Oregon State University

COLLEGE OF ENGINEERING

Oregon GNSS Users Group

Precise Point Positioning (PPP) Under Geomagnetic Storm Conditions

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Contents

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2012-15: OSU, B.S.

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2015-17: OSU, M.S.

↓

2017-23: UoN, Ph.D. (United Kingdom)

↓

2023-Present: OSU, Assistant Research Professor

GNSS Positioning Methods

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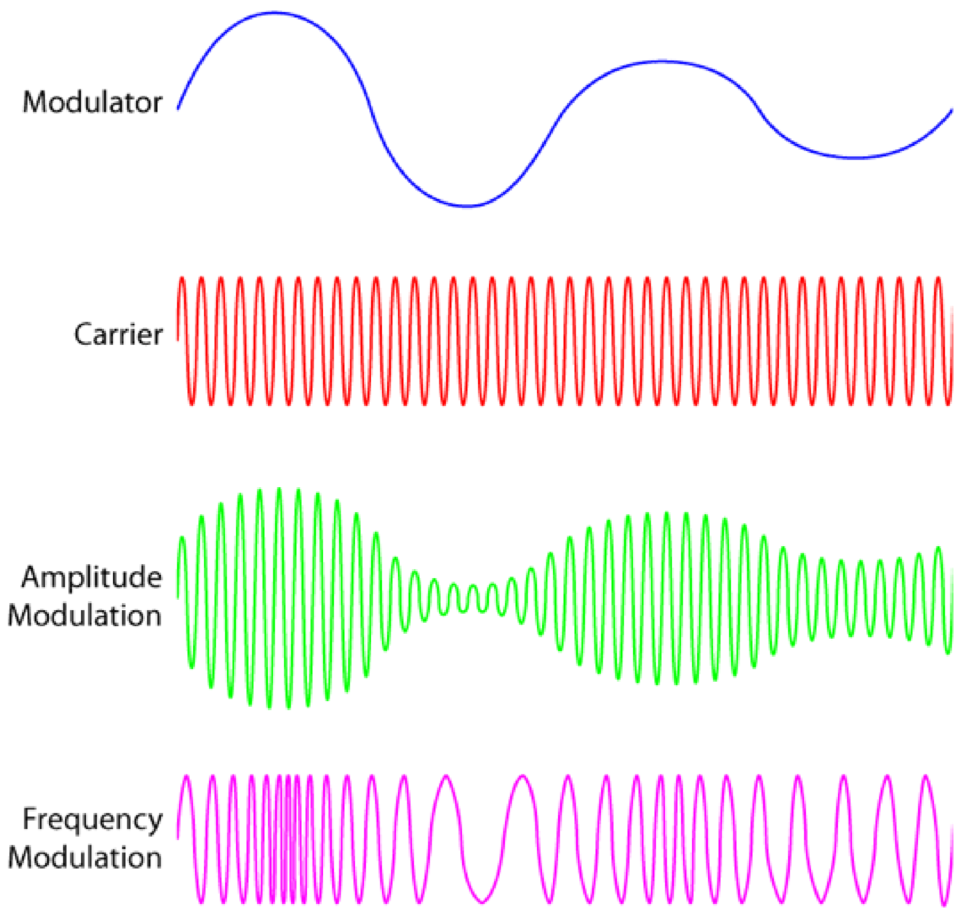
This presentation's topic
Dual-Frequency (DF) Code and Carrier Phase

Positioning concept	Accuracy (1-sigma)	Convergence time	Coverage area
SF SPP	< 10 m	Instantaneous	Global
SF PPP (GIM-based)	1–2 dm	< 10 min	Global
DF PPP (ionosphere-float)	< 1 dm	30 min (static) 60 min (kin.)	Global
Single-baseline (code-based) DGNSS	1–5 m	Instantaneous	Regional/local
Wide area DGNSS	0.5–2m	Instantaneous	Regional
SF RTK-short baseline	< 1 dm	10 min	Local
DF RTK-short baseline	< 1 dm	Instantaneous to few min	Local
Network RTK	< 1 dm	< 10 min	Regional
SF PPP-RTK (precise iono corrections)	< 1 dm	< 10 min	Regional
DF PPP-RTK	< 1 dm	30 min (static)	Global
(ionosphere-float)	< 1 dm	90 min (kin.)	Regional

Springer GNSS Handbook (2017), Chapter 21

GNSS Signals



Source: https://www.quora.com/Whats-the-difference-between-amplitude-modulation-and-phase-modulation

→ Information → GNSS **Code**: Binary Chips [dm-m-level measurement noise]

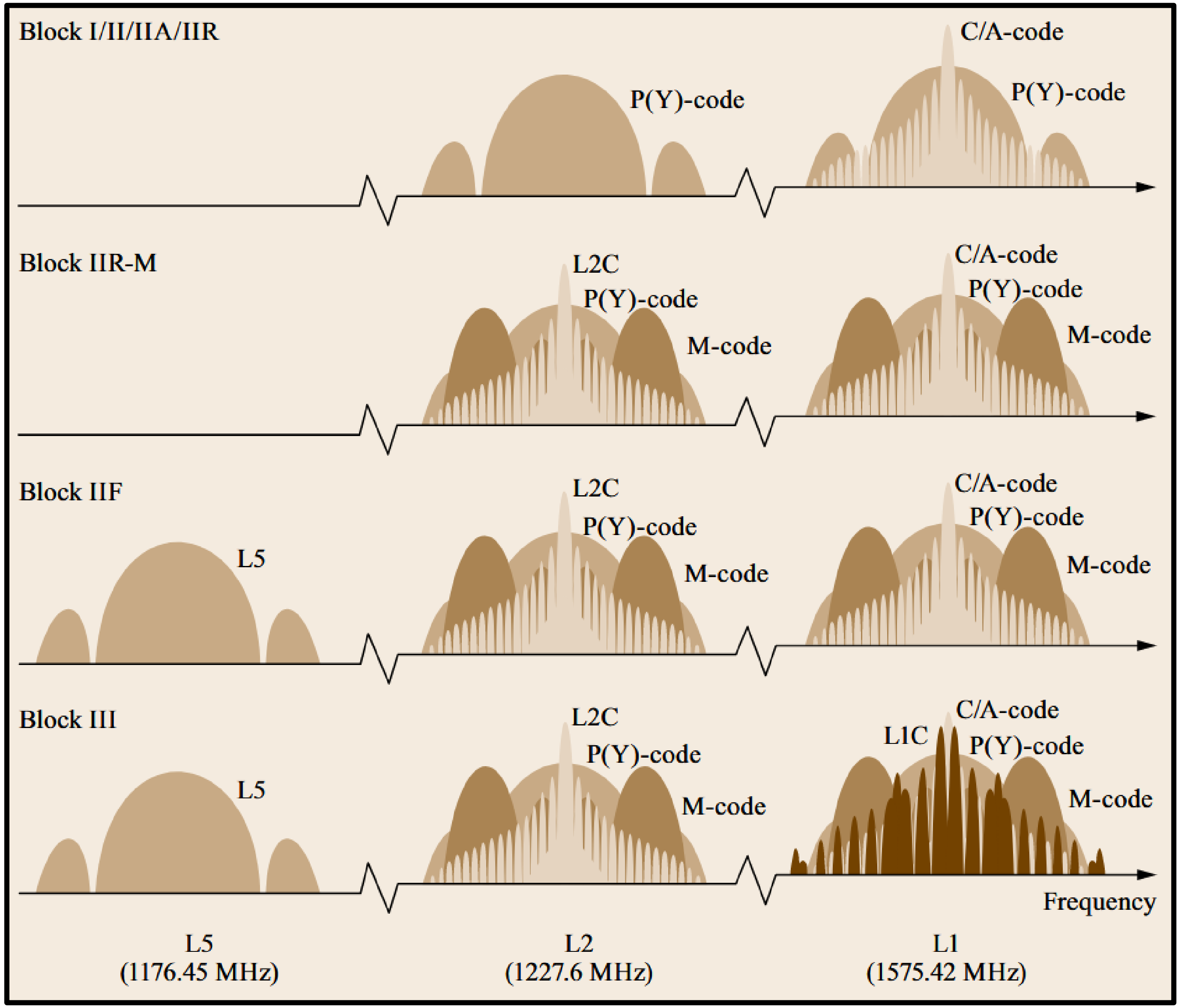
→ Signal that carries no information by itself → GNSS **Carrier**: RHCP wave [mm-level or better measurement noise]

→ AM signal carries amplitude modulated information

→ GNSS: phase modulation BPSK

→ FM signal carries frequency modulated information

GPS Signal Evolution and Current Status



Source: Springer GNSS Handbook (2017), Chapter 7

National PNT Advisory Board

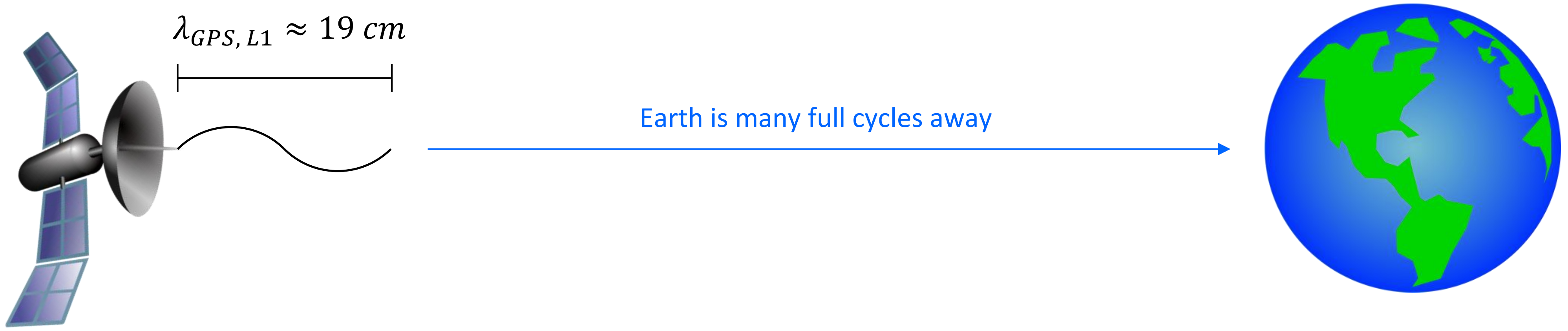
Satellite Block	Quantity	Average Age (yrs.)	Oldest
GPS IIR	6 (4*)	22.8	27.3
GPS IIR-M	7 (1*)	17.1	19.1
GPS IIF	11 (1*)	10.5	14.1
GPS III	6	4.8	5.8

