

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Numerical solutions of equations

Exercise A, Question 1

Question:

Use interval bisection to find the positive square root of $x^2 - 7 = 0$, correct to one decimal place.

Solution:

$$x^2 - 7 = 0$$

So roots lies between 2 and 3 as $f(2) = -3$ and $f(3) = +2$ Using table method.

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$\frac{f(a+b)}{2}$
2	-3	3	+2	2.5	-0.75
2.5	-0.75	3	+2	2.75	0.5625
2.5	-0.75	2.75	0.5625	2.625	-0.109375
2.625	-0.109375	2.75	0.5625	2.6875	0.2226562
2.625	-0.109375	2.6875	0.2226562	2.65625	0.055664
2.625	-0.109375	2.65625	0.055664	2.640625	-0.0270996

Hence $x^2 - 7 = 0$ when $x = 2.6$ to 1 decimal place

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Exercise A, Question 2

Question:

- a** Show that one root of the equation $x^3 - 7x + 2 = 0$ lies in the interval $[2, 3]$.
- b** Use interval bisection to find the root correct to two decimal places.

Solution:

a $f(2) = 8 - 14 + 2 = -4$ $f(x) = x^3 - 7x + 2$

$f(3) = 27 - 21 + 2 = +8$

Hence change of sign, implies roots between 2 and 3.

- b** Using table method.

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$\frac{f(a+b)}{2}$
2	-4	3	+8	2.5	0.125
2	-4	2.5	0.125	2.25	-2.359375
2.25	-2.359375	2.5	0.125	2.375	-1.2285156
2.375	-1.2285156	2.5	0.125	2.4375	-0.5803222
2.4375	-0.5803222	2.5	0.125	2.46875	-0.2348938
2.46875	-0.2348938	2.5	0.125	2.484375	-0.0567665
2.484375	-0.0567665	2.5	0.125	2.4921875	0.0336604
2.484375	-0.0567665	2.4921875	0.0336604	2.4882813	-0.0116673

Hence $x = 2.49$ to 2 decimal places.

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Exercise A, Question 3

Question:

a Show that the largest positive root of the equation $0 = x^3 + 2x^2 - 8x - 3$ lies in the interval $[2, 3]$.

b Use interval bisection to find this root correct to one decimal place.

Solution:

a $f(2) = 8 + 8 - 16 - 3 = -3$ $f(x) = x^3 + 2x^2 - 8x - 3$

$$f(3) = 27 + 18 - 24 - 3 = 18$$

Change of sign implies root in interval $[2,3]$

b

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
2	-3	3	18	2.5	5.125
2	-3	2.5	5.125	2.25	0.51562
2	-3	2.25	0.515625	2.125	-1.37304
2.125	-1.3730469	2.25	0.515625	2.1875	-0.46215

Hence solution = 2.2 to 1 decimal place

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Exercise A, Question 4

Question:

a Show that the equation $f(x) = 1 - 2\sin x$ has one root which lies in the interval $[0.5, 0.8]$.

b Use interval bisection four times to find this root. Give your answer correct to one decimal place.

Solution:

a $f(0.5) = +0.0411489$

$$f(0.8) = -0.4347121$$

Change of sign implies root between 0.5 and 0.8

b

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$\frac{f(a+b)}{2}$
0.5	0.0411489	0.8	-0.4347121	0.65	-0.2103728
0.5	0.0411489	0.65	-0.2103728	0.575	-0.0876695
0.5	0.0411489	0.575	-0.0876696	0.5375	-0.0239802

0.5 to 1 decimal place.

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Exercise A, Question 5

Question:

a Show that the equation $0 = \frac{x}{2} - \frac{1}{x}$, $x > 0$, has a root in the interval $[1, 2]$.

b Obtain the root, using interval bisection two times. Give your answer to two significant figures.

