HA1-853: Applying the Laws of Logarithms

In this lesson, you will learn a new function called the logarithmic function, or "log" for short. The logarithmic function is used in several scientific applications.

The notation is $y = \log_b x$, where b is the base and x is the argument. This is read as "y equals log base b of x."

Note: The argument cannot be negative or zero, but the result of taking the log of a positive number may be positive, negative, or zero.

Logarithm	If b and x are positive numbers ($b \neq 1$ and $b > 0$), $\log_b x = y$ if and only if $b^y = x$. *The can be any positive number except 1, since every power of 1 is 1.*

Properties of Logarithms:

1. $\log_b M = \log_b N$ if and only if $M = N$.
2. $\log_b 1 = 0$ if and only if $1 = b^0$.
3. $\log_b b = 1$ if and only if $b = b^1$.
4. $\log x = a$ if and only if $x = 10^a$.

Note: A log written without any base means $\log_{10}$.

The log function is related to the exponential function and, because of this, there are properties of logs that are similar to properties of exponents. Here are some examples.

Exponential Form	Logarithmic Form
$2^3 = 8$	$\log_2 8 = 3$
$2^4 = 16$	$\log_2 16 = 4$
$2^0 = 1$	$\log_2 1 = 0$
$2^{-1} = \frac{1}{2}$	$\log_2 \frac{1}{2} = -1$
$2^k = N$	$\log_2 N = k$

Example 1 Write each equation in exponential form: A) $\log_6 36 = 2$ B) $\log(0.001) = -3$

A) $\log_6 36 = 2$ **Answer:** $6^2 = 36$ Definition of Logarithm

B) $\log_{10}(0.001) = -3$ **Step 1:** Remember a log written without any base means $\log_{10}$.

$10^{-3} = 0.001$	**Step 2:** Using the definition for logarithms, $\log_b x = y$ if and only if $b^y = x$, let the base = 10, $x = 0.001$ and $y = -3$.
	Answer: $10^{-3} = 0.001$

Example 2 Write each equation in logarithmic form: A) $6^0 = 1$ B) $8^{\frac{-2}{3}} = \frac{1}{4}$

A) $6^0 = 1$	**Answer:** $\log_6 1 = 0$
B) $8^{\frac{-2}{3}} = \frac{1}{4}$	**Answer:** $\log_8 \frac{1}{4} = -\frac{2}{3}$

Example 3 Simplify each logarithm: A) $\log_5 25$ B) $\log_2 8\sqrt{2}$ C) $\log_2 0.125 = x$

A) $\log_5 25 = x$	**Step 1:** Rewrite the equation as an exponential equation.
$5^x = 25$ $5^x = 5^2$	**Step 2:** Find a common base.
$5^x = 5^2$ $x = 2$	**Step 3:** Set the exponents equal to each other.
	Answer: $x = 2$
B) $\log_2 8\sqrt{2} = x$	**Step 1:** Rewrite the equation as an exponential equation.
$2^x = 8\sqrt{2}$ $2^x = 2^3 \cdot 2^{1/2}$ $2^x = 2^{7/2}$	**Step 2:** Find a common base. Remember the laws of exponents states $a^m \cdot a^n = a^{m+n}$.
$x = \frac{7}{2}$	**Step 3:** Set the exponents equal to each other.
	Answer: $x = \frac{7}{2}$
C) $\log_2 0.125 = x$	**Step 1:** Rewrite the equation as an exponential equation.
$2^x = 0.125$ $2^x = \frac{1}{8}$ $2^x = \frac{1}{2^3}$	**Step 2:** Find a common base.

$2^x = 2^{-3}$ $x = -3$	**Step 3:**	Set the exponents equal to each other.
	Answer:	$x = -3$

The laws of exponents can be used to derive the laws of logarithms. Let's review the laws of exponents.

$$a^m \cdot a^n = a^{m+n} \qquad (ab)^n = a^n b^n$$

$$\frac{a^m}{a^n} = a^{m-n} \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$(a^m)^n = a^{mn}$$

Laws of Logarithms

Let b be the base of a logarithmic function ($b > 0$, $b \neq 1$). Let n and m be positive numbers.