*Not set healthy

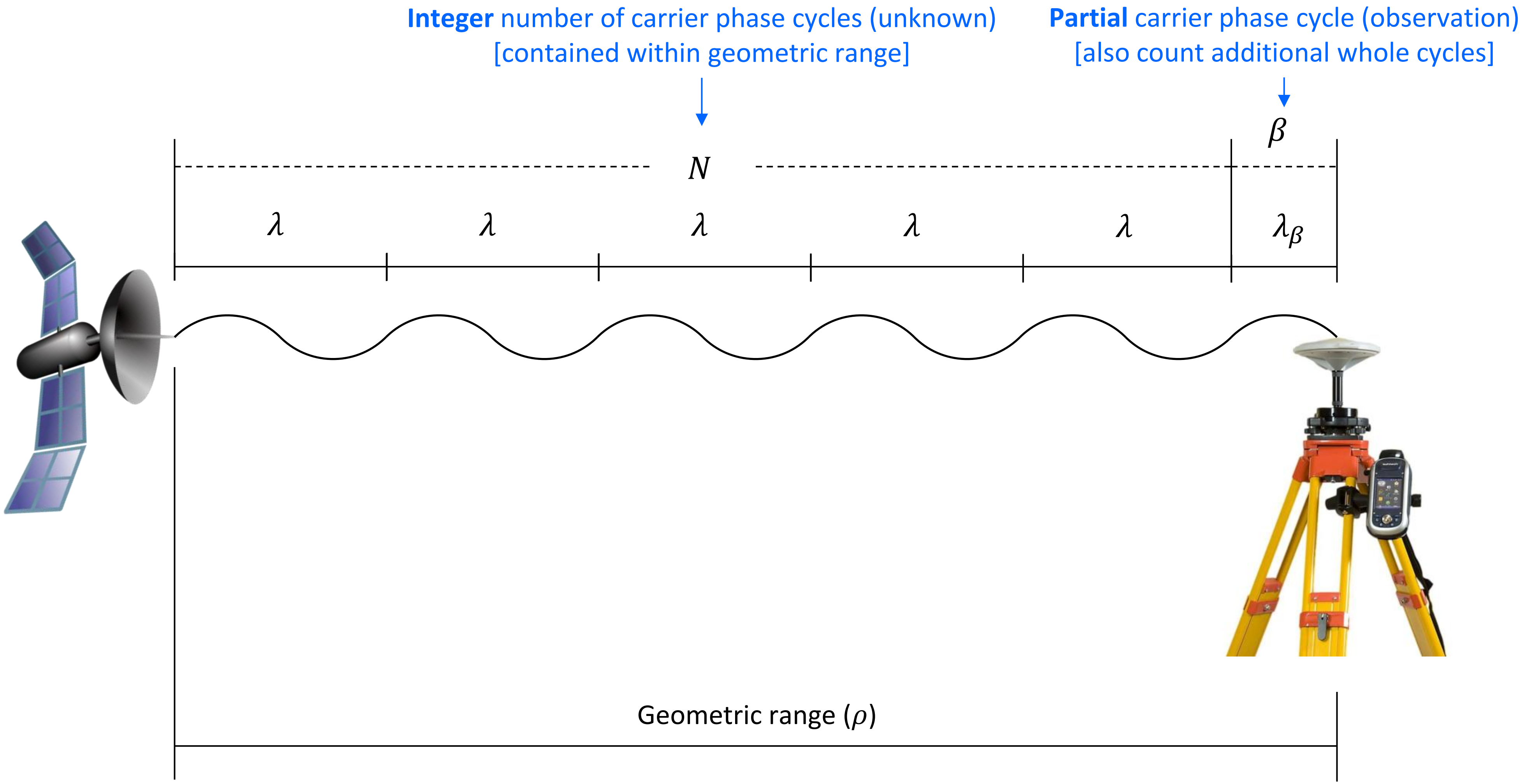
As of: 18 Nov 24

Source: https://www.gps.gov/governance/advisory/meetings/2024-12/delapena.pdf

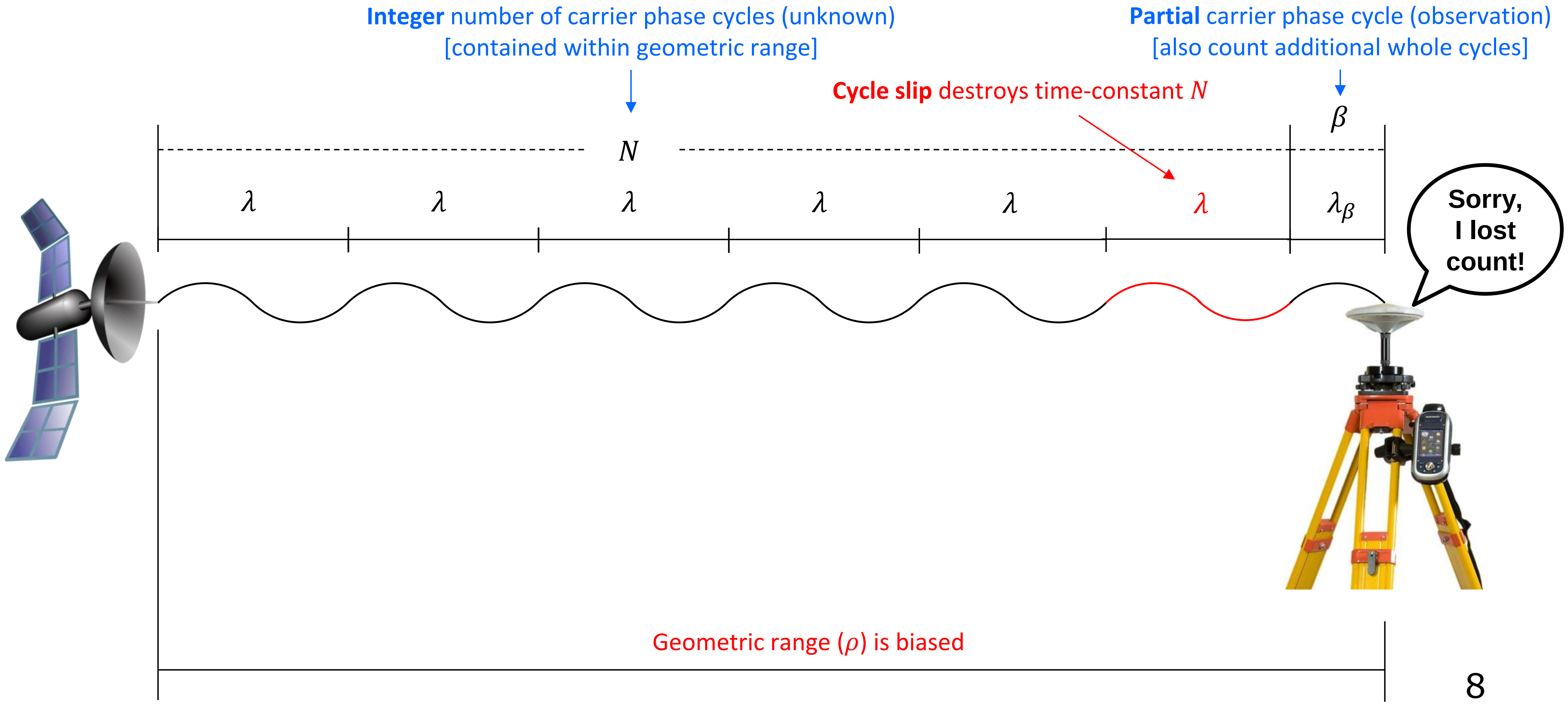
Carrier Phase



Carrier Phase



Carrier Phase



Carrier Phase Cycle Slips

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- Individual satellite detection and repair using standard dual-frequency linear combinations
  - Melbourne–Wübbena Wide-Lane (MWWL) (Melbourne 1985; Wübbena and Hannover 1985)
  - Geometry-free combination (contains ionospheric residual)

$$MWWL = L_{WL} - P_{NL}$$

$$L_{WL} = \frac{L_A \cdot f_A - L_B \cdot f_B}{f_A - f_B}$$

$$P_{NL} = \frac{P_A \cdot f_A + P_B \cdot f_B}{f_A + f_B}$$

✓ Cycle slip sensitive
…but **noisy**

$$L_{GF} = L_A - L_B$$

$$I_{res} = \alpha \left(\frac{1}{f_A^2} - \frac{1}{f_B^2} \right)$$

✓ Cycle slip sensitive
…but sensitive to **ionosphere**

TurboEdit (Blewitt, 1990) → Higher-order time-differencing (Liu, 2011) → Smoothing (Cai et al., 2013)

Carrier Phase Cycle Slips

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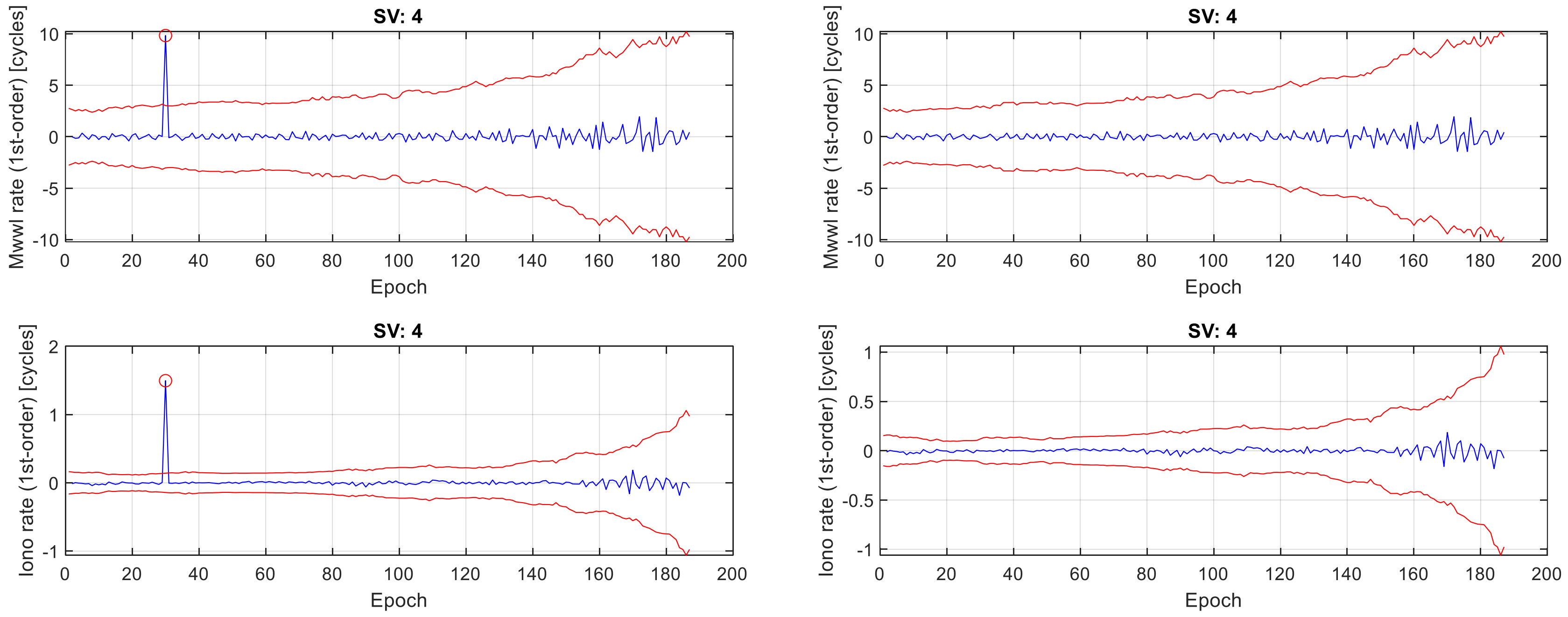
- Individual satellite detection and repair using standard dual-frequency linear combinations
  - Melbourne–Wübbena Wide-Lane (MWWL) (Melbourne 1985; Wübbena and Hannover 1985)
  - Geometry-free combination (contains ionospheric residual)

Before Repair *(Biased measurement)* | **After Repair** *(Unbiased measurement)*



TurboEdit (Blewitt, 1990) → Higher-order time-differencing (Liu, 2011) → Smoothing (Cai et al., 2013)

Model Error Detection

- Evaluate all measurements (satellites) together in a per-epoch "least squares adjustment"
  - Carrier phase model errors are assumed to be cycle slips

Kalman Filter Measurement Update

1. Innovation (Observed-minus-Computed)

$$V_k = Z_k - C_k$$

2. Innovation Covariance

$$S_k = H_k P_{k-1} H^T + R_k$$

3. Kalman Gain

$$K_k = P_{k-1} H^T \cdot S_k^{-1}$$

4. State Update

$$\Delta x_k = K_k V_k$$
$$x_k = x_{k-1} + \Delta x_k$$

5. Covariance Update

$$P_k = (I - K_k H_k) P_{k-1} (I - K_k H_k)^T + K_k R_k K_k^T$$

6. Measurement Residual

$$r_k = (C_k + H_k \Delta x_k) - Z_k$$

7. Residual Covariance

$$V_{res} = H_k P_{k-1} H^T + R_k$$

8. Standardized Residual

$$s_{res} = \left| \frac{V_{res}^{-1} r_k}{\sqrt{diag(V_{res}^{-1})}} \right|$$

Model error?

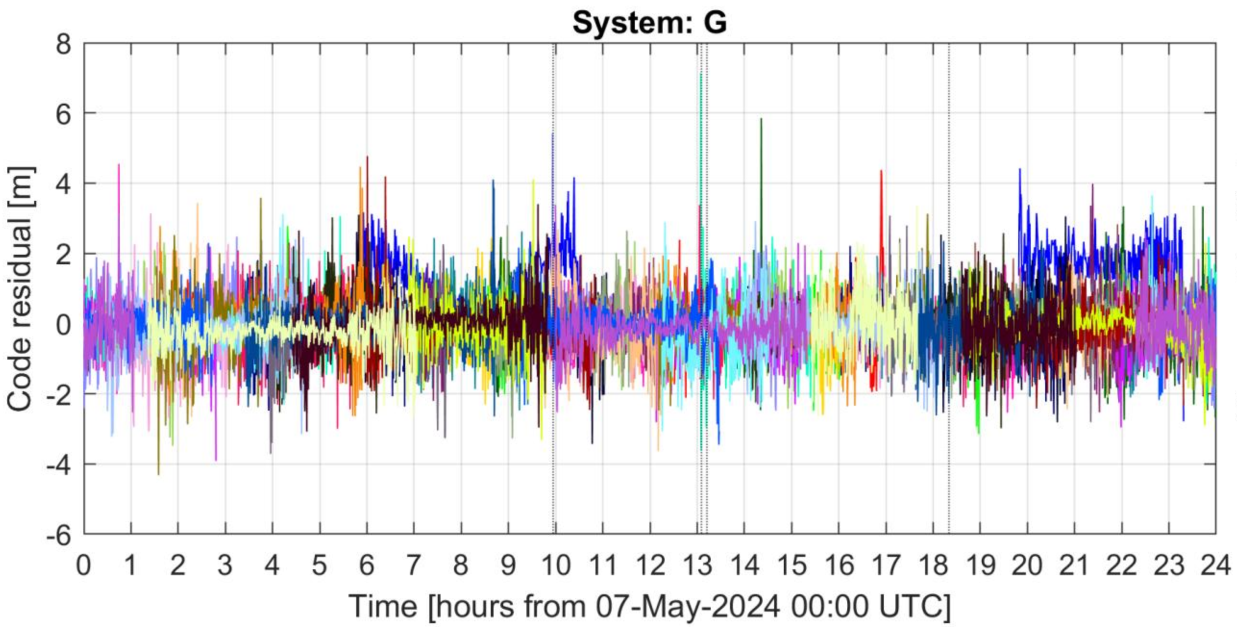
Yes

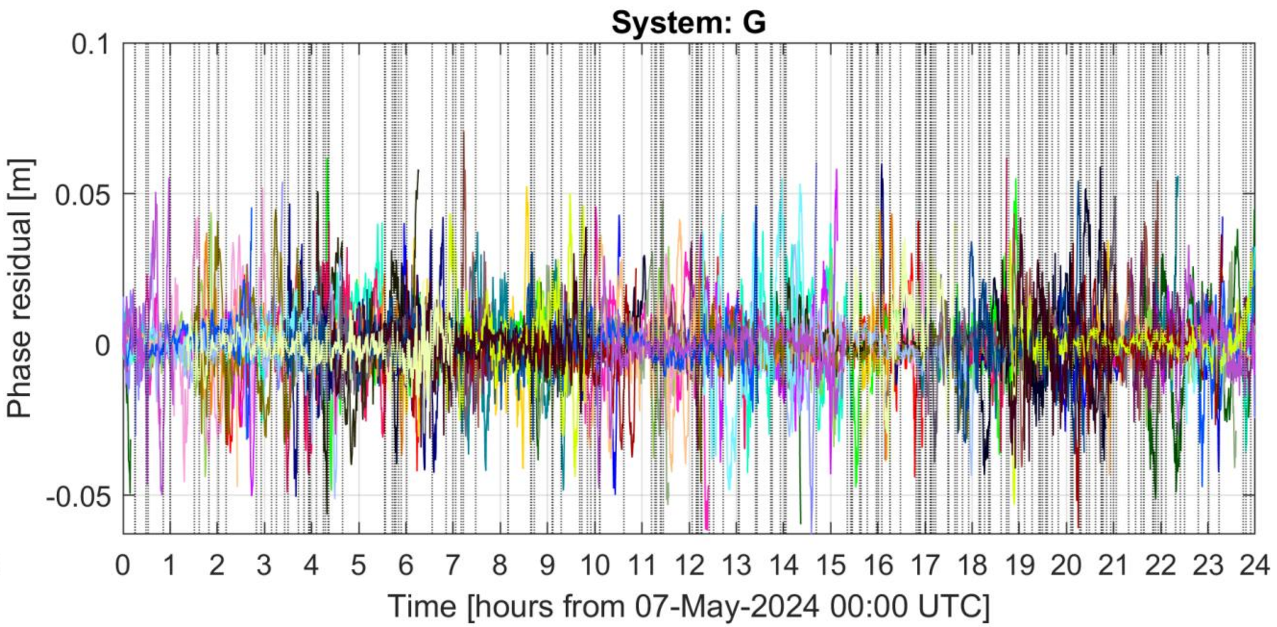
Recompute update

Next epoch

Model Error Detection

- Evaluate all measurements (satellites) together in a per-epoch "least squares adjustment"
  - Carrier phase model errors are assumed to be cycle slips