Solution:

a $f(1) = -0.5$ $p = \frac{1}{2} + x - \frac{1}{x}$

$f(2) = +0.5$

Change of sign implies root between interval $[1, 2]$

b

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
1	-0.5	2	+0.5	1.5	0.0833
1	-0.5	1.5	0.083	1.25	-0.175
1.25	-0.175	1.5	0.083	1.375	-0.0397727
1.375	-0.0397727	1.5	0.083	1.4375	0.0230978

Hence $x = 1.4$ to 2 significant figures

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Exercise A, Question 6

Question:

$$f(x) = 6x - 3^x$$

The equation $f(x) = 0$ has a root between $x = 2$ and $x = 3$. Starting with the interval $[2, 3]$ use interval bisection three times to give an approximation to this root.

Solution:

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
2	3	3	-9	2.5	-0.588457
2	3	2.5	-0.5884572	2.25	1.65533
2.25	1.6553339	2.5	-0.5884572	2.375	0.66176
2.375	0.6617671	2.5	-0.5884572	2.4375	0.0708
2.4375	0.0709769	2.5	-0.5844572	2.46875	-0.2498
2.4375	0.0709769	2.46875	-0.2498625	2.453125	-0.08726
2.4375	0.0709769	2.453125	-0.0872613	2.4453125	-0.0076

2.4 correct to 1 decimal place.

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Exercise B, Question 1

Question:

a Show that a root of the equation $x^3 - 3x - 5 = 0$ lies in the interval $[2, 3]$.

b Find this root using linear interpolation correct to one decimal place.

Solution:

$$\mathbf{a} \quad f(2) = 8 - 6 - 5 = -3 \quad f(x) = x^3 - 3x - 5$$

$$f(3) = 27 - 9 - 5 = +13$$

Change of size therefore root in interval $[2, 3]$

b Using linear interpolation and similar triangle taking x_1 as the first root.

$$\frac{3 - x_1}{x_1 - 2} = \frac{3}{13} \quad x = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

so

$$13(3 - x_1) = 3(x_1 - 2)$$

$$39 - 13x_1 = 3x_1 - 6$$

$$16x_1 = 45$$

$$x_1 = 2.8125 \quad f(x_1) = 8.8098$$

Using interval $(2, 2.8125)$

$$\frac{2.8125 - x_2}{x_2 - 2} = \frac{3}{8.8098}$$

$$x_2 = 2.606 \quad f(x_2) = 4.880$$

Using interval $(2, 2.606)$

$$\frac{2.606 - x_3}{x_3 - 2} = \frac{3}{4.880}$$

$$x_3 = 2.375 \quad f(x_3) = 1.276$$

Using interval $(2, 2.375)$

$$\frac{2.375 - x_4}{x_4 - 2} = \frac{3}{1.276}$$

$$x_4 = 2.112 \quad f(x_4) = -1.915$$

Using interval $(2.112, 2.375)$

$$\frac{2.375 - x_5}{x_5 - 2.112} = \frac{1.915}{1.276}$$

$$= 2.218 \quad f(x_5) = -0.736$$

Using interval $(2.218, 2.375)$

$$\begin{aligned}\frac{2.375 - x_6}{x_6 - 2.218} &= \frac{0.736}{1.276} \\ &= 2.318 \quad f(x_6) = 0.494\end{aligned}$$

Using interval (2.218, 2.318)

$$\begin{aligned}\frac{2.318 - x_7}{x_7 - 2.218} &= \frac{0.736}{0.494} \\ &= 2.25 \quad f(x_7) = -0.229\end{aligned}$$

2.3 to 1 decimal place.

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Exercise B, Question 2

Question:

a Show that a root of the equation $5x^3 - 8x^2 + 1 = 0$ has a root between $x = 1$ and $x = 2$.

b Find this root using linear interpolation correct to one decimal place.