1. $\log_b (n \cdot m) = \log_b n + \log_b m$
2. $\log_b \left(\frac{n}{m}\right) = \log_b n - \log_b m$
3. $\log_b a^n = n \cdot \log_b a$
4. $\log_b b^x = x$
5. $b^{\log_b x} = x$

Example 4 Express $\log_6 n^2m^3$ in terms of $\log_6 n$ and $\log_6 m$.

$\log_6 n^2m^3 = \log_6 n^2 + \log_6 m^3$	**Step 1:**	Separate the logarithm by adding the log of each factor.
$= 2\log_6 n + 3\log_6 m$	**Step 2:**	Make the exponents of the variables become the coefficients of the logarithms.
	Answer:	$= 2\log_6 n + 3\log_6 m$

Example 5 Express $\log_2 \sqrt{\frac{n}{m^5}}$ in terms of $\log_2 n$ and $\log_2 m$.

$\log_2 \sqrt{\frac{n}{m^5}} = \log_2 \left(\frac{n}{m^5}\right)^{\frac{1}{2}}$	**Step 1:**	Rewrite the radical as an exponent.
$= \frac{1}{2}\log_2 \left(\frac{n}{m^5}\right)$	**Step 2:**	Make that exponent a coefficient of the logarithm.

$= \frac{1}{2}(\log_2 n - \log_2 m^5)$	**Step 3:**	Separate the logarithm by subtracting the log of the divisor from the log of the dividend.
$= \frac{1}{2}(\log_2 n - 5\log_2 m)$	**Step 4:**	Make the exponent of m^5 a coefficient of its logarithm.
	Answer:	$\frac{1}{2}(\log_2 n - 5\log_2 m)$

Example 6 If $\log\ 2 \approx 0.30$ and $\log\ 3 \approx 0.48$, find the following: A) $\log 18$ B) $\log\left(\frac{1}{\sqrt[3]{2}}\right)$

A) $\log 18$ Since $18 = 2 \cdot 3^2$, $\log 18 = \log 2 + \log 3^2$ $= \log 2 + 2\log 3$	**Step 1:**	Rewrite 18 using the factors of 2 and 3. Separate the logarithm by adding the log of each factor. Make the exponents of the variables become the coefficients of the logarithms
$\approx 0.30 + 2(0.48)$	**Step 2:**	Substitute approximate values.
	Answer:	$\log 18 \approx 1.26$
B) $\log\left(\frac{1}{\sqrt[3]{2}}\right)$ Since $\frac{1}{\sqrt[3]{2}} = \frac{1}{2^{\frac{1}{3}}}$ $= 2^{\frac{-1}{3}}$, $\log\left(\frac{1}{\sqrt[3]{2}}\right) = \log 2^{\frac{-1}{3}}$ $= -\frac{1}{3}\log 2$	**Step 1:**	Factor $\frac{1}{\sqrt[3]{2}}$ using 2 as the only factor. Make the exponents of the variable become the coefficient of the logarithms.
$\approx -\frac{1}{3}(0.30)$	**Step 2:**	Substitute the approximate value.
	Answer:	$\log\left(\frac{1}{\sqrt[3]{2}}\right) \approx -0.10$

We have talked about the common log, log base ten. There is another special log that is used primarily in natural science and it is called the **natural logarithm function**. Its base is the irrational number e, which has the approximate value 2.71828. The natural logarithm of x is sometimes denoted by $\mathbf{\log_e} x$, but more often by $\mathbf{\ln}\ x$.

