Chapter 9
Trigonometric Ratios and Functions

Section 9-3
Trigonometric Functions of Any Angle

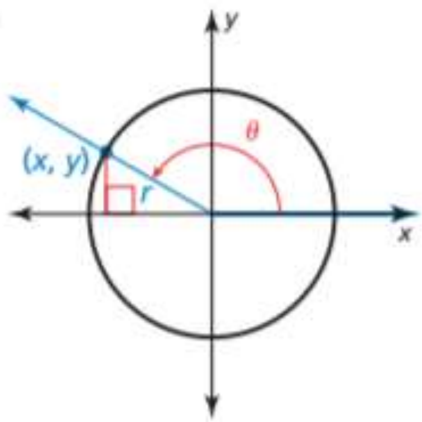
Trigonometric Functions of Any Angle

You can generalize the right-triangle definitions of trigonometric functions so that they apply to any angle in standard position.

Core Concept

General Definitions of Trigonometric Functions

Let θ be an angle in standard position, and let (x, y) be the point where the terminal side of θ intersects the circle $x^2 + y^2 = r^2$. The six trigonometric functions of θ are defined as shown.



$\sin \theta = \frac{y}{r}$ $\qquad$ $\csc \theta = \frac{r}{y}, y \neq 0$

$\cos \theta = \frac{x}{r}$ $\qquad$ $\sec \theta = \frac{r}{x}, x \neq 0$

$\tan \theta = \frac{y}{x}, x \neq 0$ $\qquad$ $\cot \theta = \frac{x}{y}, y \neq 0$

These functions are sometimes called *circular functions.*

EXAMPLE 1 **Evaluating Trigonometric Functions Given a Point**

Let $(-4, 3)$ be a point on the terminal side of an angle θ in standard position. Evaluate the six trigonometric functions of θ.

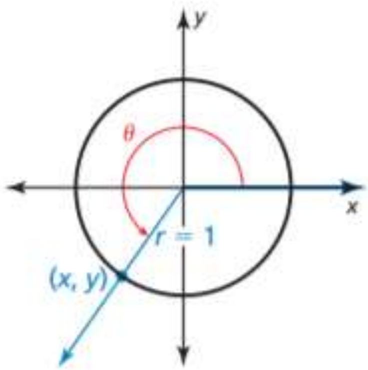
Core Concept

The Unit Circle

The circle $x^2 + y^2 = 1$, which has center (0, 0) and radius 1, is called the **unit circle**. The values of sin θ and cos θ are simply the y-coordinate and x-coordinate, respectively, of the point where the terminal side of θ intersects the unit circle.

$$\sin \theta = \frac{y}{r} = \frac{y}{1} = y$$

$$\cos \theta = \frac{x}{r} = \frac{x}{1} = x$$



It is convenient to use the unit circle to find trigonometric functions of **quadrantal angles**. A quadrantal angle is an angle in standard position whose terminal side lies on an axis. The measure of a quadrantal angle is always a multiple of 90º, or $\frac{\pi}{2}$ radians.

EXAMPLE 2 Using the Unit Circle

Use the unit circle to evaluate the six trigonometric functions of $\theta = 270º$.

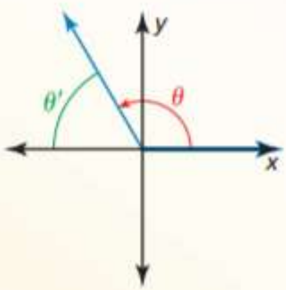
Reference Angles

Core Concept

Reference Angle Relationships

Let θ be an angle in standard position. The **reference angle** for θ is the acute angle θ' formed by the terminal side of θ and the x-axis. The relationship between θ and θ' is shown below for nonquadrantal angles θ such that $90^\circ < \theta < 360^\circ$ or, in radians, $\frac{\pi}{2} < \theta < 2\pi$.

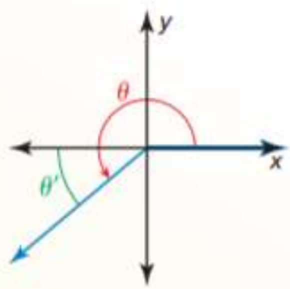
Quadrant II



Degrees: $\theta' = 180^\circ - \theta$

Radians: $\theta' = \pi - \theta$

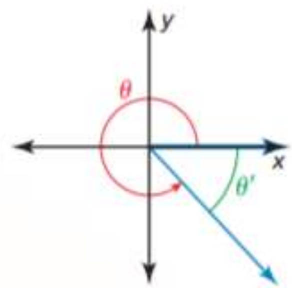
Quadrant III



Degrees: $\theta' = \theta - 180^\circ$

Radians: $\theta' = \theta - \pi$

Quadrant IV



Degrees: $\theta' = 360^\circ - \theta$

Radians: $\theta' = 2\pi - \theta$

EXAMPLE 3 **Finding Reference Angles**

Find the reference angle θ' for (a) $\theta = \frac{5\pi}{3}$ and (b) $\theta = -130^\circ$.

Reference angles allow you to evaluate a trigonometric function for any angle θ. The sign of the trigonometric function value depends on the quadrant in which θ lies.

Core Concept

Evaluating Trigonometric Functions

Use these steps to evaluate a trigonometric function for any angle θ:

Step 1 Find the reference angle θ'.

Step 2 Evaluate the trigonometric function for θ'.

Step 3 Determine the sign of the trigonometric function value from the quadrant in which θ lies.

Signs of Function Values

	Quadrant II	Quadrant I
$\sin\theta$, $\csc\theta$	+	+
$\cos\theta$, $\sec\theta$	−	+
$\tan\theta$, $\cot\theta$	−	+
	Quadrant III	**Quadrant IV**
$\sin\theta$, $\csc\theta$	−	−
$\cos\theta$, $\sec\theta$	−	+
$\tan\theta$, $\cot\theta$	+	−

EXAMPLE 4 Using Reference Angles to Evaluate Functions

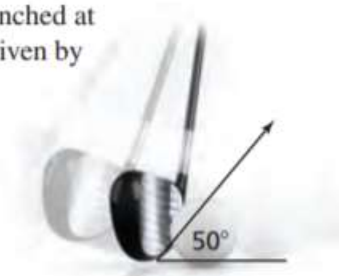
Evaluate (a) $\tan(-240°)$ and (b) $\csc \frac{17\pi}{6}$.

EXAMPLE 5 Solving a Real-Life Problem

The horizontal distance d (in feet) traveled by a projectile launched at an angle θ and with an initial speed v (in feet per second) is given by

$$d = \frac{v^2}{32}\sin 2\theta. \qquad \text{Model for horizontal distance}$$

Estimate the horizontal distance traveled by a golf ball that is hit at an angle of 50° with an initial speed of 105 feet per second.



INTERPRETING MODELS

This model neglects air resistance and assumes that the projectile's starting and ending heights are the same.

Section 9-3 Homework # 3-37 odd and 49, 51