Solution:

$$\mathbf{a} \quad f(1) = 5 - 8 + 1 = -2 \quad f(x) = 5x^3 - 8x^2 + 1$$

$$f(2) = 40 - 32 + 1 = +9$$

Therefore root in interval $[1, 2]$ as sign change.

b Using linear interpolation.

$$\frac{2 - x_1}{x_1 - 1} = \frac{2}{9}$$

$$x_1 = 1.818 \quad f(x_1) = 4.612.$$

Using interval $(1, 1.818)$

$$\frac{1.818 - x_2}{x_2 - 1} = \frac{2}{4.612}$$

$$x_2 = 1.570 \quad f(x_2) = 0.647$$

Using interval $(1, 1.570)$

$$\frac{1.570 - x_3}{x_3 - 1} = \frac{2}{0.647}$$

$$x_3 = 1.139 \quad f(x_3) = -1.984$$

Using interval $(1.139, 1.570)$

$$\frac{1.570 - x_4}{x_4 - 1.139} = \frac{1.984}{0.647}$$

$$x_4 = 1.447 \quad f(x_4) = -0.590$$

Use interval $(1.447, 1.570)$

$$\frac{1.570 - x_5}{x_5 - 1.447} = \frac{0.590}{0.647}$$

$$= 1.511 \quad f(x_5) = -0.0005.$$

Ans 1.5 correct to 1 decimal place.

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Exercise B, Question 3

Question:

a Show that a root of the equation $\frac{3}{x} + 3 = x$ lies in the interval $[3, 4]$.

b Use linear interpolation to find this root correct to one decimal place.

Solution:

a $f(3) = 1$ $f(x) = \frac{3}{x} + 3 - x$

$$f(4) = -0.25$$

Hence root as sign change in interval $[3, 4]$

b Using linear interpolation

$$\frac{4 - x_1}{x_1 - 3} = \frac{0.25}{1}$$

$$x_1 = 3.8 \quad f(x_1) = -0.011$$

Using interval $[3, 3.8]$

$$\frac{3.8 - x_2}{x_2 - 3} = \frac{0.011}{1}$$

$$x_2 = 3.791 \quad f(x_2) = -0.0004579$$

Ans = 3.8 to 1 decimal place

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Exercise B, Question 4

Question:

a Show that a root of the equation $2x \cos x - 1 = 0$ lies in the interval $[1, 1.5]$.

b Find this root using linear interpolation correct to two decimal places.

Solution:

a $f(1) = 0.0806$

$$f(1.5) = -0.788$$

Hence root between $(1, 1.5)$ as sign change

b Using linear interpolation

$$\frac{1.5 - x_1}{x_1 - 1} = \frac{0.788}{1}$$
$$x_1 = 1.280 \quad f(1.280) = -0.265$$

Use interval $[1, 1.28]$

$$\frac{1.28 - x_2}{x_2 - 1} = \frac{0.265}{1}$$
$$x_2 = 1.221 \quad f(1.221) = -0.164$$

Use interval $[1, 1.221]$

$$\frac{1.221 - x_3}{x_3 - 1} = \frac{0.164}{1}$$
$$x_3 = 1.190 \quad f(1.190) = -0.115$$

Use interval $[1, 1.190]$

$$\frac{1.190 - x_4}{x_4 - 1} = \frac{0.115}{1}$$
$$x_4 = 1.170 \quad f(1.170) = 0.088$$

Use interval $[1, 1.170]$

$$\frac{1.170 - x_5}{x_5 - 1} = \frac{0.088}{1}$$
$$x_5 = 1.156 \quad f(1.156) = -0.068$$

Root 1.10 to 2 decimal places.

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Exercise B, Question 5

Question:

a Show that the largest possible root of the equation $x^3 - 2x^2 - 3 = 0$ lies in the interval $[2, 3]$.

b Find this root correct to one decimal place using interval interpolation.