The exercises in this part of the lesson are just like the previous exercises. The only difference is that the symbol **ln** x is used instead of $\mathbf{log}_b\, x$. The following example illustrates:

Working with base 2 logs	**Working with base *e* logs**
1. If $\log_2 x = 5$, then $x = 2^5$.	1. If $\ln x = 5$, then $x = e^5$.
2. If $2^x = 7$, then $x = \log_2 7$.	2. If $e^x = 7$, then $x = \ln 7$.
3. $\log_2 2^5 = 5$ and $2^{\log_2 7} = 7$	3. $\ln e^5 = 5$ and $e^{\ln 7} = 7$

Logarithms and natural logarithms are also related in this way:

$\log_e e = 1$, following Properties of Logarithms 3.

$\log_e = \ln$, following the definition of natural logarithm.

$\ln e = 1$, substituting ln for $\log_e$ for first sentence.

Example 7 Simplify: $\ln \frac{1}{e^2}$

$\ln \frac{1}{e^2} = \ln e^{-2}$	**Step 1:** Write the logarithm as e to an exponent.
$= -2 \ln e$	**Step 2:** Rewrite the exponent of e as a coefficient of the logarithm.
$= -2(1)$ $= -2$	**Step 3:** Substitute $\ln e = 1$
	Answer: $\ln \frac{1}{e^2} = -2$

Example 8 If $\ln 5 \approx 1.61$, $\ln 4 \approx 1.39$, and $\ln 3 \approx 1.10$, find each of the following:

A) $\ln \sqrt[3]{16}$ B) $\ln \frac{25}{48}$

A) $\ln \sqrt[3]{16}$

$\sqrt[3]{16} = \sqrt[3]{4^2}$ $\ln \sqrt[3]{16} = \ln \sqrt[3]{4^2}$	**Step 1:** Rewrite $\sqrt[3]{16}$ as an exponent of base 4. Take the natural log of each side.
$= \ln (4^2)^{1/3}$ $= \frac{1}{3} \ln 4^2$ $= \left(\frac{1}{3}\right)(2) \ln 4$ $= \left(\frac{2}{3}\right) \ln 4$	**Step 2:** Make the exponents become the coefficient of the logarithm.

$\approx \left(\frac{2}{3}\right)(1.39)$ $\approx \frac{2.78}{3}$ ≈ 0.93	**Step 3:** Substitute the given value for $\ln 4$.
	Answer: $\ln \sqrt[3]{16} \approx 0.93$

B) $\ln \frac{25}{48}$

$25 = 5^2$ $48 = 4^2 \cdot 3$	**Step 1:** Rewrite 25 and 48 using the factors 5, 4, and 3.
$\ln \frac{25}{48} = \ln 25 - \ln 48$	**Step 2:** Rewrite using the laws of logarithms
$= \ln 5^2 - \ln 4^2 \cdot 3$ $= \ln 5^2 - (\ln 4^2 + \ln 3)$ $= 2\ln 5^2 - (2\ln 4 + \ln 3)$	**Step 3:** Substitute factorizations for 25 and 48 and apply laws of logarithms.
$\approx 2(1.61) - [2(1.39) + 1.10]$ $\approx 3.22 - (2.78 + 1.10)$ ≈ -0.66	**Step 4:** Substitute given values and simplify.
	Answer: $\ln \frac{25}{48} \approx -0.66$

Because logarithms and exponents are related, some interesting things happen when they are together in the same problem. Let's look at two properties which show the canceling effect of logs and exponents together.

Properties of Logarithms that demonstrate the relationships between Logarithms and Exponents:

1. $\log_b b^x = x$
2. $b^{\log_b x} = x$

Example 9 Simplify the following: A) $\log_8 8^x$ B) $9^{\log_9 x}$

A) $\log_8 8^x$

$\log_8 8^x = x\log_8 8$	**Step 1:** Rewrite the exponent as the coefficient.
$= x(1)$	**Step 2:** $\log_8 8 = 1$, so substitute.

Answer: $\log_8 8^x = x$

B) $9^{\log_9 x}$

Answer: Using Properties of Logarithms (#2 above), $9^{\log_9 x} = x$, so the solution is x.

