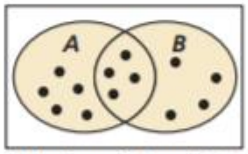
Chapter 10
Probability

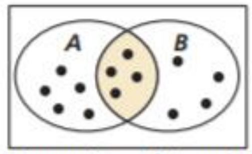
Section 10-4
Probability of Disjoint and Overlapping Events

Compound Events

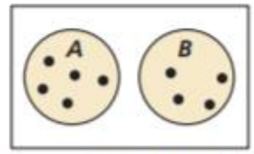
When you consider all the outcomes for either of two events A and B, you form the *union* of A and B, as shown in the first diagram. When you consider only the outcomes shared by both A and B, you form the *intersection* of A and B, as shown in the second diagram. The union or intersection of two events is called a **compound event**.



Union of A and B



Intersection of A and B



Intersection of A and B is empty.

To find $P(A \text{ or } B)$ you must consider what outcomes, if any, are in the intersection of A and B. Two events are **overlapping** when they have one or more outcomes in common, as shown in the first two diagrams. Two events are **disjoint**, or **mutually exclusive**, when they have no outcomes in common, as shown in the third diagram.

STUDY TIP

If two events A and B are overlapping, then the outcomes in the intersection of A and B are counted *twice* when $P(A)$ and $P(B)$ are added. So, $P(A \text{ and } B)$ must be subtracted from the sum.

Core Concept

Probability of Compound Events

If A and B are any two events, then the probability of A or B is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

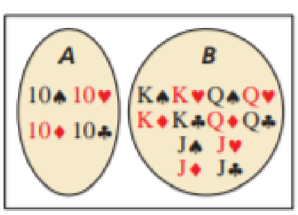
If A and B are disjoint events, then the probability of A or B is

$$P(A \text{ or } B) = P(A) + P(B).$$

EXAMPLE 1 Finding the Probability of Disjoint Events

A card is randomly selected from a standard deck of 52 playing cards. What is the probability that it is a 10 *or* a face card?

SOLUTION



Let event A be selecting a 10 and event B be selecting a face card. From the diagram, A has 4 outcomes and B has 12 outcomes. Because A and B are disjoint, the probability is

$P(A \text{ or } B) = P(A) + P(B)$	Write disjoint probability formula.
$= \frac{4}{52} + \frac{12}{52}$	Substitute known probabilities.
$= \frac{16}{52}$	Add.
$= \frac{4}{13}$	Simplify.
$\approx 0.308.$	Use a calculator.

COMMON ERROR

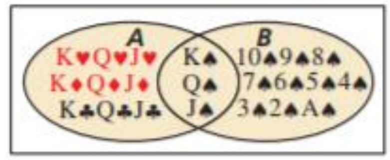
When two events *A* and *B* overlap, as in Example 2, *P*(*A* or *B*) does not equal *P*(*A*) + *P*(*B*).

EXAMPLE 2 Finding the Probability of Overlapping Events

A card is randomly selected from a standard deck of 52 playing cards. What is the probability that it is a face card *or* a spade?

SOLUTION

Let event *A* be selecting a face card and event *B* be selecting a spade. From the diagram, *A* has 12 outcomes and *B* has 13 outcomes. Of these, 3 outcomes are common to *A* and *B*. So, the probability of selecting a face card or a spade is



$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ — Write general formula.

$= \frac{12}{52} + \frac{13}{52} - \frac{3}{52}$ — Substitute known probabilities.

$= \frac{22}{52}$ — Add.

$= \frac{11}{26}$ — Simplify.

$\approx 0.423.$ — Use a calculator.

EXAMPLE 3 Using a Formula to Find *P*(*A* and *B*)

Out of 200 students in a senior class, 113 students are either varsity athletes or on the honor roll. There are 74 seniors who are varsity athletes and 51 seniors who are on the honor roll. What is the probability that a randomly selected senior is both a varsity athlete *and* on the honor roll?

SOLUTION

Let event *A* be selecting a senior who is a varsity athlete and event *B* be selecting a senior on the honor roll. From the given information, you know that $P(A) = \frac{74}{200}$, $P(B) = \frac{51}{200}$, and $P(A \text{ or } B) = \frac{113}{200}$. The probability that a randomly selected senior is both a varsity athlete *and* on the honor roll is *P*(*A* and *B*).

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ — Write general formula.