Solution:

$$\text{a } f(2) = 8 - 8 - 3 = -3 \quad f(x) = x^3 - 2x^2 - 3$$

$$f(3) = 27 - 18 - 3 = 6$$

Hence root lies in interval $[2, 3]$ and $\forall x \in x \geq 3 f(x) < 0$.

b Using linear interpolation

$$\frac{3 - x_1}{x_1 - 2} = \frac{6}{3}$$

$$x_1 = 2.333 \quad f(x_1) = -1.185$$

$$\frac{3 - x_2}{x_2 - 2.333} = \frac{6}{1.185}$$

$$x_2 = 2.443 \quad f(x_2) = -0.356$$

$$\frac{3 - x_3}{x_3 - 2.443} = \frac{6}{0.356}$$

$$x_3 = 2.474 \quad f(x_3) = -0.095$$

$$\frac{3 - x_4}{x_4 - 2.474} = \frac{6}{0.095}$$

$$x_4 = 2.482$$

Hence root = 2.5 to 1 d.p

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Exercise B, Question 6

Question:

$$f(x) = 2^x - 3x - 1$$

The equation $f(x) = 0$ has a root in the interval $[3, 4]$.

Using this interval find an approximation to x .

Solution:

Let root be α

$$f(3) = -2$$

$$f(4) = 3$$

$$\frac{4 - \alpha}{\alpha - 3} = \frac{3}{2}$$

$\alpha = 3.4$ is the approximation.

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Exercise C, Question 1

Question:

Show that the equation $x^3 - 2x - 1 = 0$ has a root between 1 and 2. Find the root correct to two decimal places using the Newton–Raphson process.

Solution:

$$f(1) = -2 \quad f(x) = x^3 - 2x - 1$$

$$f(2) = 3 \quad f(2) = 3 \text{ is correct}$$

Hence root in interval $[1,2]$ as sign change

$$f(x) = x^3 - 2x - 1$$

$$f'(x) = 3x^2 - 2$$

Let $x_0 = 2$.

$$\text{Then } x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 2 - \frac{3}{10}$$

$$x_1 = 1.7$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 1.88 - \frac{1.885}{8.6032}$$

$$= 1.661$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 1.661 - \frac{0.2597}{6.2767}$$

$$= 1.6120$$

$$x_4 = 1.620 - \frac{f(1.620)}{f'(1.620)}$$

$$x_4 = 1.62 - \frac{0.0115}{5.8732}$$

$$= 1.618$$

Solution = 1.62 to 2 decimal places

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Exercise C, Question 2

Question:

Use the Newton–Raphson process to find the positive root of the equation $x^3 + 2x^2 - 6x - 3 = 0$ correct to two decimal places.

Solution:

$$f(0) = -3 \quad f(x) = x^3 + 2x^2 - 6x - 3$$

$$f(1) = 1 + 2 - 6 - 3 = -6$$

$$f(2) = 8 + 8 - 12 - 3 = 1$$

Hence root in interval $[1,2]$

Using Newton Raphson

$$f(x) = x^3 + 2x^2 - 6x - 3$$

$$f'(x) = 3x^2 + 4x - 6$$

$$x_0 = 2$$

$$\text{Then } x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2 - \frac{1}{14}$$

$$= 1.92857$$

$$x_2 = 1.92857 - \frac{0.0404494}{12.872427}$$

$$= 1.92857 - 0.00314$$

$$= 1.9254$$

Root = 1.93 to 2 decimal places.

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Exercise C, Question 3

Question:

Find the smallest positive root of the equation $x^4 + x^2 - 80 = 0$ correct to two decimal places. Use the Newton–Raphson process.

Solution:

$$f(x) = x^4 + x^2 - 80$$

$$f'(x) = 4x^3 + 2x$$

$$\text{Let } x_0 = 3 \quad f(3) = 10$$

$$\text{So } x_1 = 3 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 3 - \frac{10}{114}$$
$$= 2.912$$

$$\text{Then } x_2 = 2.912 - \frac{0.1768}{104.388}$$
$$= 2.908$$

Hence root = 2.91 to 2 decimal places.

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Exercise C, Question 4

Question:

Apply the Newton–Raphson process to find the negative root of the equation $x^3 - 5x + 2 = 0$ correct to two decimal places.