Problem Set

Evaluate the expression:

1. $\log_2 1$ **2.** $\ln e$ **3.** $\log 10$

Rewrite each expression as the sum or difference of simple logarithms:

4. $\ln (6x)$ **5.** $\log (xyz)$ **6.** $\log_2 \left(\frac{x}{y}\right)$ **7.** $\log y^{12}$ **8.** $\log \left(\frac{1}{5}\right)$

Solve the following:

9. Evaluate the expression:

$$\log_3 81$$

10. Use the properties of logarithms to simplify:

$$8^{\log_8 x}$$

11. Evaluate the expression given:
ln 2 = 0.693 and ln 3 = 1.099.

$$\ln \left(\frac{2}{3}\right)$$

12. Evaluate the expression given:
ln 2 = 0.693, ln 3 = 1.099, and ln 5 = 1.609

$$\ln 30$$

13. Rewrite the expression as the sum or difference of simple logarithms. Simplify where possible.

$$\ln (e^3 x^2)$$

14. Use the properties of logarithms to simplify:

$$\ln \left(\frac{x^4}{e^3}\right)$$

15. Rewrite the expression as the sum or difference of simple logarithms.

$$\ln \left(\frac{2}{xy}\right)$$

16. Rewrite the expression as the sum or difference of simple logarithms. Simplify where possible.

$$\ln (x^3 e^2)$$

17. Rewrite the expression as the sum or difference of simple logarithms. Simplify where possible.

$$\ln (x^2 y^3)$$

18. Rewrite the expression as the sum or difference of simple logarithms. Simplify where possible.

$$\log \sqrt{\frac{x}{y}}$$

19. Evaluate the expression given.

$\log_5 7 = 1.209$ and $\log_5 3 = 0.683$

$$\log_5 \left(\frac{49}{3}\right)$$

20. Evaluate the expression given.

$\log_5 2 = 0.431$ and $\log_5 3 = 0.683$

$$\log_5 \left(\frac{2}{9}\right)$$

HA1-855: Solving Exponential Equations

You have worked with variables in expressions and in equations. In this lesson, you will learn to solve equations that have one or more variables in an exponent. Two types of equations have exponents. In the first type, the bases are the same. Let's look at a few examples of this type. If $2^x = 8$, you can solve the equation by converting both sides to the same base. Because $8 = 2^3$, you can say that $2^x = 2^3$. Now that the bases are equal, the exponents must also be equal, so $x = 3$. The preceding argument comes from the following rule: If $a^x = a^y$, then $x = y$.

Let's apply this rule to two examples.

Example 1 Solve: $2^{x+1} = 4^{x-1}$

$2^{x+1} = (2^2)^{x-1}$	**Step 1:**	Express both sides of the equation in the same base. *Note: Remember, the Law of Exponents for a Power of a Power is* $(a^m)^n = a^{mn}$ *for all positive integers.*
$2^{x+1} = 2^{2x-2}$	**Step 2:**	Simplify the right side of the equation by multiplying the exponents.
$x + 1 = 2x - 2$	**Step 3:**	Because the bases are equal, set the exponents equal.
$x + 3 = 2x$ $3 = x$	**Step 4:**	Solve the equation.
	Answer:	$x = 3$

Example 2 Solve: $3^a = \frac{1}{81}$

$3^a = \frac{1}{81}$ $= \frac{1}{3^4}$ $= 3^{-4}$	**Step 1:**	If possible, express both sides of the equation in the same base.
$3^a = 3^{-4}$	**Step 2:**	Because the bases are equal, set the exponents equal.
$a = -4$	**Step 3:**	Solve the equation.
	Answer:	$a = -4$

Unfortunately, not all exponential equations can be rewritten with the same base. When this happens, you can eliminate the variable from the exponent by using the properties of logarithms. Let's try to solve $3^x = 8$. Because you cannot convert to equivalent bases, you must convert the exponential form to logarithmic form. Therefore, $3^x = 8$ becomes $\log_3 8 = x$, so $x = \log_3 8$

Remember, $\log_3 8 = \frac{\log_8}{\log_3}$.