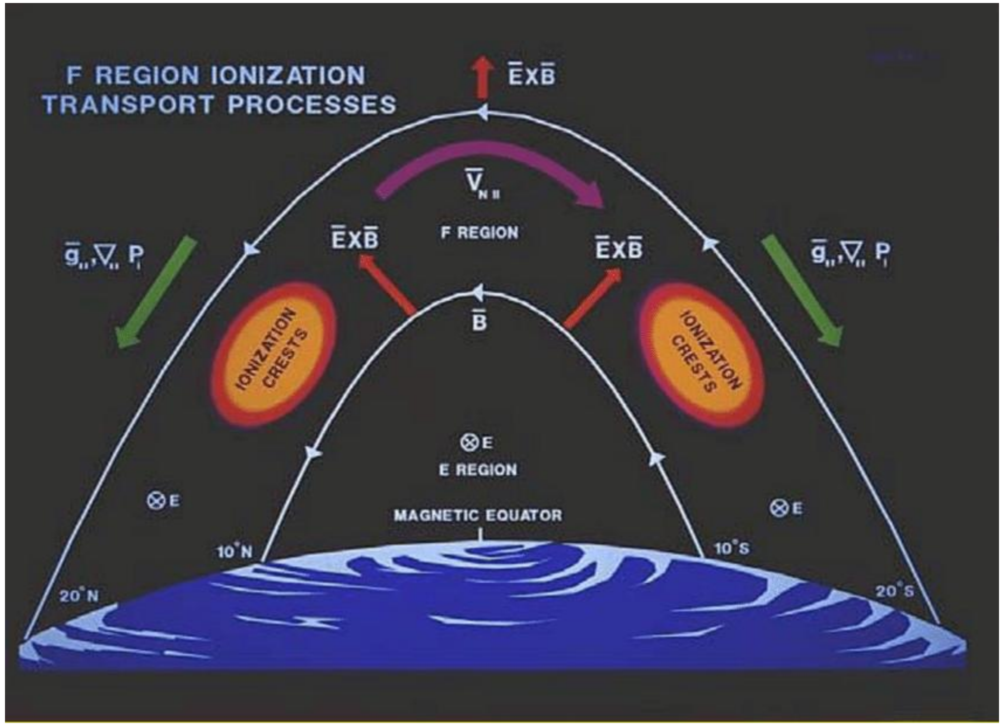




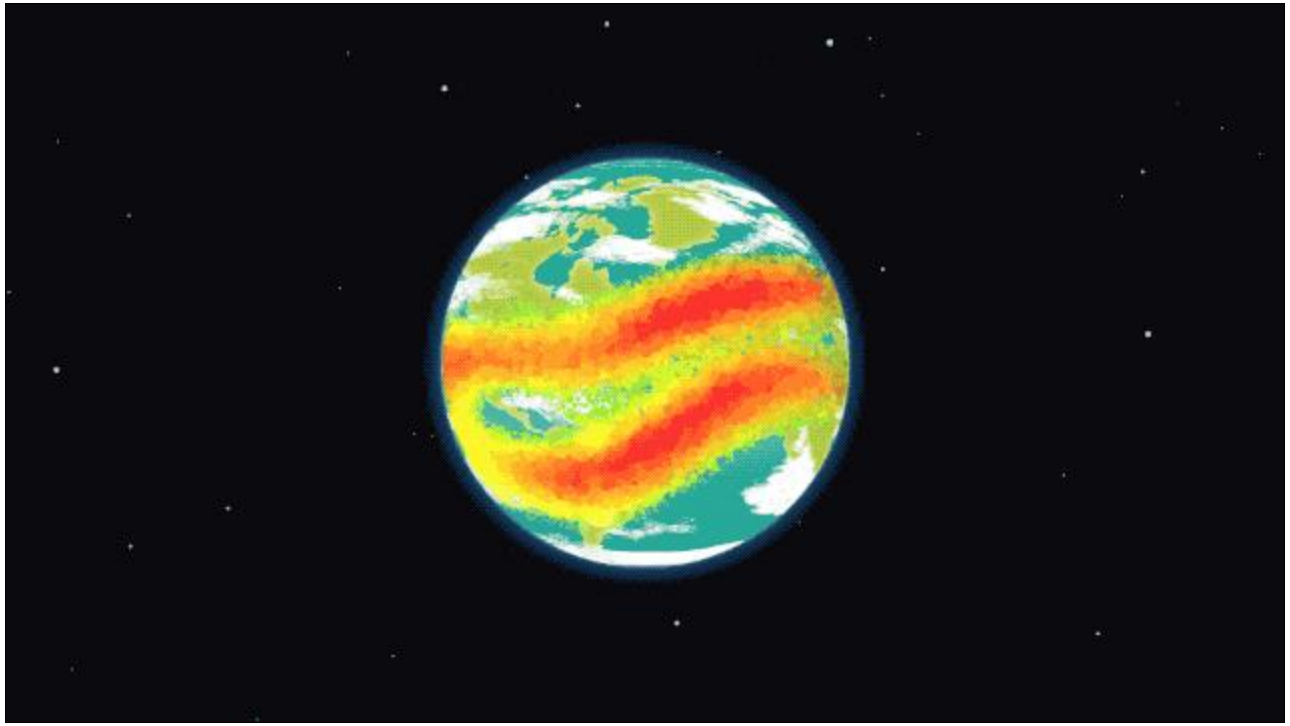
- Mean **code** residual ≈ 50-cm
- Mean **phase** residual ≈ 5-mm

Geomagnetic Activity

- Equatorial Ionospheric Anomaly (EIA)
  - Daily amplification of ionospheric activity at low latitude (equatorial regions), near geomagnetic equator
  - Primarily driven by fountain (Appleton) effect, interaction of free ions with Earth's electric and magnetic fields



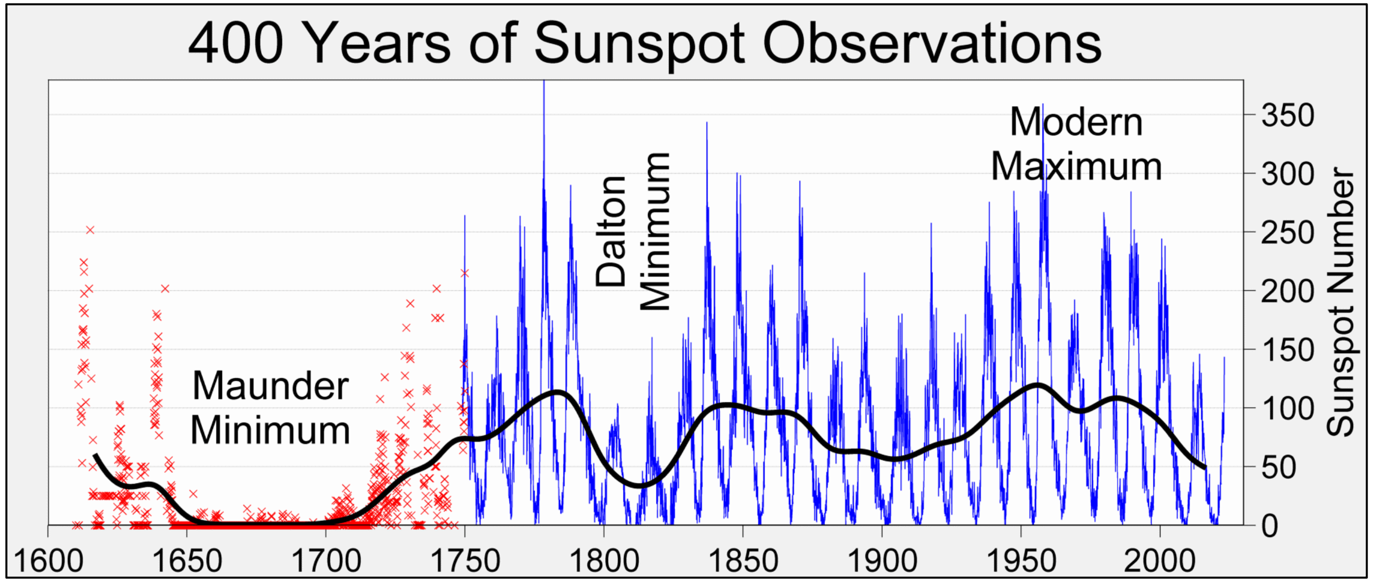
Source: Susi (2017)



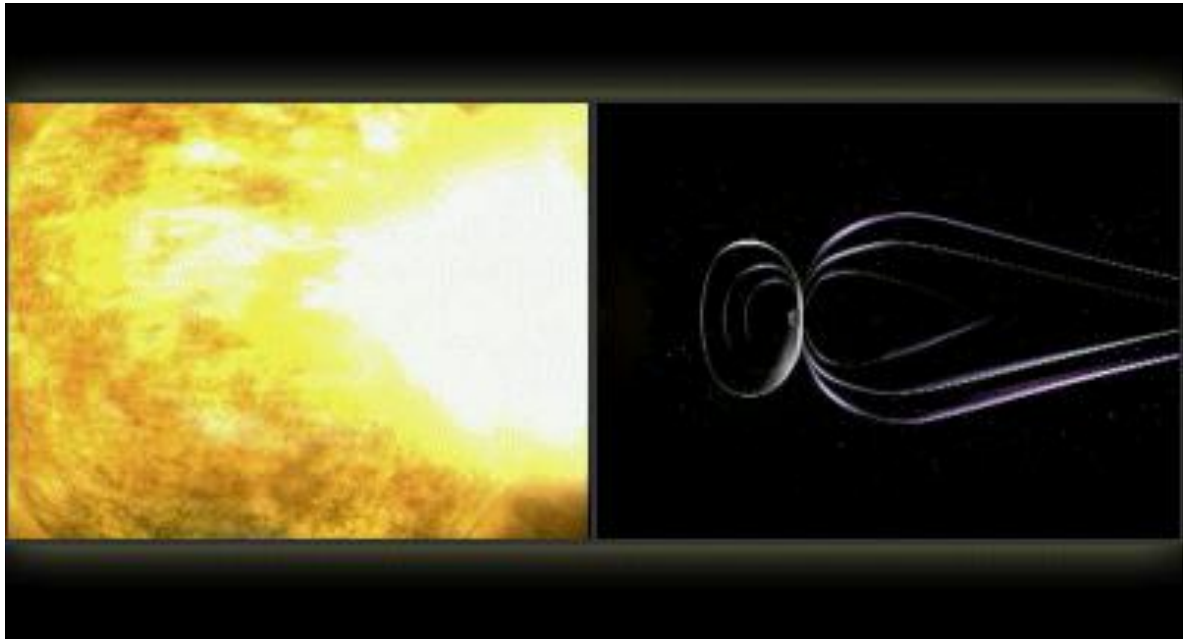
Source: https://gssc.esa.int/navipedia/index.php/Klobuchar_Ionospheric_Model

Geomagnetic Storms

- Correlated with solar activity (11-year cycle between maximums)
  - More free ions and rapidly changing magnetic field lines
  - Challenging for radio-frequency systems, especially GNSS signals (**more cycle slips**)



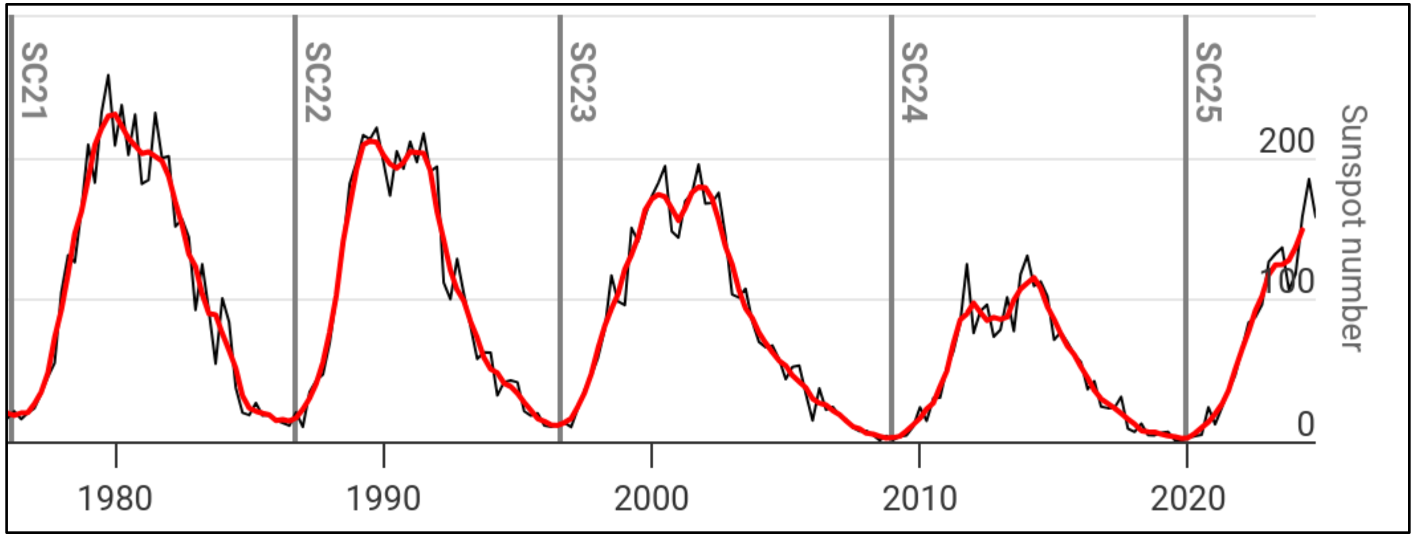
Credit: Robert Rohde



Source: https://www.esa.int/

Recent Geomagnetic Conditions

- Currently, at the peak, or approaching peak, of Solar Cycle (SC) 25
  - Previous cycle was lower activity



Source: https://www.spaceweatherlive.com/en/solar-activity/solar-cycle/historical-solar-cycles.html

Recent Geomagnetic Conditions

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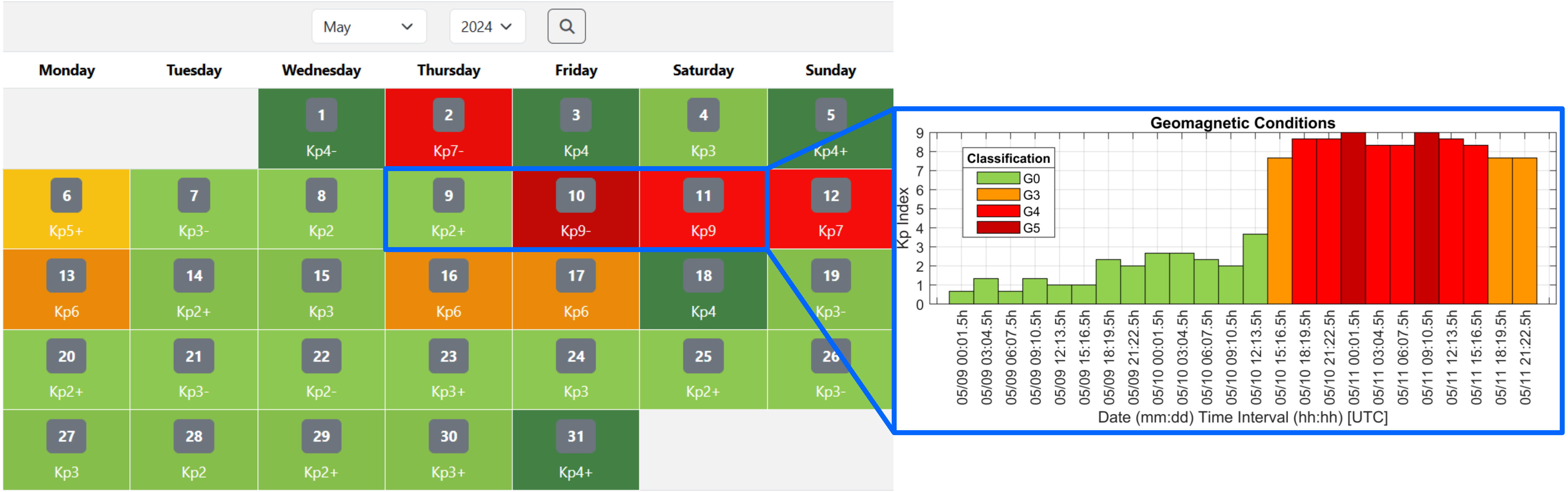
Mother's Day Storm (May 10-13, 2024)