Solution:

$$f(x) = x^3 - 5x + 2$$

$$f'(x) = 3x^2 - 5$$

$$f(0) = 2$$

$$f(-1) = -1 + 5 + 2 = 6$$

$$f(-2) = -8 + 10 + 2 = 4$$

$$f(-3) = -27 + 15 + 2 = -10$$

Hence root between interval $[-2, -3]$

Let $x_0 = -2$

$$\text{Then } x_1 = -2 - \frac{f(x_0)}{f'(x_0)}$$

$$= -2 - \frac{4}{7}$$

$$= -2.5714$$

$$x_2 = -2.571 - \frac{f(x_1)}{f'(x_1)}$$

$$= -2.571 - \frac{2.1394}{14.83}$$

$$= -2.4267$$

$$x_3 = -2.4267 - \frac{0.1570}{12.6662}$$

$$= -2.4267 - 0.01234$$

$$= -2.439$$

$$x_4 = -2.439 - \frac{0.00163}{12.846}$$

$$= -2.4391$$

Root = -2.44 correct to 2 decimal places.

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Exercise C, Question 5

Question:

Show that the equation $2x^3 - 4x^2 - 1 = 0$ has a root in the interval $[2, 3]$. Taking 3 as a first approximation to this root, use the Newton–Raphson process to find this root correct to two decimal places.

Solution:

$$f(x) = 2x^3 - 4x^2 - 1.$$

$$f(2) = 16 - 16 - 1 = -1$$

$$f(3) = 54 - 36 - 1 = 17$$

Sign change implies root in interval $[2, 3]$

$$f'(x) = 6x^2 - 8x$$

$$\text{Let } x_0 = 3$$

$$\text{Then } x_1 = 3 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3 - \frac{17}{30}$$

$$= 2.43$$

$$x_2 = 2.43 - \frac{f(2.43)}{f'(2.43)}$$

$$= 2.43 - \frac{4.078}{16.05}$$

$$= 2.43 - 0.254$$

$$= 2.179$$

$$x_3 = 2.179 - \frac{f(2.179)}{f'(2.179)}$$

$$= 2.179 - \frac{0.6998}{11.056}$$

$$= 2.179 - 0.063296$$

$$= 2.116$$

$$x_4 = 2.116 - \frac{f(2.116)}{f'(2.116)}$$

$$= 2.116 - \frac{0.0388}{9.937} = 2.112$$

$$x_5 = 2.112 - \frac{f(2.112)}{f'(2.112)}$$

$$= 2.112 - \frac{-0.00084}{9.8672}$$

$$= 2.112$$

Ans = 2.11 correct to 2 decimal place.

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Exercise C, Question 6

Question:

$$f(x) = x^3 - 3x^2 + 5x - 4$$

Taking 1.4 as a first approximation to a root, x , of this equation, use Newton–Raphson process once to obtain a second approximation to x . Give your answer to three decimal places.

Solution:

$$\begin{aligned}f(x) &= x^3 - 3x^2 + 5x - 4 \\f'(x) &= 3x^2 - 6x + 5\end{aligned}$$

$$\text{Let } x_0 = 1.4$$

Using Newton Raphson

$$\begin{aligned}x_1 &= 1.4 - \frac{f(1.4)}{f'(1.4)} \\&= 1.4 - \frac{-0.136}{2.48} \\&= 1.4 + 0.0548 \\&= 1.455 \text{ to 3 decimal places}\end{aligned}$$

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Exercise C, Question 7

Question:

Use the Newton–Raphson process twice to find the root of the equation $2x^3 + 5x = 70$ which is near to $x = 3$. Give your answer to three decimal places.