Using a calculator, we find that $x \approx 1.893$.

Now let's try an example. In this example, take the log of both sides to eliminate the variable from the exponent. Then, follow the usual procedures to solve for x.

Example 3 Solve: $4^{x+2} = 6^x$

$\log 4^{x+2} = \log 6^x$	**Step 1:** Because you cannot express both sides of the equation in the same base, take the log of each side.
$(x+2)\log 4 = x\log 6$	**Step 2:** Extract the exponents as factors. Keep $x + 2$ in parentheses.
$x\log 4 + 2\log 4 = x\log 6$	**Step 3:** Distribute log 4 to x and to 2.
$2\log 4 = x\log 6 - x\log 4$	**Step 4:** Collect the terms with x in them on the same side of the equation.
$2\log 4 = x(\log 6 - \log 4)$	**Step 5:** Because the terms on the right side are like terms, factor x out of each term.
$\frac{2\log 4}{\log 6 - \log 4} = x$	**Step 6:** Divide both sides by the coefficient of x in order to isolate x.
$x \approx 6.838$	**Step 7:** Find a decimal approximation with a calculator.
	Answer: $x \approx 6.838$

Here's an example with a natural logarithm:

Example 4 Solve: $5^x = e^{x-4}$

$\ln 5^x = \ln e^{x-4}$	**Step 1:** Because we cannot express both sides of the equation in the same base, take the log of each side.
$x\ln 5 = (x-4)\ln e$	**Step 2:** Extract the exponents as factors. Keep $x - 4$ in parentheses.
$x\ln 5 = x - 4$	**Step 3:** $\ln e = 1$
$4 = x - x\ln 5$	**Step 4:** Collect the terms with x in them on the same side of the equation.
$4 = x(1 - \ln 5)$	**Step 5:** Because the terms on the right side are like terms, factor x out of each term.
$\frac{4}{1 - \ln 5} = x$	**Step 6:** Divide both sides by the coefficient of x in order to isolate x.
$x \approx -6.563$	**Step 7:** Find a decimal approximation with a calculator.
	Answer: $x \approx -6.563$

Example 5 \$6,000 is invested at 7.5% interest compounded monthly. In how many years will the investment reach \$18,000?

$A = P\left(1 + \frac{r}{n}\right)^{nt}$	**Step 1:** Write the formula for compound interest.

$18000 = 6000\left(1 + \frac{0.075}{12}\right)^{12t}$	**Step 2:** Substitute the variable with given values: $A = 18{,}000$, $P = 6{,}000$, $r = 7.5\%$ or 0.075, $n = 12$ because the interest is compounded twelve times per year, and t is the unknown.
$3 = (1 + 0.00625)^{12t}$	**Step 3:** Divide both sides by 6,000 and reduce the fraction in parentheses.
$\log 3 = \log (1.00625)^{12t}$	**Step 4:** Take the log of each side.
$\log 3 = 12t\log 1.00625$	**Step 5:** Extract the exponent on the right, $12t$, as a factor.
$\frac{\log 3}{12\log 1.00625} = t$	**Step 6:** Divide both sides by the coefficient of t.
$t \approx 14.694$	**Step 7:** Find the decimal approximation with a calculator.
	Answer: The $6,000 dollar investment will reach $18,000 dollars in approximately 14.7 years.

Here's a problem involving exponential decay:

Example 6 The decay constant of a neutron (before it decays into even smaller subatomic particles) is 0.0011325 for seconds. If a physicist has 15,000,000 neutrons, how many does she have after 10,000 seconds?

$A_t = A_0 e^{-kt}$	**Step 1:** Write the formula for exponential decay.
$A_t = 15,000,000 e^{-(0.0011325)(10,000)}$	**Step 2:** Substitute variables for given values: $A_0 = 15{,}000{,}000$; $k = 0.0011325$; $t = 10{,}000$.
$A_t = 15,000,000 e^{-11.325}$	**Step 3:** Multiply the factors in the exponent.
$A_t \approx (15,000,000)(0.1207 \times 10^{-4})$	**Step 4:** Raise e to the product in step 3.
$A_t \approx 181$	**Step 5:** Multiply 15,000,000 by the result.
	Answer: Approximately 181 neutrons. Therefore, after about 2.8 hours, approximately 181 neutrons remain from the initial 15,000,000.