Source: https://www.spaceweatherlive.com/en/archive/2024/05.html

Monitoring Space Weather

- NOAA Space Weather Prediction Center (SWPC), warning two days prior to storm arrival at Earth

Source: www.swpc.noaa.gov

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GNSS Positioning

- Fundamental starting place: **raw measurements** (undifferenced/uncombined observations)
  - Some parameters can be emphasized or eliminated by linear combinations of these observations
  - Problem can become quite complex with multiple: receivers, satellites, frequencies, channels, epochs, + more

GNSS Code and Carrier Phase Observations

$$P_j^s = \rho_r^s + c \cdot \Delta t_r + m_r^s \cdot T^Z + f_1^2/f_j^2 \cdot I_1 + B_{r,j} - B_j^s + \epsilon_j^s$$
$$L_j^s = \rho_r^s + c \cdot \Delta t_r + m_r^s \cdot T^Z - f_1^2/f_j^2 \cdot I_1 + \lambda_j \cdot \left(b_{r,j} - b_j^s\right) + \lambda_j \cdot N_j^s + \varepsilon_j^s$$

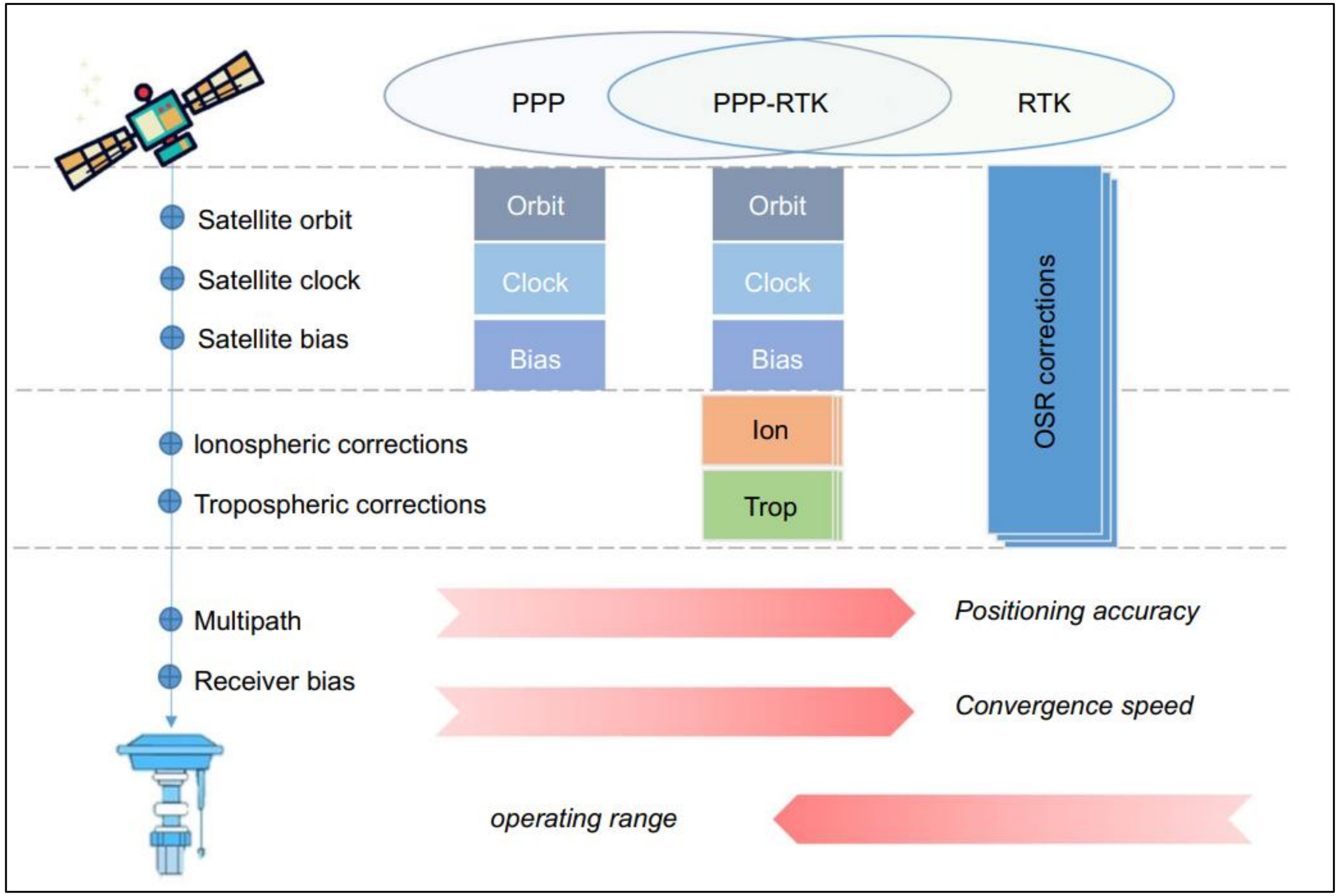
Differential Positioning | *Absolute Positioning* | *Precision Timing* | *Atmospheric Modeling*

3D Distance Formula

$$\rho_r^1 = \sqrt{(x_r - x^1)^2 + (y_r - y^1)^2 + (z_r - z^1)^2}$$
$$\rho_r^2 = \sqrt{(x_r - x^2)^2 + (y_r - y^2)^2 + (z_r - z^2)^2}$$

Precise Point Positioning (PPP)

- Single-receiver "stand-alone" GNSS positioning technique
  - Position accuracy: **mm-level stationary** (static); **cm-level non-stationary** (kinematic)



Source: Li et al. (2022): https://doi.org/10.1186/s43020-022-00089-9

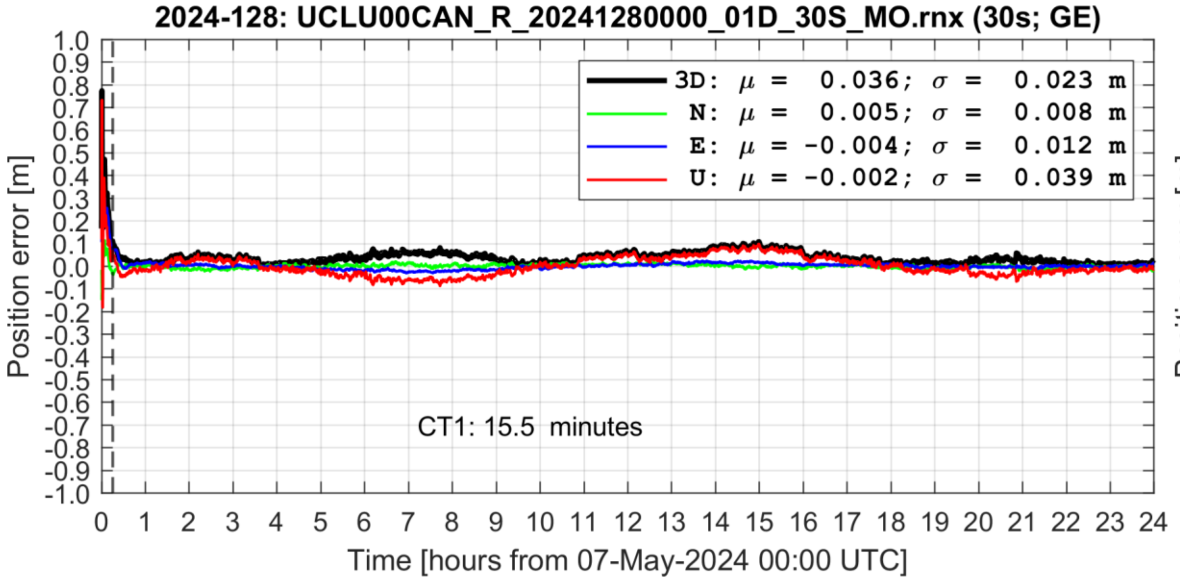
Type		Accuracy	Latency
Broadcast	orbits	~100 cm	real time
	Sat. clocks	~5 ns RMS ~2.5 ns SDev	
Ultra-Rapid (predicted half)	orbits	~5 cm	real time
	Sat. clocks	~3 ns RMS ~1.5 ns SDev	
Ultra-Rapid (observed half)	orbits	~3 cm	3 – 9 hours
	Sat. clocks	~150 ps RMS ~50 ps SDev	
Rapid	orbits	~2.5 cm	17 – 41 hours
	Sat. & Stn. clocks	~75 ps RMS ~25 ps SDev	
Final	orbits	~2.5 cm	12 – 19 days
	Sat. & Stn. clocks	~75 ps RMS ~20 ps SDev	

Source: igs.org (International GNSS Service)

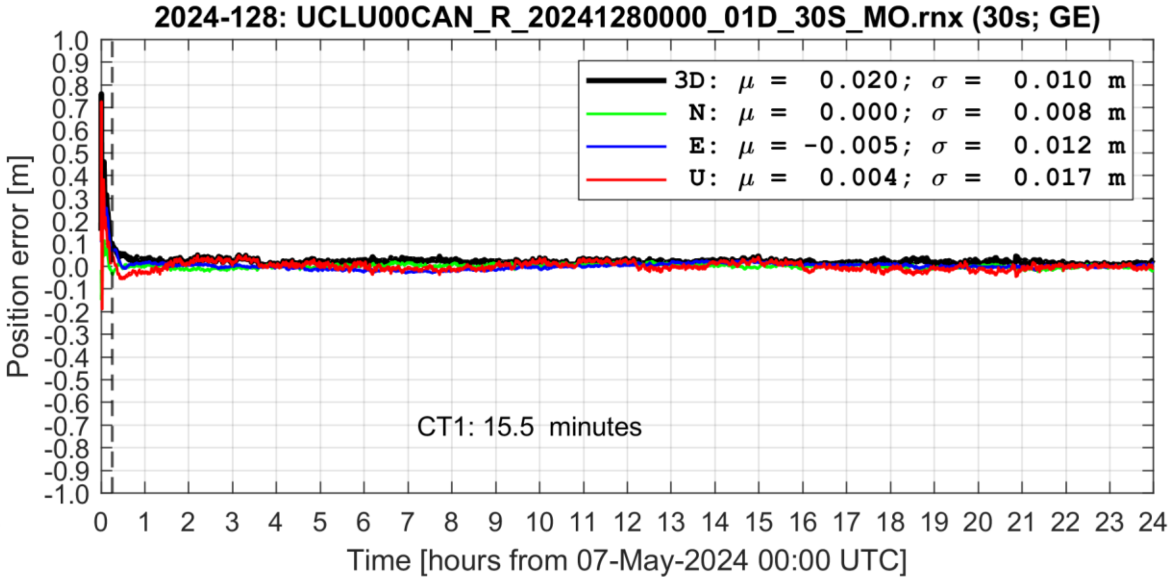
Why PPP?

- Relative positioning requires at least two GNSS receivers
    - Also, eliminates interesting effects to be studied

Exclude Ocean Tide Model

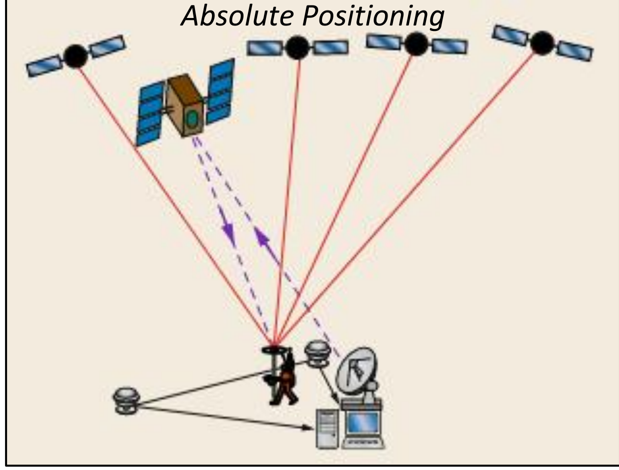


Include Ocean Tide Model



Ocean tide model and data source:
Greene et al. (2024): https://doi.org/10.21105/joss.06018.
Hart-Davis, et al. (2021): [doi:10.5194/essd-13-3869-2021](https://doi.org/10.5194/essd-13-3869-2021).

PPP Observation Model



Springer GNSS Handbook (2017), Ch. 21

Pseudorange Observations at an Epoch

$$P_j^s = \rho_r^s + c \cdot \Delta t_r + m_r^s \cdot T^Z + f_1^2/f_j^2 \cdot I_1 + B_{r,j} - B_j^s + \epsilon_j^s$$

Dual-Frequency Pseudorange
Single-Receiver and Single-Satellite

$$P_j = \rho + c \cdot \Delta t + m \cdot T^Z + f_1^2/f_j^2 \cdot I_1 + B_j^r - B_j^s + \epsilon_j; j = \{1,2\}$$

$$P_1 = \rho + c \cdot \Delta t + m \cdot T^Z + f_1^2/f_1^2 \cdot I_1 + B_1^r - B_1^s + \epsilon_1; j = \{1\}$$
$$P_2 = \rho + c \cdot \Delta t + m \cdot T^Z + f_1^2/f_2^2 \cdot I_1 + B_2^r - B_2^s + \epsilon_2; j = \{2\}$$

Ionosphere-Free Linear Combination

$$P_{IF} = \alpha_{IF} \cdot P_1 + \beta_{IF} \cdot P_2; \alpha_{IF} = 1 - \beta_{IF}$$
$$\alpha_{IF} = \frac{f_1^2}{f_1^2 - f_2^2}; \beta_{IF} = \frac{f_2^2}{f_1^2 - f_2^2}$$

PPP Observation Model

Group Frequency Independent Terms

$$G = \rho + c \cdot \Delta t + m \cdot T^Z$$

Dual-Frequency Ionosphere-free Pseudorange

$$\alpha_{IF} \cdot P_1 = \alpha_{IF} \cdot (G + f_1^2/f_1^2 \cdot I_1 + B_1^r - B_1^s + \epsilon_1)$$
$$\beta_{IF} \cdot P_2 = \beta_{IF} \cdot (G + f_1^2/f_2^2 \cdot I_1 + B_2^r - B_2^s + \epsilon_2)$$
$$\vdots$$
$$\alpha_{IF} \cdot P_1 = +\frac{f_1^2}{f_1^2 - f_2^2} \cdot (G + f_1^2/f_1^2 \cdot I_1 + B_1^r - B_1^s + \epsilon_1)$$
$$\beta_{IF} \cdot P_2 = -\frac{f_2^2}{f_1^2 - f_2^2} \cdot (G + f_1^2/f_2^2 \cdot I_1 + B_2^r - B_2^s + \epsilon_2)$$

Geometry

$$G_{IF} = \alpha_{IF} \cdot G + \beta_{IF} \cdot G$$
$$G_{IF} = G \cdot (\alpha_{IF} + \beta_{IF})$$
$$G_{IF} = G \cdot \left(\frac{f_1^2}{f_1^2 - f_2^2} - \frac{f_2^2}{f_1^2 - f_2^2}\right)$$
$$G_{IF} = G \cdot (1)$$
$$G_{IF} = G$$