Solution:

$$f(x) = 2x^3 + 5x - 70$$

$$f'(x) = 6x^2 + 5$$

$$\text{Let } x_0 = 3$$

Using Newton Raphson

$$x_1 = 3 - \frac{f(3)}{f'(3)}$$

$$= 3 - \frac{-1}{59}$$

$$= 3.02$$

$$x_2 = 3.02 - \frac{f(3.02)}{f'(3.02)}$$

$$= 3.02 - \frac{0.1872}{59.72}$$

$$= 3.017 \text{ to 3 decimal places.}$$

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Exercise D, Question 1

Question:

Given that $f(x) = x^3 - 2x + 2$ has a root in the interval $[-1, -2]$, use interval bisection on the interval $[-1, -2]$ to obtain the root correct to one decimal place.

Solution:

$$\begin{aligned} f(x) &= x^3 - 2x + 2 \\ f(-1) &= -1 + 2 + 2 = +3 \\ f(-2) &= -8 + 4 + 2 = -2 \end{aligned}$$

Hence root in interval $[-1, -2]$ as sign change

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$\frac{f(a+b)}{2}$
-1	+3	-2	-2	-1.5	+1.625
-1.5	1.625	-2	-2	-1.75	0.141
-1.75	0.141	-2	-2	-1.875	-0.842
-1.75	0.141	-1.875	-0.841	-1.8125	-0.329
-1.75	0.141	-1.8125	-0.329	-1.78125	

Hence solution is -1.8 to 1 decimal place.

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Exercise D, Question 2

Question:

Show that the equation $x^3 - 12x - 7.2 = 0$ has one positive and two negative roots. Obtain the positive root correct to three significant figures using the Newton-Raphson process.

Solution:

$$f(x) = x^3 - 12x - 7.2 = 0$$

$f(0) = -7.2$	$f(-1) = 3.8$
$f(1) = -18.2$	$f(-2) = 8.8$
$f(2) = -23.2$	$f(-3) = 1.8$
$f(3) = -16.2$	$f(-4) = -23.2$
$f(4) = 8.8$	

positive root between [3, 4]

negative roots between [0, -1], [-3, -4] Let $x_0 = 4$

$$\text{Using } x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

where $f(x) = x^3 - 12x - 7.2$

$$f'(x) = 3x^2 - 12$$

$$\text{So } x_1 = 4 - \frac{8.8}{36}$$

$$x_1 = 3.756 \text{ to 3d.p.}$$

$$x_2 = 3.756 - \frac{0.716}{30.322}$$

$$x_2 = 3.732$$

$$x_3 = 3.732 - \frac{0.011}{30.323}$$

$$x_3 = 3.7316$$

Hence root = 3.73 to 3 significant figures

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Exercise D, Question 3

Question:

Find, correct to one decimal place, the real root of $x^3 + 2x - 1 = 0$ by using the Newton–Raphson process.

Solution:

$$f(x) = x^3 + 2x - 1$$

$$f(0) = -1$$

$$f(1) = 2$$

Hence root interval $[0, 1]$

Using $f(x) = x^3 + 2x - 1$

$$f'(x) = 3x^2 + 2 \quad \text{and} \quad x_0 = 1$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 1 - \frac{2}{5}$$

$$x_1 = 0.6$$

$$x_2 = 0.6 - \frac{0.416}{3.08}$$

$$x_2 = 0.465$$

$$x_3 = 0.465 - \frac{0.031}{2.647}$$

$$x_3 = 0.453$$

Hence root is 0.5 to 1 decimal place.

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Numerical solutions of equations

Exercise D, Question 4

Question:

Use the Newton–Raphson process to find the real root of the equation $x^3 + 2x^2 + 4x - 6 = 0$, taking $x = 0.9$ as the first approximation and carrying out one iteration.

Solution:

$$f(x) = x^3 + 2x^2 + 4x - 6$$

$$f'(x) = 3x^2 + 4x + 4$$

$$f(0.9) = -0.051$$

$$f'(0.9) = 10.03$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_1)}$$

$$= 0.9 - \frac{-0.051}{10.03}$$

$$= 0.905 \text{ to 3 decimal places}$$

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Exercise D, Question 5

Question:

Use linear interpolation to find the positive root of the equation $x^3 - 5x + 3 = 0$ correct to one decimal place.