Example 7 Plutonium has a half-life of 24,000 years. Nuclear engineers stored 92 grams of plutonium in an underground facility in the year 2000. If the decay constant is 0.000028881, how much plutonium remains in the year 20000?

$A_t = A_0 e^{-kt}$	**Step 1:** Write the formula for exponential decay.
$A_t = 92 e^{-(0.2888 \times 10^{-4})(18,000)}$	**Step 2:** Substitute variables for given values: $A_0 = 92$; $k = 0.000028881$; $t = 20000 - 2000 = 18{,}000$.
$A_t = 92 e^{-0.519858}$	**Step 3:** Multiply the factors in the exponent.

$A_t \approx (92)(0.59460)$	**Step 4:** Raise e to the product in step 3.
$A_t \approx 54.7$	**Step 5:** Multiply 92 by the result.
	Answer: Approximately 55 grams of plutonium remain after 18 millenniums.

Problem Set

Solve:

1. $5^x = \frac{1}{125}$ **2.** $2^x = 8$ **3.** $2^x = \frac{1}{8}$ **4.** $4^x = 16$ **5.** $6^x = 36$

6. $6^x = \frac{1}{36}$ **7.** $2^x = \frac{1}{64}$ **8.** $4^{2x+4} = 16^{3x}$ **9.** $6^{x+3} = 36^x$ **10.** $2^{5x+6} = 16^{2x}$

Solve the equations below and round your answers to the nearest thousandth:

11. $4^x = 14$ **12.** $14^x = 4$ **13.** $e^{x-5} = 13$ **14.** $e^{x-1} = 17$

15. $4^{x+1} = 20^x$ **16.** $6^{x+1} = 30^x$ **17.** $11^{x-2} = 9^x$

18. If $1,000 is deposited at an interest rate of 3% compounded monthly, how long will it take for the investment to grow to $2,000? Use the formula below and round your answer to the nearest tenth.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$ where:

A = amount of investment after t years
P = original amount invested
r = interest rate
n = number of times compounded per year
t = time (number of years)

19. The value of the constant of growth for a certain bacteria is 0.08 when time is recorded in minutes. Approximately how long will it take for the bacteria to double in quantity? Use the formula below and round your answer to the nearest whole number.

$$A_t = A_o e^{kt}$$ where:

A_t = final amount
A_o = initial amount
k = constant
t = time

20. In Harrison County, Mississippi, the population has grown at an average rate of 12% a year. If the rate of growth remains constant, how many years will it take for an area with a current population of 25,000 residents to reach a population of 45,000? Use the formula below and round your answer to the nearest tenth.

$$P_t = P_o e^{kt}$$ where:

P_t = population after t years
P_o = current population
k = constant
t = time

HA1-856: Translating Exponential and Logarithmic Equations

To continue the discussion of the logarithmic and exponential functions, let's examine their relationship more closely. You know they are related due to the similar properties they have. Logarithmic and exponential functions are inverse functions of each other.

Let $y = \log_b x$

The inverse of $y = \log_b x$ is
$$\log_b y = x$$
$$y = b^x$$

Therefore, $y = \log_b x$ and $\log_b y = x$ are inverse functions.

To understand this relationship, let's look at some other inverse functions that are more familiar.

Let $y = x^2$. The inverse of $y = x^2$ is $y^2 = x$.

Therefore, you can make a general statement: $A^n = B$ if and only if $\sqrt[n]{B} = A$.