$\frac{113}{200} = \frac{74}{200} + \frac{51}{200} - P(A \text{ and } B)$ — Substitute known probabilities.

$P(A \text{ and } B) = \frac{74}{200} + \frac{51}{200} - \frac{113}{200}$ — Solve for *P*(*A* and *B*).

$P(A \text{ and } B) = \frac{12}{200}$ — Simplify.

$P(A \text{ and } B) = \frac{3}{50}$, or 0.06 — Simplify.

A card is randomly selected from a standard deck of 52 playing cards. Find the probability of the event.

1. selecting an ace *or* an 8

2. selecting a 10 *or* a diamond

3. **WHAT IF?** In Example 3, suppose 32 seniors are in the band and 64 seniors are in the band or on the honor roll. What is the probability that a randomly selected senior is both in the band and on the honor roll?

Using More Than One Probability Rule

In the first four sections of this chapter, you have learned several probability rules. The solution to some real-life problems may require the use of two or more of these probability rules, as shown in the next example.

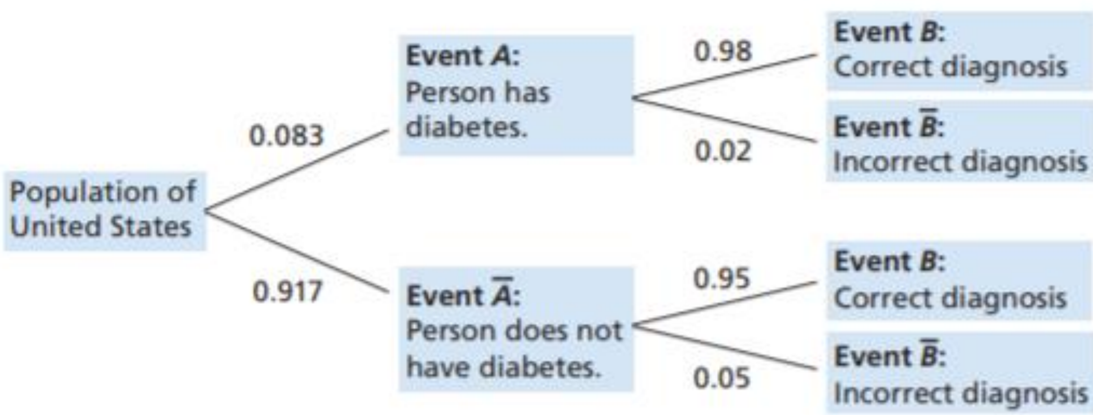
EXAMPLE 4 Solving a Real-Life Problem

The American Diabetes Association estimates that 8.3% of people in the United States have diabetes. Suppose that a medical lab has developed a simple diagnostic test for diabetes that is 98% accurate for people who have the disease and 95% accurate for people who do not have it. The medical lab gives the test to a randomly selected person. What is the probability that the diagnosis is correct?

SOLUTION

Let event A be "person has diabetes" and event B be "correct diagnosis." Notice that the probability of B depends on the occurrence of A, so the events are dependent. When A occurs, $P(B) = 0.98$. When A does not occur, $P(B) = 0.95$.

A probability tree diagram, where the probabilities are given along the branches, can help you see the different ways to obtain a correct diagnosis. Use the complements of events A and B to complete the diagram, where $\overline{A}$ is "person does not have diabetes" and $\overline{B}$ is "incorrect diagnosis." Notice that the probabilities for all branches from the same point must sum to 1.



To find the probability that the diagnosis is correct, follow the branches leading to event B.

$$P(B) = P(A \text{ and } B) + P(\overline{A} \text{ and } B) \quad \text{Use tree diagram.}$$

$$= P(A) \cdot P(B|A) + P(\overline{A}) \cdot P(B|\overline{A}) \quad \text{Probability of dependent events}$$

$$= (0.083)(0.98) + (0.917)(0.95) \quad \text{Substitute.}$$

$$\approx 0.952 \quad \text{Use a calculator.}$$

▶ The probability that the diagnosis is correct is about 0.952, or 95.2%.

4. In Example 4, what is the probability that the diagnosis is *incorrect*?

5. A high school basketball team leads at halftime in 60% of the games in a season. The team wins 80% of the time when they have the halftime lead, but only 10% of the time when they do not. What is the probability that the team wins a particular game during the season?

Section 10-4 Homework #3-17 odd