✓*Geometry-preserving*

Ionosphere

$$I_{IF} = \alpha_{IF} \cdot (f_1^2/f_1^2 \cdot I_1) + \beta_{IF} \cdot (f_1^2/f_2^2 \cdot I_1)$$
$$I_{IF} = \frac{f_1^2}{f_1^2 - f_2^2}\left(\frac{f_1^2}{f_1^2} \cdot I_1\right) - \frac{f_2^2}{f_1^2 - f_2^2} \cdot \left(\frac{f_1^2}{f_2^2} \cdot I_1\right)$$
$$I_{IF} = \frac{f_1^2}{f_1^2 - f_2^2} \cdot I_1 - \frac{f_1^2}{f_1^2 - f_2^2} \cdot I_1$$
$$I_{IF} = \alpha_{IF} \cdot I_1 - \alpha_{IF} \cdot I_1$$
$$I_{IF} = 0$$

✓*Ionosphere-free*

PPP Observation Model

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Ionosphere-Free Pseudorange

$$P_{IF} = \alpha_{IF} \cdot P_1 + \beta_{IF} \cdot P_2; \alpha_{IF} = 1 - \beta_{IF}$$
$$P_{IF} = G + (\alpha_{IF} \cdot B_1^r + \beta_{IF} \cdot B_2^r) - (\alpha_{IF} \cdot B_1^s + \beta_{IF} \cdot B_2^s) + \epsilon_{IF}$$
$$P_{IF} = G + DCB_{r,IF} - DCB_{IF}^s + \epsilon_{IF}$$
$$P_{IF} = G + DCB_{r,IF} - DCB_{IF}^s + \epsilon_{IF}$$
$$\boldsymbol{P_{IF} = \rho + c \cdot \Delta t + m \cdot T^Z + \epsilon_{IF}}$$

Ionosphere-Free Carrier Phase

$$L_{IF} = \alpha_{IF} \cdot L_1 + \beta_{IF} \cdot L_2$$
$$\boldsymbol{L_{IF} = \rho + c \cdot \Delta t + m \cdot T^Z + \lambda_{IF} \cdot N_{IF} + \varepsilon_{IF}}$$

Standard dual-frequency PPP is based on these expressions

Model is sensitive to cycle slips which affect the ionosphere-free (non-integer) ambiguity

Typical PPP Configuration

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Error Source	Product	Rate
Satellite orbit	SP3	5-minute
Satellite clock	CLK	30-second
Satellite code & phase bias	BIA	Daily
Earth orientation	ERP	Daily
Phase center offset (PCO) & variation (PCV)	igs20.atx	[N/A]

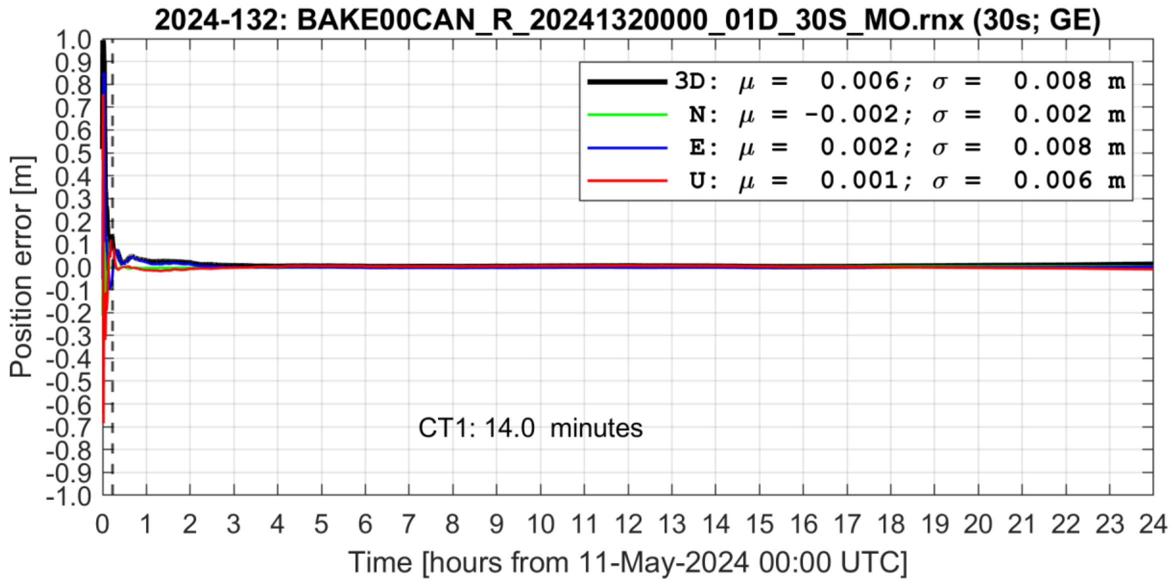
Unknown Parameters	Initial Noise [m]	Process Noise $[m/\sqrt{s}]$
Position	1e2	0 (static) 1e2 (kinematic)
Receiver clock (WLS initialization)	1e5	1e3
Troposphere (ZWD)	0.1	3e-5
Ambiguity	1e5	0
System time offset	1e2	1e-5

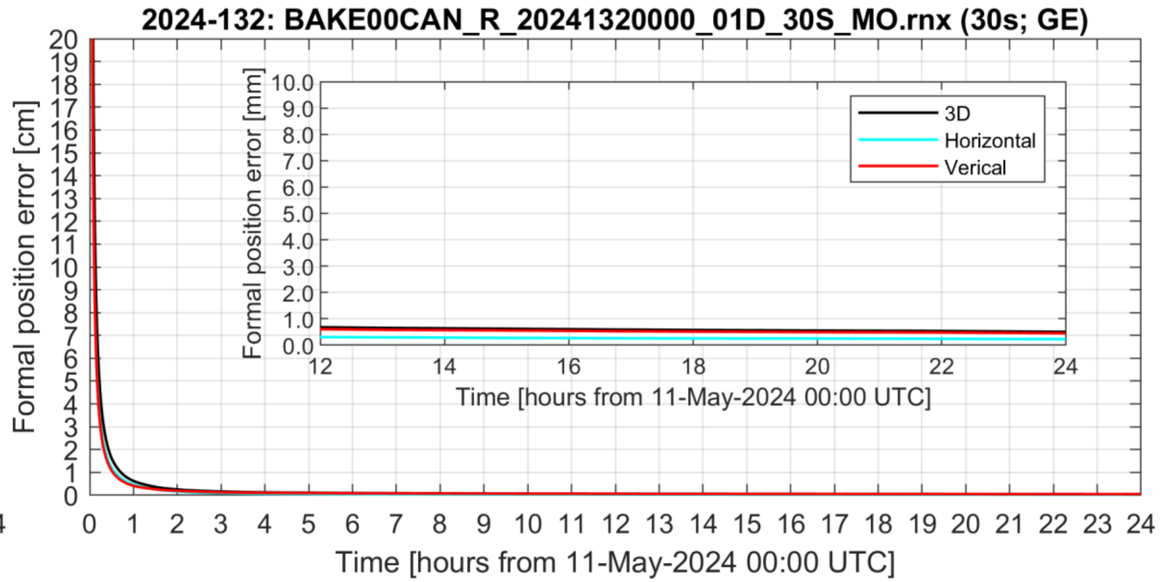
Pre-Processing	Method(s)	Evaluation(s)
Elevation mask	Constant	$Elev \geq 7.5\ deg$
Cycle slip detection and repair	MWWL and IONO	$MWWL > s_M \cdot \sigma$ $IONO > s_I \cdot \sigma$
Ambiguity reinitializations	MWWL & failed repairs	$MWWL > s \cdot \sigma$ Satellites in epoch

Measurements and Model Components	Value/Type	Notes
Functional model	Dual-frequency ionosphere-free	Eliminates 1st-order ionospheric delay
Constellations	GPS (G), GLONASS (R), Galileo (E), BeiDou (C)	Default: GE
Ref. noise (σ_{ref})	At zenith (best-case)	$\sigma_{code} = 60\ cm$ $\sigma_{phas} = 2\ mm$
Stochastic model	Satellite elevation	$\sigma_{ref}/\sin(\text{Elev})$
Error propagation	Amplified by combination	G: $\sigma_{IF} \approx 3 \cdot \sigma_{raw}$ E: $\sigma_{IF} \approx 2.6 \cdot \sigma_{raw}$

Static PPP (Storm)

- Strong static model **does not respond** to **extreme** geomagnetic storm conditions
  - Maintains mm-level position precision after initial convergence interval





Kinematic PPP (Calm)

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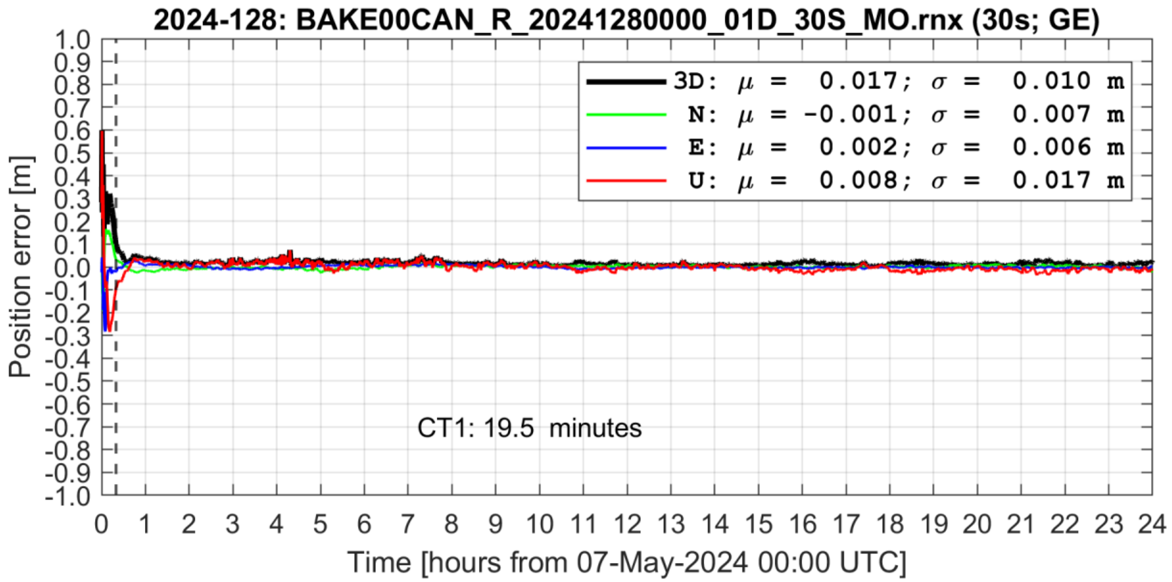
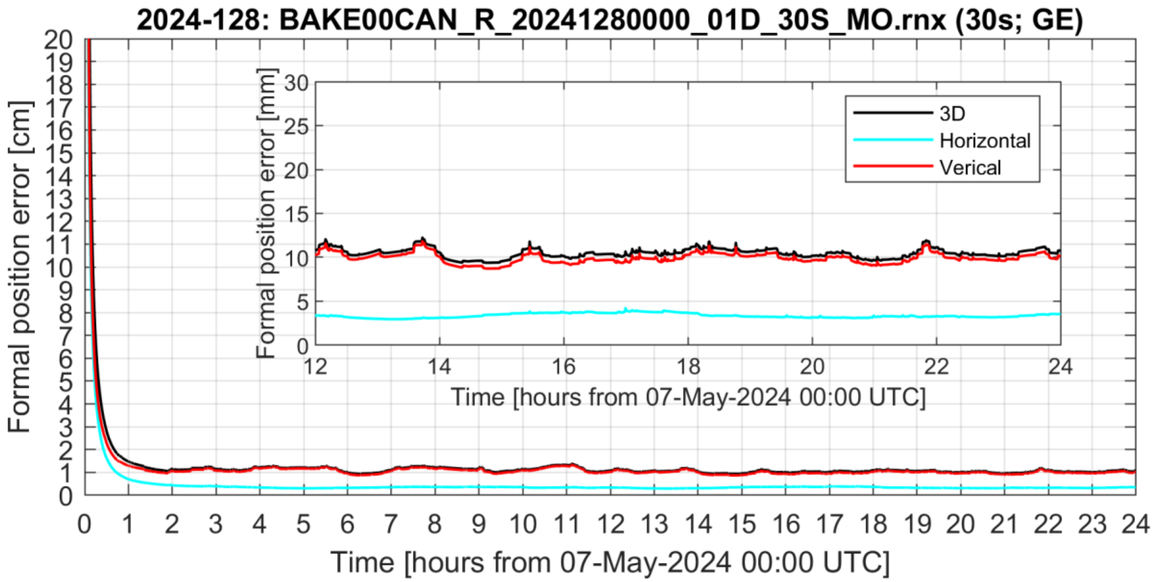
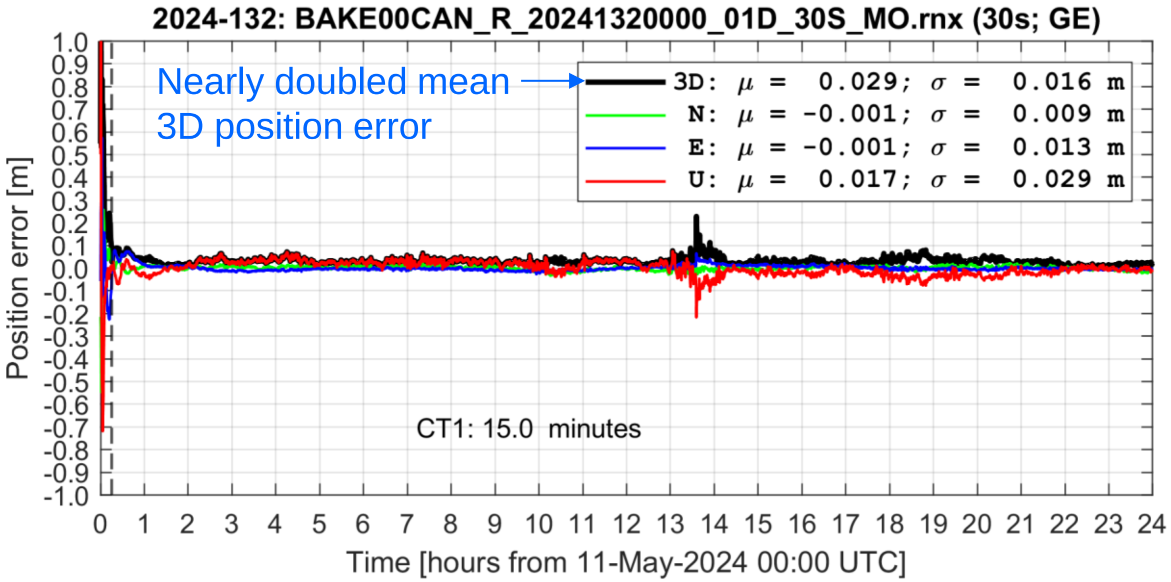
- Weaker kinematic model **maintains cm-level accuracy** under calm geomagnetic conditions
  - Mean 3D position error equal to 1.7-cm a few days prior to storm