Example 1 Solve each equation:

A) $\sqrt[8]{256} = x$ B) $\sqrt[5]{-1024} = x$

A) $\sqrt[8]{256} = x$

$$\sqrt[8]{256} = x$$
$$x^8 = 256$$
$$x^8 = 128 \cdot 2$$
$$x^8 = 64 \cdot 2 \cdot 2$$
$$x^8 = 8 \cdot 8 \cdot 2 \cdot 2$$
$$x^8 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$
$$x = 2$$

Step 1: Find the prime factors of 256.

Step 2: Because eight x factors are on the left and eight 2 factors are on the right, the eighth root of both sides yields one x on the left and one 2 on the right.

Answer: $x = 2$

B) $\sqrt[5]{-1024} = x$

$$\sqrt[5]{-1024} = x$$
$$x^5 = -1024$$
$$x^5 = 256 \cdot -4$$
$$x^5 = 64 \cdot -4 \cdot -4$$
$$x^5 = 16 \cdot -4 \cdot -4 \cdot -4$$
$$x^5 = -4 \cdot -4 \cdot -4 \cdot -4 \cdot -4$$
$$x = -4$$

Step 1: Find the prime factors of –1024.

Step 2: Because five x factors are on the left and five –4 factors are on the right, the fifth root of both sides yields one x on the left and one –4 on the right.

Answer: $x = -4$

Let's look at some examples of a logarithm and its inverse.

Example 2 Solve each equation:

A) $\log_4 2 = x$ B) $\log_x 5 = -1$ C) $\log_{\frac{1}{9}} \sqrt[4]{3} = x$ D) $\log_9 x = \frac{3}{2}$

A) $\log_4 2 = x$

$4^x = 2$	**Step 1:** Rewrite the equation in exponential form.
$(2^2)^x = 2$	**Step 2:** Find a common base.
$2^{2x} = 2^1$ $2x = 1$ $x = \frac{1}{2}$	**Step 3:** Set the exponents equal to each other and solve for x.
	Answer: $x = \frac{1}{2}$

B) $\log_x 5 = -1$

$x^{-1} = 5$	**Step 1:** Rewrite the equation in exponential form.
$\frac{1}{x} = 5$ $1 = 5x$ $\frac{1}{5} = x$	**Step 2:** Solve.
	Answer: $x = \frac{1}{5}$

C) $\log_{\frac{1}{9}} \sqrt[4]{3} = x$

$\left(\frac{1}{9}\right)^x = \sqrt[4]{3}$	**Step 1:** Rewrite the equation in exponential form.
$\left(\frac{1}{3^2}\right)^x = 3^{1/4}$ $(3^{-2})^x = 3^{1/4}$	**Step 2:** Find a common base.
$3^{-2x} = 3^{1/4}$ $-2x = \frac{1}{4}$	**Step 3:** Set the exponents equal to each other and solve.
$-8x = 1$ $x = -\frac{1}{8}$	**Answer:** $x = -\frac{1}{8}$

D) $\log_9 x = \frac{3}{2}$ $x = 9^{3/2}$	**Step 1:** Rewrite the equation in exponential form
$x = \sqrt{9^3}$ $x = 27$	**Step 2:** Solve.
	Answer: $x = 27$

These last examples have been about translating a logarithmic expression into an exponential one in order to evaluate the log expression. However, you can translate logs to exponents for a variety of reasons.

Log Equation	Exponential Equation
$\log_b x = y$	$b^y = x$

Example 3 Translate each exponential equation into a logarithmic equation:

A) $3^{\sqrt{x}} = 2x - 1$ B) $e^{x-4} = y^5$

A) $3^{\sqrt{x}} = 2x - 1$	
$\log_3 (2x - 1) = \sqrt{x}$	**Answer:** The exponent has a base of 3. Take the log (base 3) of both sides.
B) $e^{x-4} = y^5$	
$\ln y^5 = x - 4$	**Answer:** The exponent with a constant base has a base of e. Take the natural log of both sides.

Example 4 Translate each log equation into an exponential equation.

