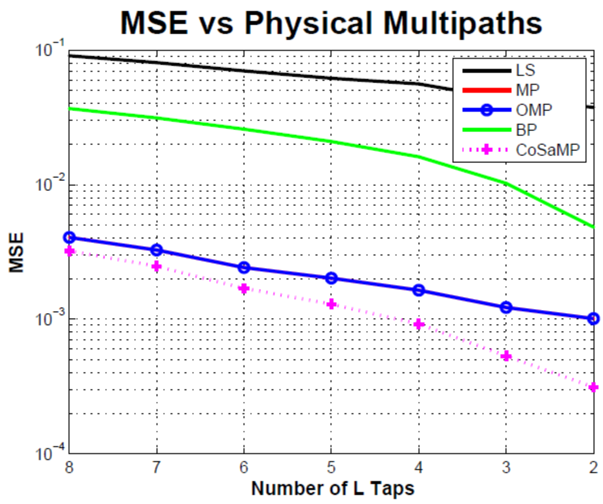

Fig. 12: MSE vs physical multipaths

We have the following observations:

- As the number of non-zero distinct multi paths increase, the performance of all the above mentioned algorithms deteriorates. This is as expected and in accordance with the theory. This is due to larger error involved in the minimization of L_1 norm in case of basis pursuit(BP) and CoSaMP while the residual is of higher magnitude for more number of multipaths in case of greedy algorithms. This explains their degradation performance.

V. CONCLUSION

In this paper, we have shown that BP algorithm based on the convex program that solves a linear programming problem has a higher stability than OMP algorithm. This is its inherent advantage over the greedy counterpart. However the BP method is computationally extremely complex and not at all suitable to be implemented in hardware. On the other hand, since OMP is much faster and easier to implement in hardware, therefore it is the choice for real time applications. As regards to performance, it cannot be said whether BP or OMP algorithm is uniformly superior to the other. The performance depends on many factors as enlisted in section I. The proposed sparse channel estimation algorithm using Compressive Sampling Matching Pursuit (CoSaMP) has the advantages of greedy algorithm and convex program. It is much more stable compared to MP or OMP and at the same time computationally much less challenging compared to Basis Pursuit. So, it is the preferred choice of sparse channel estimation technique for wireless channels whose characteristics change rapidly from one OFDM frame to another.

VI. ACKNOWLEDGEMENT

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VII. REFERENCES

[1]. W.F. Schreiber, “Advanced television systems for terrestirial bradcasting: Some problems and some proposed solutions", in Proc. IEEE, June 1995, vol. 83, pp 958-981

[2]. S. Ariyavisitakul, N.R. Sollenberger and L.J. Greenstein, “Tap selectable decision-feedback equalization", IEEE Tran. on Commun., vol. 45, pp. 1497-1500, Dec 1997.

[3]. M. Kocic et al., “Sparse equalization for real-time digital underwater acoustic communications", in Proc. OCEANS'95, San Diego, CA, Oct 1995, pp. 1417-1500.

[4]. W.U.Bajwa, J.Haupt, G.Raz and R.Nowak “Compressed channel sensing", Proc. Conf. Inf. Sci. Syst., pp.5 -10 2008

[5]. R. Gribonval, E. Bacry, S. Mallat, P. Depalle and X. Rodet "Analysis of sound signals with high resolution matching pursuit", Proc. Int. Symp. Time-Frequency Time-Scale Anal., pp.125-128 1996

[6]. S.G.Mallat and Z.Zhang, “Matching pursuits with time-frequency dictionaries", IEEE Tran. on ASSP, vol. 41, no. 12, pp. 3397-3415, Dec 1993.

[7]. Shane F. Cotter and Bhaskar D. Rao, “Sparse channel estimati via matching pursuit with application to equalization", IEEE Tran. on Communications, vol. 50, no. 3, pp. 374-377, 2002.

[8]. M. R. Raghavendra and K. Giridhar “Improving channel estimation in OFDM systems for sparse multipath channels", IEEE Signal Processing Letters, vol. 12, pp.52 -55 2005

[9]. Peter Stoica, Yngve Selen and Jian Li, “On information criteria and the generalized likelihood ratio test of model order selection", IEEE Signal Processing Letters, vol. 11, no. 10, Oct 2004.

[10]. G. Karabulut and A. Yongacoglu “Sparse channel estimation using orthogonal matching pursuit algorithm", Proc. IEEE Veh. Technol. Conf. (VTC), vol. 6, pp.3880 -3884 2004

[11]. J. A. Tropp and A. C. Gilbert, “Signal recovery from random measurements via orthogonal matching pursuit", IEEE Transaction on Information Theory, vol. 53(12), pp. 4655-4666, 2007.

[12]. Scott Shaobing Chen, David L. Donoho and Michael A. Saunders, “Atomic decomposition by basis pursuit", SIAM Journal on Scientific Computing, vol. 20, no. 1, pp. 33-61, 1999.

[13]. W.U.Bajwa, J.Haupt, G.Raz and R.Nowak “Compressed channel sensing : A New Approach to estimating Spase Multipath Channels", Proceedings of the IEEE Vol. 98, No. 6, June 2010

[14]. E. Candes and T. Tao, “The Dantzig selector: Statistical estimation when p is much larger than n", Annals of Statistics, vol. 35, pp. 2392-2404, 2006.

[15]. R. DeVore and V. N. Temlyakov, “Some remarks on greedy algorithms", Adv. Comput. Math., vol. 5, pp. 173-187, 1996.

[16]. S. Kunis and H. Rauhut, “Random sampling of sparse trigonometric polynomials II - orthogonal matching pursuit versus basis pursuit", Found. Comput. Math., vol. 8, pp. 737-763, Dec. 2008.

[17]. D. Needell and R. Vershynin, “Uniform uncertainty principle and signal recovery via Regularized Orthogonal Matching Pursuit", Jul. 2007.

[18]. D. Needell and J. A. Tropp, “CoSaMP: Iterative signal recovery from incomplete and inaccurate samples", Applied and Computational Harmonic Analysis, vol. 26(3), pp. 301-321, 2008.

[19]. Guan Gui, QunWan et el. “Sparse multipath channel estimation using compressive sampling matching pursuit algorithm", IEEE APWCS 2010

[20]. Christian R. Berger, Shengli Zhou, Weian Chen and Peter Willett, “Sparse channel estimation for OFDM : Over-complete dictionaries and super-resolution", Signal Processing Advances in Wireless Communications, 2009: 196-200.

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