Solution:

$$f(x) = x^3 - 5x + 3$$

$$f(1) = -1$$

$$f(2) = +1.$$

Hence positive root in interval $[1, 2]$ Using linear interpolation and x , as the 1st approximation

$$\frac{2 - x_1}{x_1 - 1} = \frac{1}{1}$$

$$2 - x_1 = x_1 - 1$$

$$2x_1 = 3$$

$$x_1 = 1.5 \quad f(x_1) = 1.125$$

Then

$$\frac{2 - x_2}{x_2 - 1.5} = \frac{1}{1.125}$$

$$x_2 = 1.882 \quad f(x_2) = 0.260$$

$$\frac{1.882 - x_3}{x_3 - 1.5} = \frac{0.260}{1.125}$$

$$x_3 = 1.810 \quad f(x_3) = -0.117$$

$$\frac{1.882 - x_4}{x_4 - 1.810} = \frac{0.260}{0.117}$$

$$= 1.832$$

root = 1.8 to 1 decimal place

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Numerical solutions of equations

Exercise D, Question 6

Question:

$$f(x) = x^3 + x^2 - 6.$$

a Show that the real root of $f(x) = 0$ lies in the interval $[1, 2]$.

b Use the linear interpolation on the interval $[1, 2]$ to find the first approximation to x .

c Use the Newton–Raphson process on $f(x)$ once, starting with your answer to **b**, to find another approximation to x , giving your answer correct to two decimal places.

Solution:

a

$$f(x) = x^3 + x^2 - 6$$

$$f(1) = -4$$

$$f(2) = 6$$

Hence root in interval $[1, 2]$

b

$$\frac{2 - x_1}{x_1 - 1} = \frac{6}{4}$$

$$x_1 = 1.4$$

c

$$x_0 = 1.4$$

$$f(x) = x^3 + x^2 - 6$$

$$f'(x) = 3x^2 + 2x$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1.4 - \frac{-1.296}{8.68}$$

$$= 1.55 \text{ to 2 decimal places}$$

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Numerical solutions of equations

Exercise D, Question 7

Question:

The equation $\cos x = \frac{1}{4}x$ has a root in the interval $[1.0, 1.4]$. Use linear interpolation once in the interval $[1.0, 1.4]$ to find an estimate of the root, giving your answer correct to two decimal places.

Solution:

$$\cos x = \frac{1}{4}x \Rightarrow f(x) = \frac{1}{4}x - \cos x$$

$$f(1) = -0.29$$

$$f(1.4) = 0.180$$

$$\frac{1.4 - x_1}{x_1 - 1} = \frac{-0.290}{-0.180}$$

$$x_1 = 1.153$$

$$x_1 = 1.15 \text{ to 2 decimal places}$$

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Numerical solutions of equations

Exercise D, Question 8

Question:

$$f(x) = x^3 - 3x - 6$$

Use the Newton–Raphson process to find the positive root of this equation correct to two decimal places.

Solution:

$$f(x) = x^3 - 3x - 6$$

$$f'(x) = 3x^2 - 3$$

$$f(0) = -6 \quad f(1) = -7$$

$$f(2) = -3 \quad f(3) = +13$$

Hence root in interval [2, 3]

$$\text{Let } x_0 = 2$$

Then

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_1)}$$

$$= 2 - \frac{-3}{9}$$

$$x_1 = 2.333$$

$$x_2 = \frac{4.301}{16.500}$$

$$x_2 = 2.297$$

$$x_3 = 2.297 - \frac{0.228}{12.828}$$

$$x_3 = 2.279$$

$$x_4 = 2.279 - \frac{-0.000236}{12.582}$$

$$= 2.279 + 0.000019$$

$$x_4 = 2.2790$$

Ans = 2.28 to 2 decimal places