A) $\log_{15} (2x^3 + 4) = x + 7$	
$15^{x+7} = 2x^3 + 4$	**Answer:** The logarithm has a base of 15. Rewrite each as an exponent with a base of 15.
B) $\ln \sqrt{x} = 3x - 1$	
$e^{3x-1} = \sqrt{x}$	**Answer:** The logarithm has a base of e. Rewrite each side as an exponent with a base of e.

Problem Set

Translate the logarithmic equation into an exponential equation:

1. $\log_2 16 = 4$

2. $\log_3 243 = 5$

3. $\log_2 1{,}024 = 10$

4. $\log_3 729 = 6$

5. $\log_5 \frac{1}{5} = -1$

6. $\log_{\frac{1}{2}} \frac{1}{8} = 3$

7. $\log_5 \frac{1}{25} = -2$

8. $\log_{\frac{1}{2}} 1 = 0$

9. $\log_{\sqrt{2}} 16 = 8$

10. $\log_{\sqrt{3}} 27 = 6$

Translate the logarithmic equation into an exponential equation and solve for *x*:

11. $\log_4 16 = x$

12. $\log_3 27 = x$

13. $\ln e^{\sqrt{2}} = x$

14. $\log_5 25 = x$

15. $\log_2 \sqrt{2} = x$

16. $\log_7 \sqrt[3]{7} = x$

17. $\log_x 64 = 2$

Evaluate the following:

18. Evaluate the logarithmic expression by translating it into an exponential equation. Remember to start with a logarithmic equation.

$$\log_7 49$$

19. Evaluate the logarithmic expression by translating it into an exponential equation. Remember to start with a logarithmic equation.

$$\log_3 \sqrt[5]{27}$$

20. Combine into a single logarithm and translate into an exponential equation:

$$\log_2 x + \log_2 (x+1) = 1$$

HA1-853: Applying the Laws of Logarithms

1. 0 **3.** 1 **5.** $\log x + \log y + \log z$

7. $\log y^{12} = 12 \log y$ **9.** 4 **11.** –0.406

13. $3 + 2 \ln x$ **15.** $\ln 2 - \ln x - \ln y$ **17.** $2 \ln x + 3 \ln y$

19. 1.735

HA1-854: Determining the Value of Common and Natural Logarithms and Irrational Exponents

1. 1.301 **3.** 0.434 **5.** 3.045 **7.** 15.673 **9.** 0.149

11. 0.356 **13.** 0.005 **15.** –6.644 **17.** 3.415 **19.** 5.907

HA1-855: Solving Exponential Equations

1. –3 **3.** –3 **5.** 2 **7.** –6 **9.** 3

11. 1.904 **13.** 7.565 **15.** 0.861 **17.** 23.899 **19.** 9 minutes

HA1-856: Translating Exponential and Logarithmic Equations

1. $2^4 = 16$ **3.** $2^{10} = 1{,}024$ **5.** $5^{-1} = \frac{1}{5}$ **7.** $5^{-2} = \frac{1}{25}$ **9.** $(\sqrt{2})^8 = 16$

11. 2 **13.** $\sqrt{2}$ **15.** $\frac{1}{2}$ **17.** 8 **19.** $\frac{3}{5}$

HA1-857: Solving Logarithmic Equations

1. 729 **3.** 36 **5.** 5 **7.** $2\sqrt{3}$ **9.** 4

11. $\frac{1}{4}$ **13.** $\frac{3}{4}$ **15.** $\frac{1}{6}$ **17.** $\pm 2\sqrt{2}$ **19.** no solution

HA1-859: Using the Algebraic System

1. Property of Negative One **3.** Zero Product Property **5.** Direct proof

7. Associative Property **9.** Inverse Property **11.** False

13. False **15.** False **17.** Inverse Property

19. Inverse Property

HA1-866: Drawing a Line Using Slope-Intercept Form and Determining if Two Lines are Parallel or Perpendicular

1. $m = -\frac{2}{5}$ and $b = \frac{1}{5}$ **3.** $m = -3$ and $b = 6$ **5.** $m = 2$ and $b = -3$