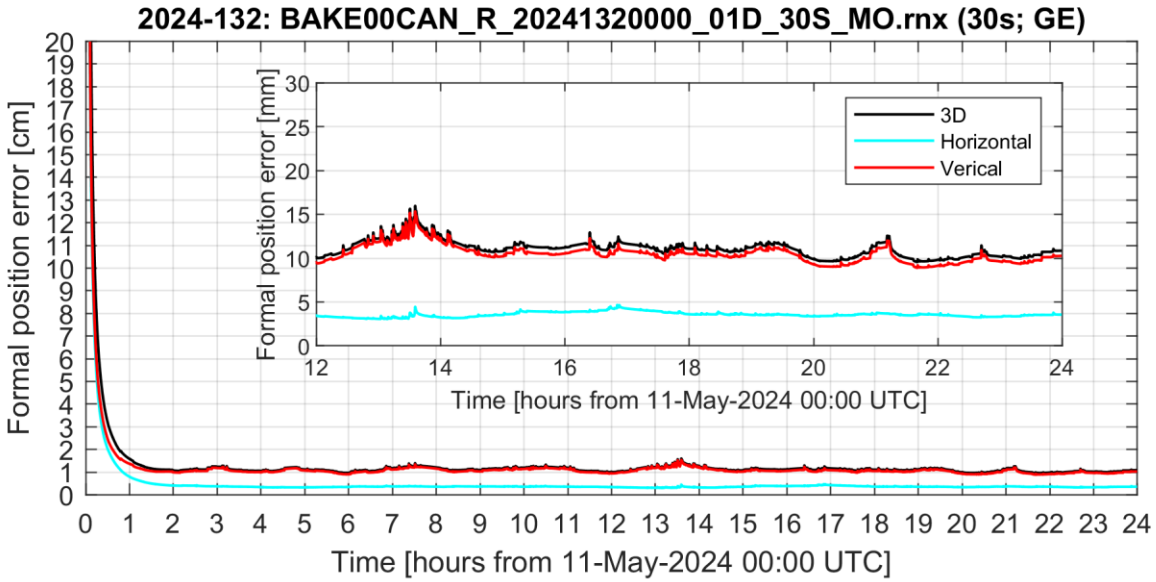




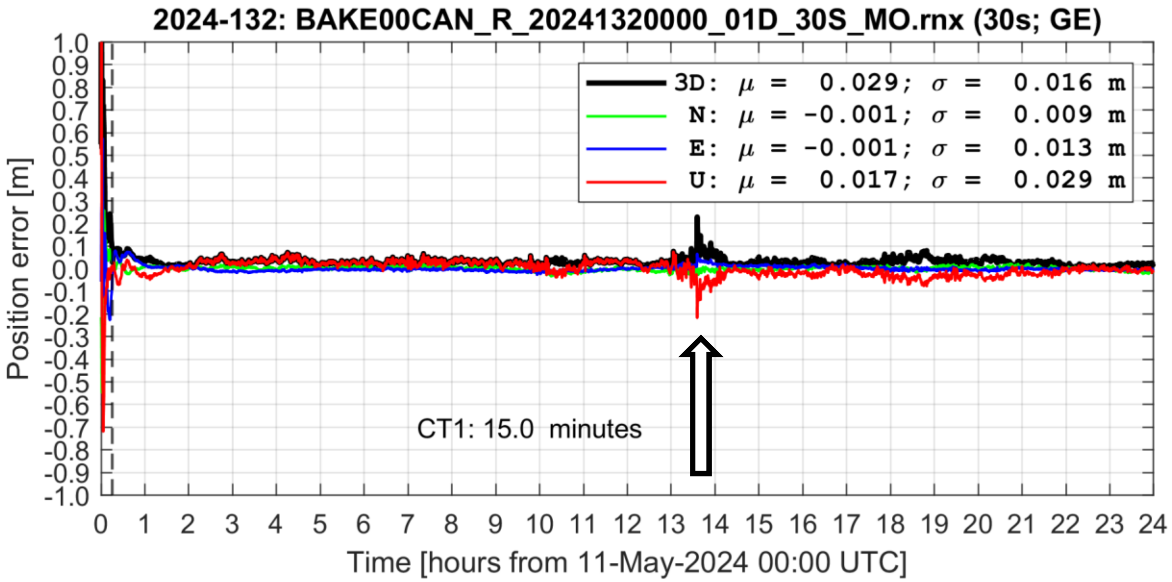
Kinematic PPP (Storm)

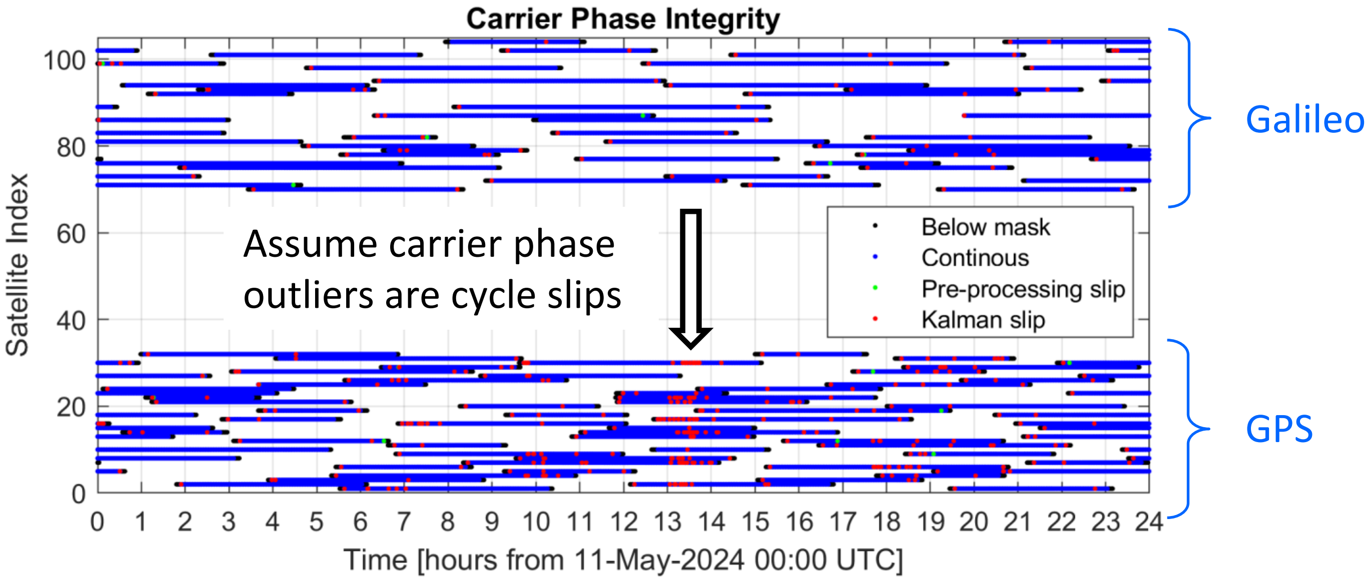
- Weaker kinematic model **responds** to **extreme** geomagnetic storm conditions
  - Position error amplification up to a few decimeters (mainly vertical component)





Kinematic PPP Analysis



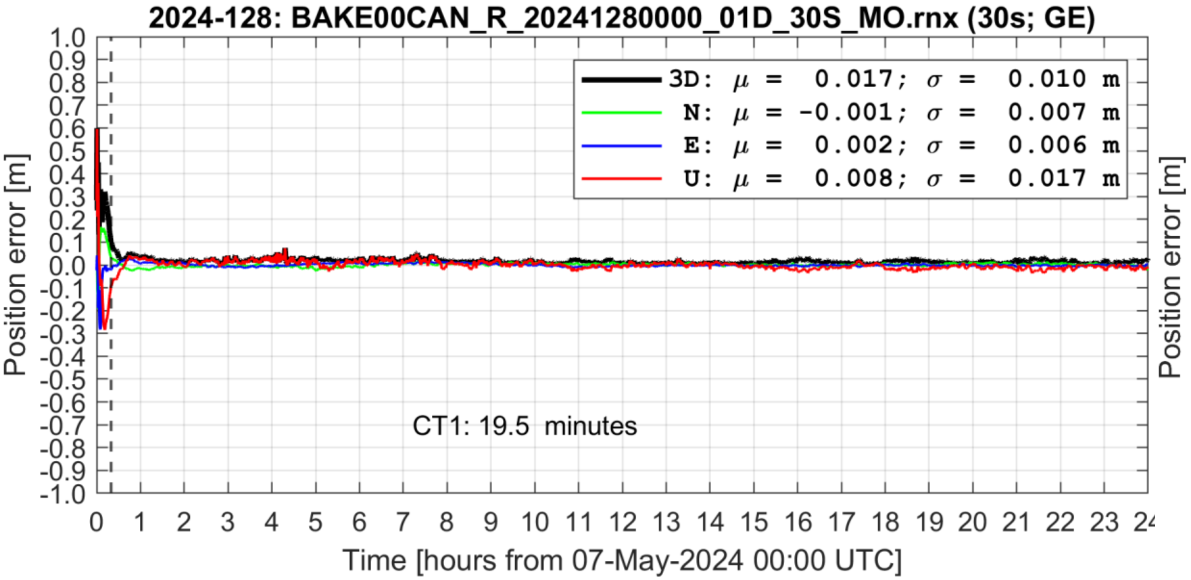


Remarks:

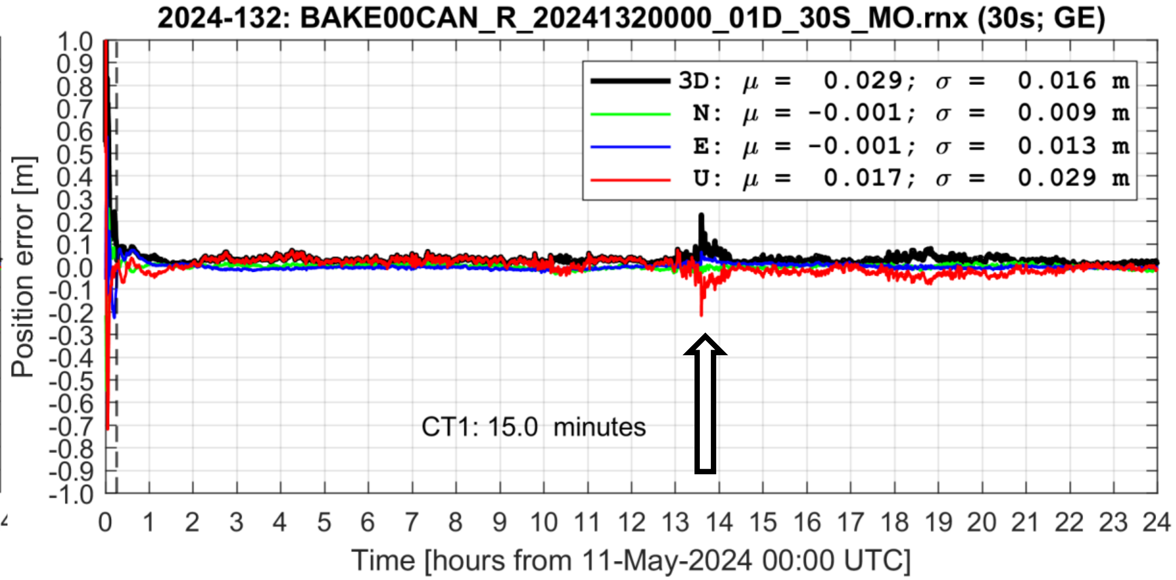
- Many GPS satellite ambiguities are reinitialized (red markers) while position error is amplified.
- Fewer Galileo satellites are reinitialized in the same interval.
- Carrier phase is less reliable despite observations made at high-elevation (mid-arc).

