

Method of Undetermined Coeffs

$$ay'' + by' + cy = q(x)$$

1st y_c $ay'' + by' + cy = 0$ $y_c = c_1 y_1 + c_2 y_2$

then for y_p we guess

if $q(x) = \text{exponential}$ $y_p \text{ exp.}$

$q(x) = \text{powers poly.}$ $y_p \text{ poly.}$

$q(x) = \sin x, \cos x$ $y_p \sin x, \cos x$

we lost consider

$$y'' - 3y' + 2y = e^x$$

$$y_c = c_1 e^x + c_2 e^{2x}$$

$y_p = A e^x$ but part of y_p ! so what do we do?

Consider

$$y' - y = e^x \quad \text{1st order}$$

4.1 $y' - y = 0$

$$m - 1 = 0$$

$$m = 1 \quad \text{so} \quad y_c = c_1 e^x$$

$$y_p = ?$$

This ODE is linear so $\mu = e^{\int -1 dx} = e^{-x}$

$$\text{so } \frac{d}{dx} (e^{-x} y) = e^{-x} \cdot e^x = 1$$

$$e^{-x} y = x + c_1$$

$$y = c_1 e^x + x e^x$$

$\uparrow$
 y_c

$\uparrow$
 y_p

so the particular solⁿ
is "bumped" up

$$y = A x e^x$$

For $y'' - 3y' + 2y = e^x$

try

$$y_p = Axe^x$$

$$y_p' = Axe^x + Ae^x$$

$$y_p'' = Axe^x + Ae^x + Ae^x$$

$$Axe^x + 2Ae^x - 3(Axe^x + Ae^x) + 2Axe^x = e^x$$

$$\cancel{Axe^x} + 2Ae^x - 3\cancel{Axe^x} - 3Ae^x + 2\cancel{Axe^x} = e^x$$

$$-Ae^x = e^x \Rightarrow A = -1$$

so $y_p = -xe^x$

∴ Q.S. $y = y_c + y_p$

$$= C_1 e^x + C_2 e^{2x} - xe^x$$

$$\text{Ex 2} \quad y'' - 2y' = -12x + 2$$

$$\underline{y_c} \quad y'' - 2y' = 0$$

$$m^2 - 2m = 0$$

$$m(m-2) = 0$$

$$m = 0, 2$$

$$\text{So } y_c = c_1 e^0 + c_2 e^{2x}$$

$$= c_1 + c_2 e^{2x}$$

For y_p would guess $y_p = Ax + B$

but B is the same AS c_1

so bump

$$y_p = Ax^2 + Bx$$

$$y_p' = 2Ax + B, \quad y_p'' = 2A$$

$$\text{Sub} \quad 2A - 2(2Ax + B) = -12x + 2$$

$$2A - 4Ax - 2B = -12x + 2$$

$$-4A = -12 \quad 2A - 2B = 2$$

$$A = 3$$

$$6 - 2B = 2$$

$$2B = 6 - 2 = 4$$

$$B = 2$$

$$y_p = 3x^2 + 2x, \quad \text{G.S. } y = c_1 + c_2 e^{2x} + 3x^2 + 2x$$

Suppose we were given

$$\text{Solve } y'' - 2y' = -12x + 2 \quad y(0) = -2$$

$$y'(0) = 4$$

then we use GS not YC

$$y(0) = c_1 + c_2 e^0 + 3(0)^2 + 2(0) = c_1 + c_2 = -2$$

$$y'(x) = 2c_2 e^{2x} + 6x + 2$$

$$y'(0) = 2c_2 + 2 = 4$$

$$c_2 = \frac{4-2}{2} = 1$$

$$c_1 + c_2 = -2 \quad \Rightarrow \quad c_1 + 1 = -2 \quad \Rightarrow \quad c_1 = -3$$

Solⁿ

$$y = -3 + e^{2x} + 3x^2 + 2x$$