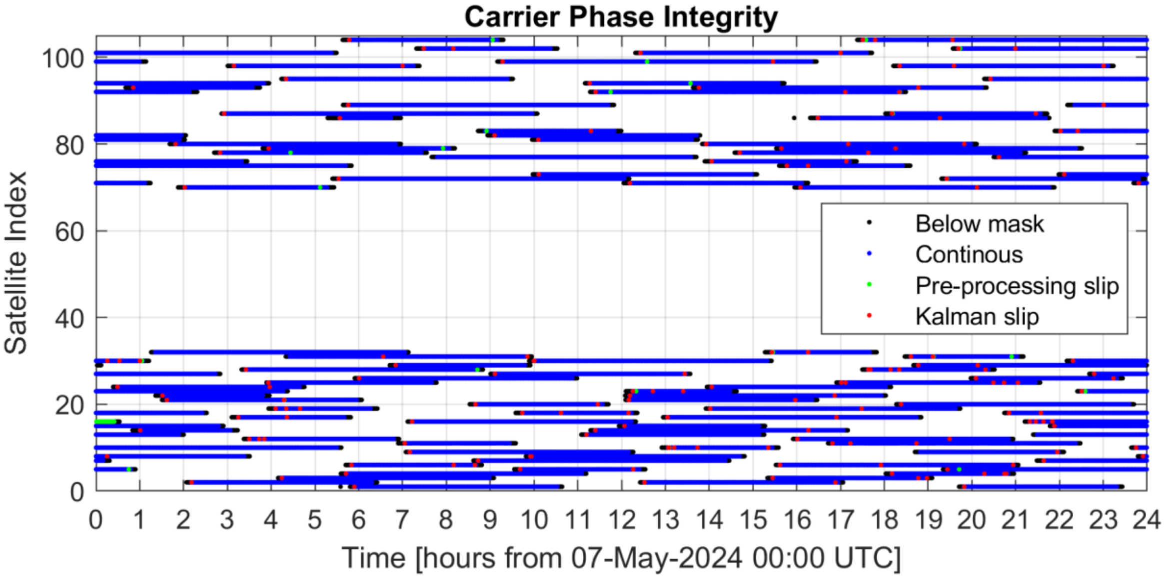
Kinematic PPP Analysis

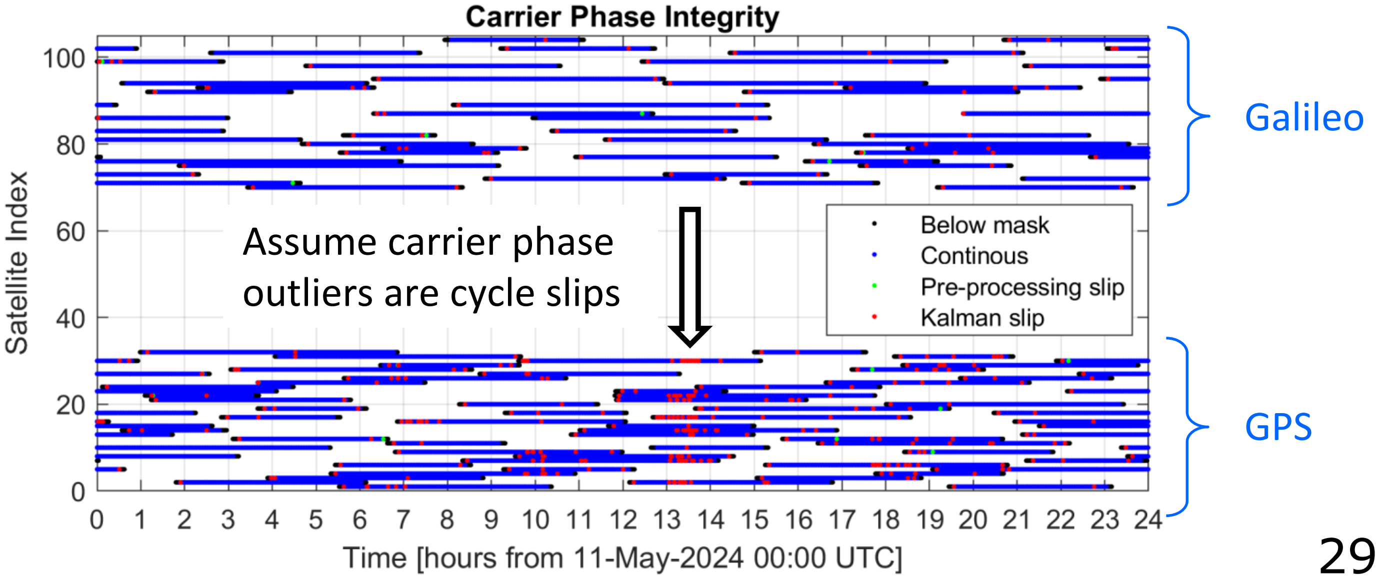
Calm



Extreme

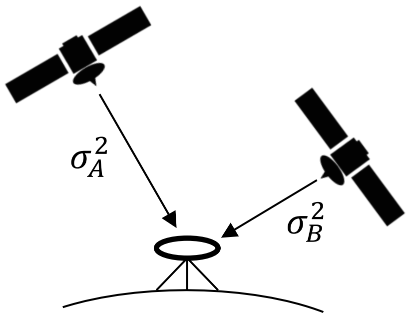






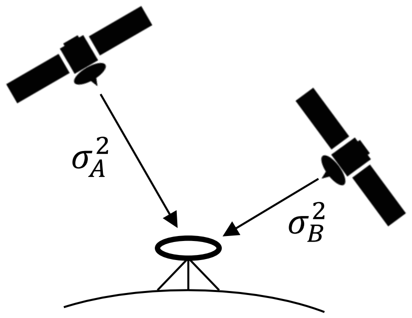
Stochastic Modeling

Constant



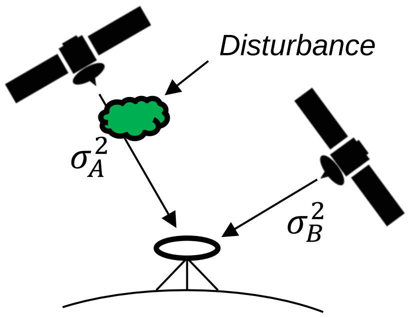
$$\boldsymbol{\sigma_A^2 = \sigma_B^2}$$

Elevation



$$\boldsymbol{\sigma_A^2 < \sigma_B^2}$$

Receiver Tracking Loop



$$\boldsymbol{\sigma_A^2 > \sigma_B^2}$$

Weighting

$$W = \begin{bmatrix} 1/\sigma_A^2 & 0 \\ 0 & 1/\sigma_B^2 \end{bmatrix}$$

$$x = (A^T W A)^{-1} A^T W y$$

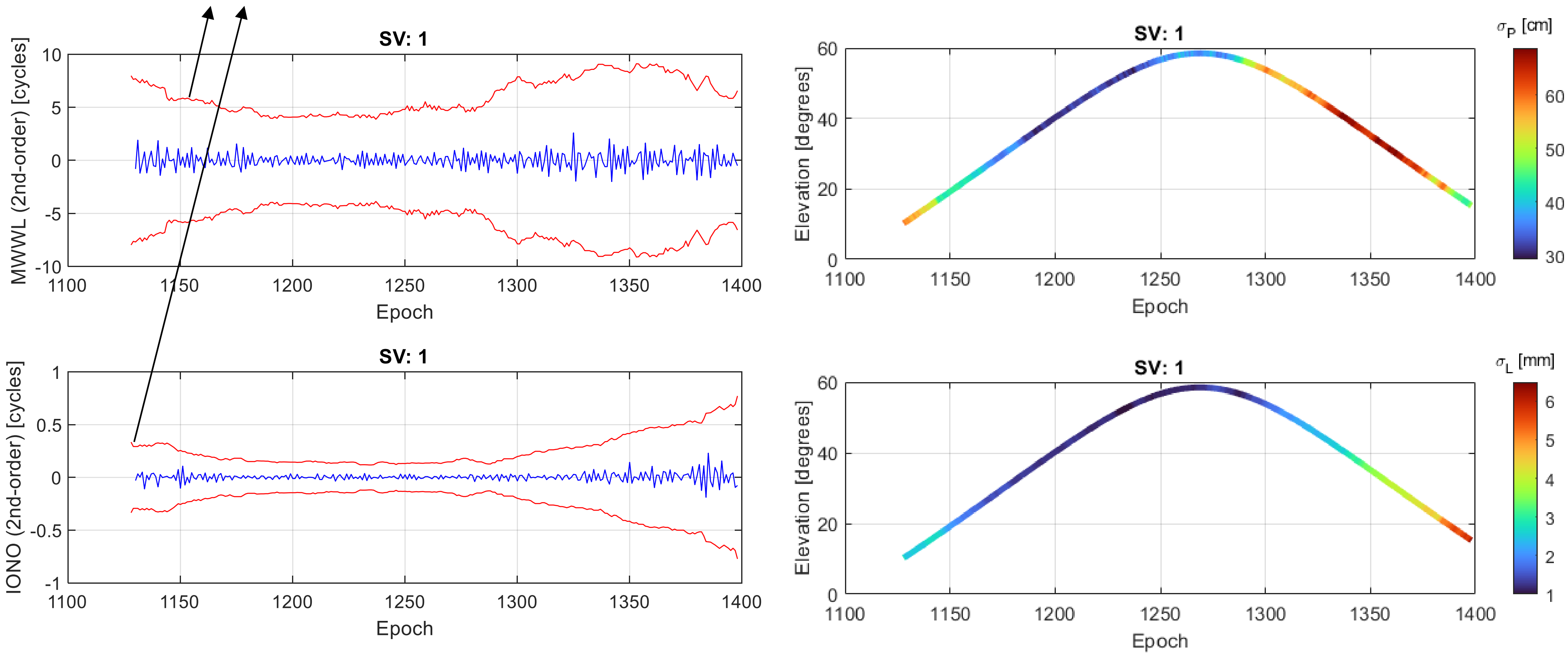
Model weighted by measurement precision

Complexity

Specialized Equipment

Data-Driven Stochastic Model

Cycle Slip Parameter Noise → *Model* → **Measurement Noise**

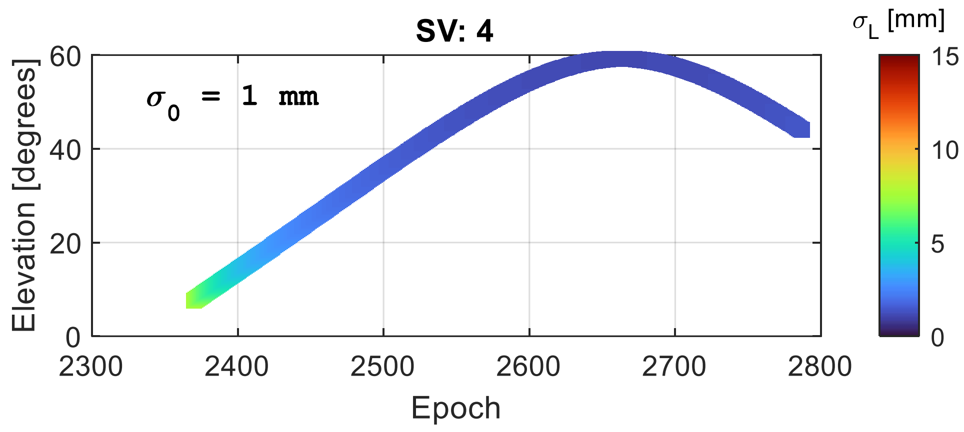


Stochastic Model Comparison

- Standard vs proposed method

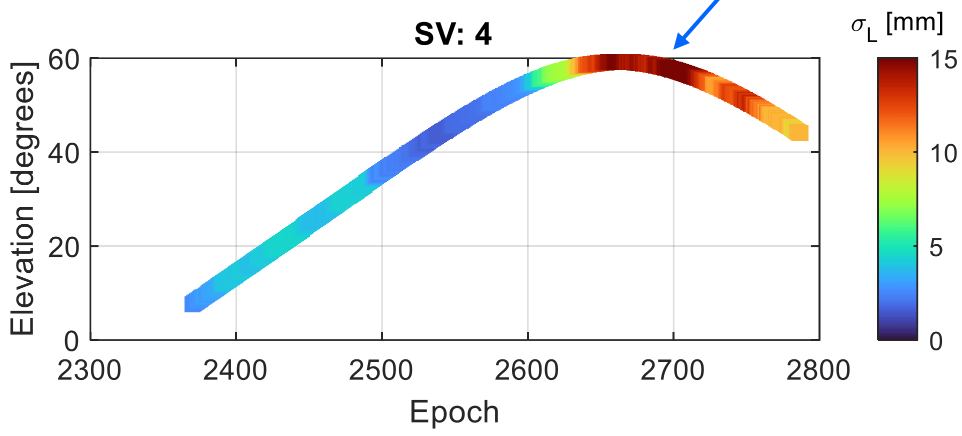
Large measurement noise despite satellite observed at high elevation

Elevation-Based



$$\sigma_A^2 = \sigma_0^2 \cdot \left({}^{1}/_{sin(E_A)}\right)^2$$
$$\sigma_B^2 = \sigma_0^2 \cdot \left({}^{1}/_{sin(E_B)}\right)^2$$

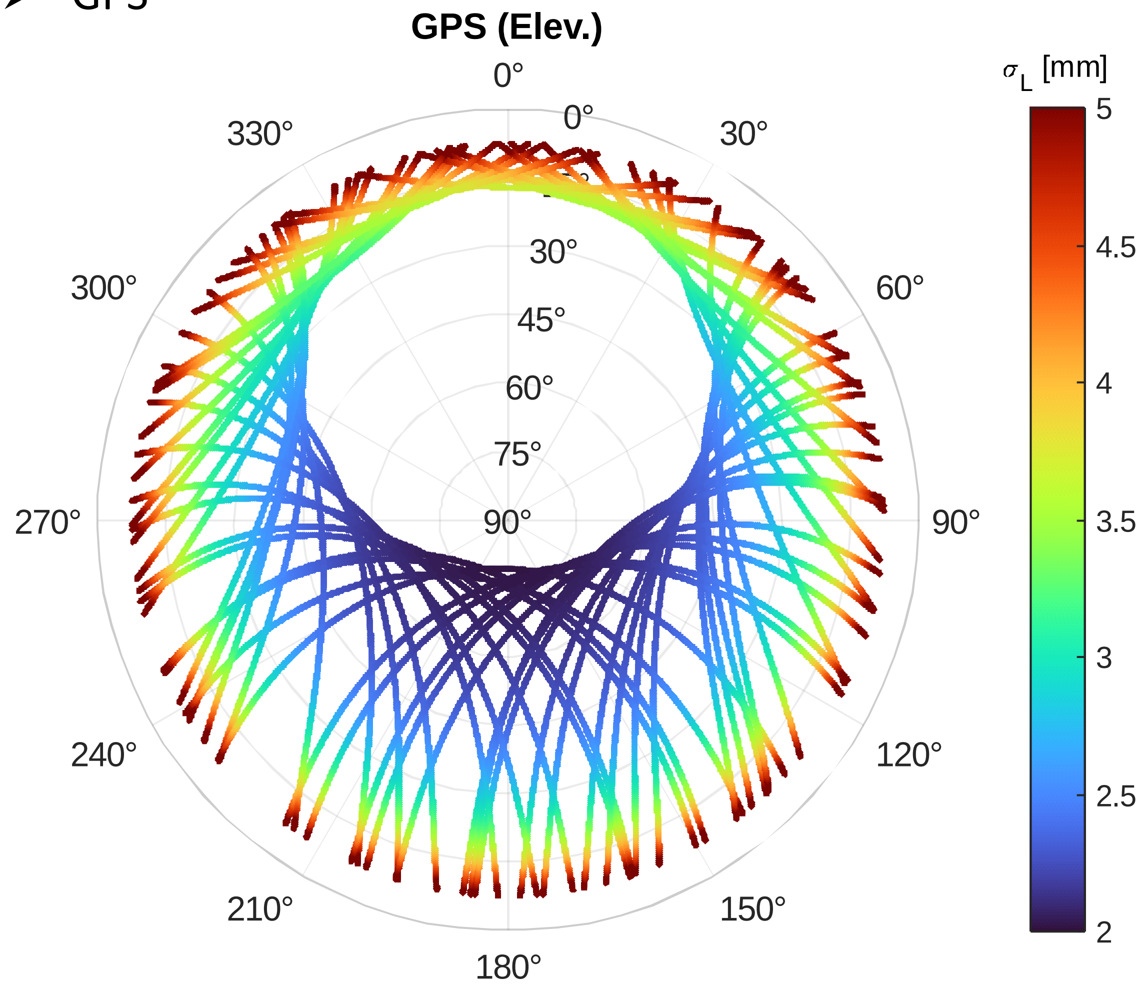
Data-Driven

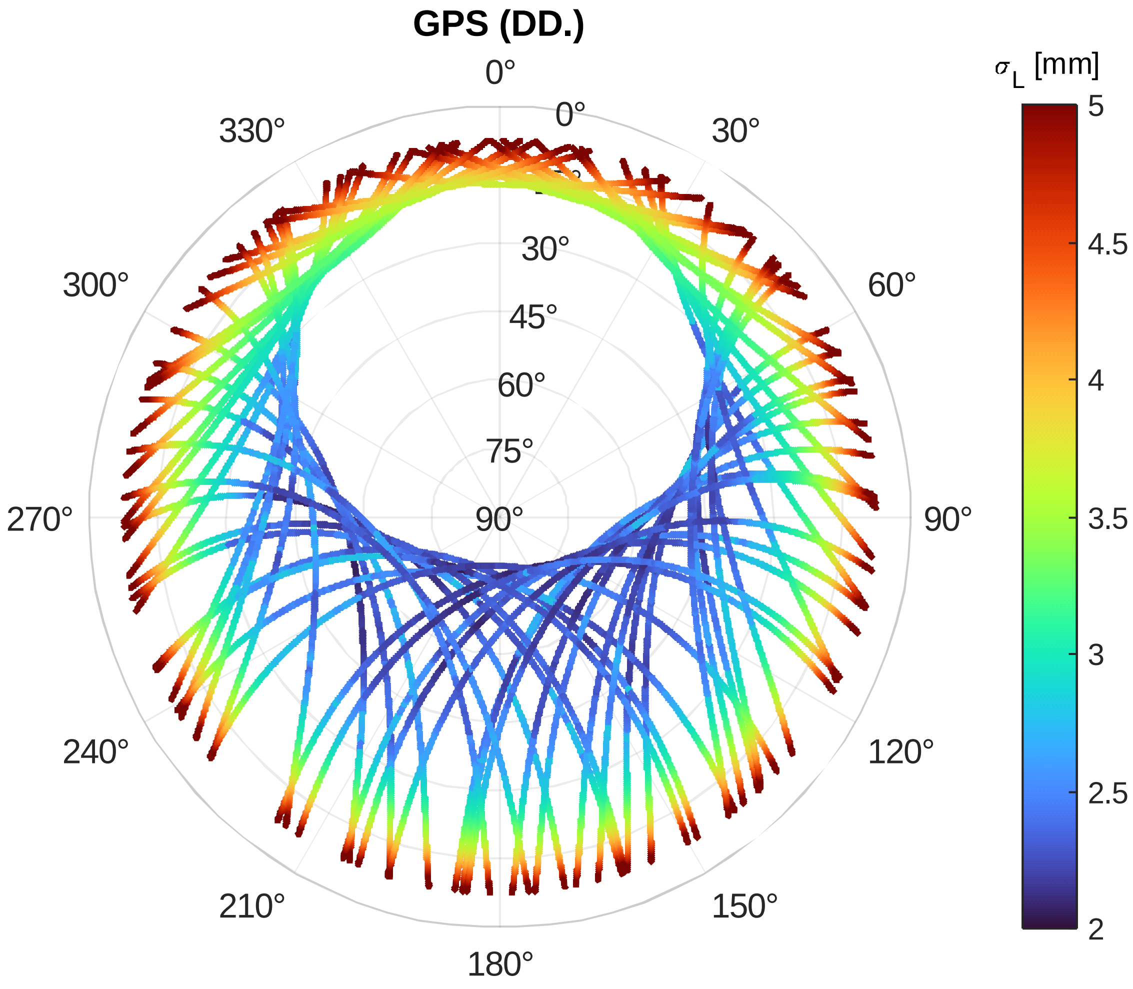


$$\sigma_A^2 = f(\sigma_{MWWL}^2, \sigma_{IONO}^2)$$
$$\sigma_B^2 = f(\sigma_{MWWL}^2, \sigma_{IONO}^2)$$

Stochastic Model Comparison

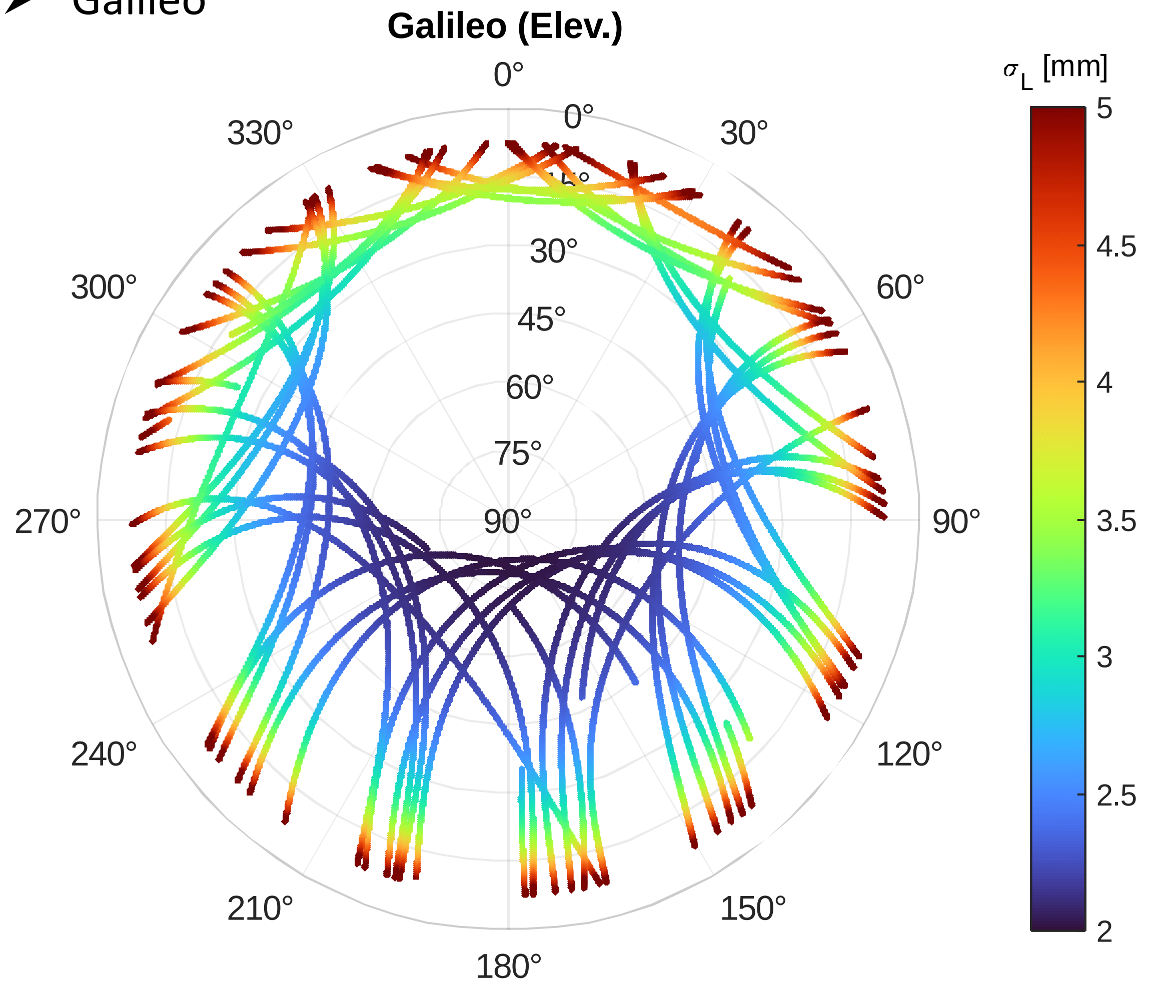
- GPS

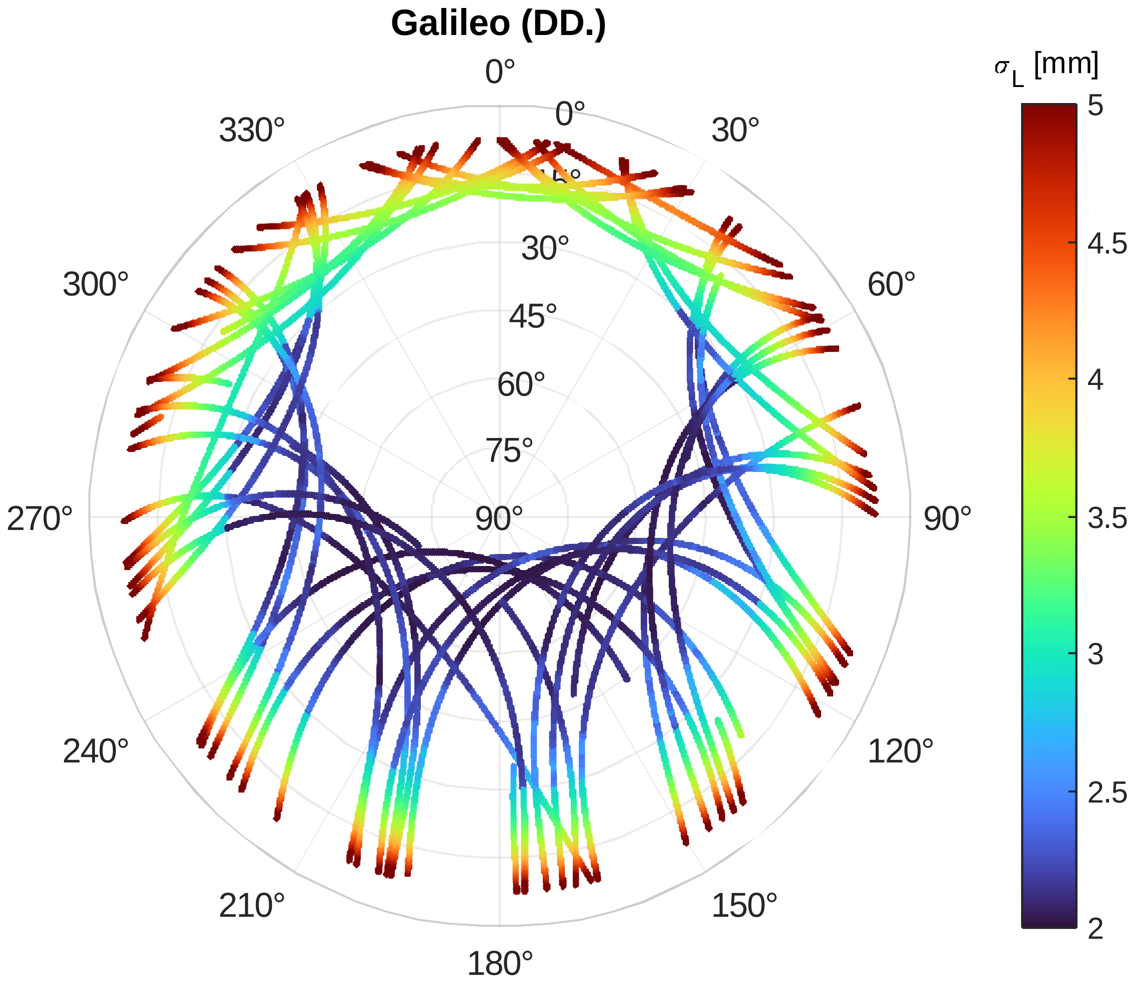




Stochastic Model Comparison

- Galileo

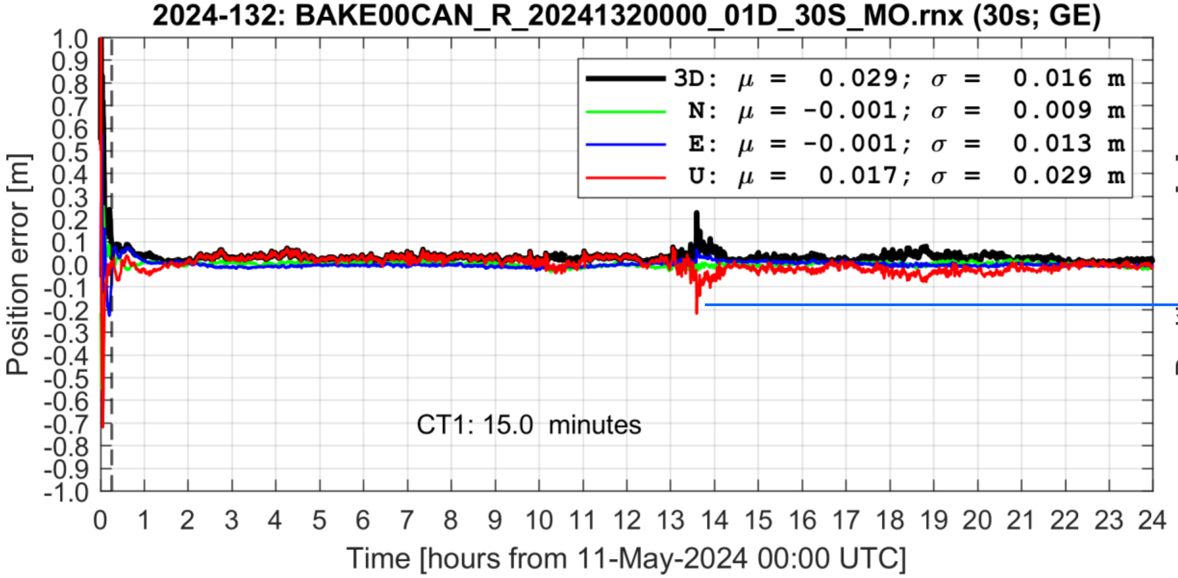




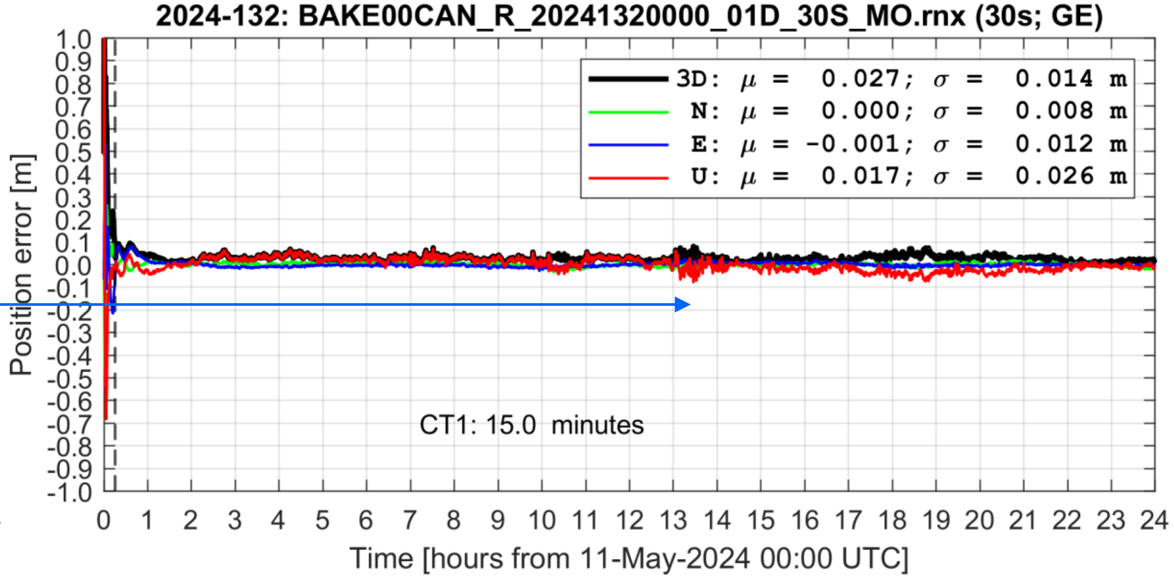
Kinematic PPP

- Data-driven stochastic model mitigates storm effects
  - Vertical error reduced from above 20-cm to below 10-cm using new approach

Elevation-Based



Data-Driven



Recommendations and Future Work

Oregon State University College of Engineering

- **Position accuracy during geomagnetic storm events**
  - Increased carrier phase biases and noise amplification
- **Geomagnetic activity monitoring**
  - Long term trends and historical activity: spaceweatherlive.com
  - Short term predictions: NOAA Space Weather Predication Center
- **Precise Point Positioning (PPP)**
  - Static PPP is (typically) stable regardless of geomagnetic conditions
  - Kinematic (multi-GNSS) PPP errors become amplified under storm conditions
  - Stochastic modeling may mitigate errors
- **Ongoing research**
  - Evaluate more stations and storm cases, then finalize stochastic modeling approach
  - Expand benefits of new techniques to real-time positioning applications

Acknowledgements

- Geospatial Center for the Arctic and Pacific (GAP): https://gcapgeospatial.org/
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  - Students: William Ohene, Hunter Mitchell

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End

- [Bonus slides are next]