Today

- Ramsey taxation - No tax on capital (Judd 1985, Chamley 1986)
- Taxing Capital? Not a Bad Idea After All! (Conesa Kitao Krueger 2009)
- Wealth Inequality (De Nardi SED Plenary 2016)

Capital Taxation

- Generally thought to be a bad idea - Judd 1985, Chamley 1986 show analytically in neoclassical growth model
- Not true in variety of circumstances:
- Conesa Kitao Krueger 2009 provides rich quanitative exploration of implications

Basic Model and Result

Based on simple example from Werning Straub (2015), Judd (1985):

- Time is indefinite and discrete and infinite:
- Two types of agents, workers and capitalists
  - Capitalists save and derive all their income fromthe returns to capital.
  - Workers supply one unit of labor inelastically and live hand to mouth.
- Government taxes the returns to capital to pay for transfers targeted to workers.

Preferences

- $\beta < 1$
- Workers have inelasic labor supply, labor endowment $n = 1$.
- Captialists do not work
- Utility is CRRA:
- Capitalist capital C_t, workers lowercase c_t.

$$\sum_{t=0}^{\infty} \beta^t u(C_t)$$
$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

Technology

Technology. Output is obtained from capital and labor using a neoclassical constant returns production function $F(k_t, n_t)$ satisfying standard conditions.[7] Capital depreciates at rate $\delta > 0$. In equilibrium $n_t = 1$, so define $f(k) = F(k, 1)$. The government consumes a constant flow of goods $g \geq 0$. We normalize both populations to unity and abstract from technological progress and population growth. The resource constraint in period t is then

$$c_t + C_t + g + k_{t+1} \leq f(k_t) + (1 - \delta)k_t.$$

There is some given positive level of initial capital, $k_0 > 0$.

Market and Taxes

Markets and Taxes. Markets are perfectly competitive, with labor being paid wage $w_t^* = F_L(k_t, n_t)$ and the before-tax return on capital being given by

$$R_t^* = f'(k_t) + 1 - \delta.$$

The after-tax return equals R_t and can be parameterized as either

$$R_t = (1 - \tau_t)(R_t^* - 1) + 1 \qquad \text{or} \qquad R_t = (1 - \mathcal{T}_t)R_t^*,$$

where τ_t is the tax rate on the net return to wealth and $\mathcal{T}_t$ the tax rate on the gross return to wealth, or wealth tax for short. Whether we consider a tax on net returns or on gross returns is irrelevant and a matter of convention. We say that capital is taxed whenever $R_t < R_t^*$ and subsidized whenever $R_t > R_t^*$.

Government

Government Budget Constraint. As in Judd (1985), the government cannot issue bonds and runs a balanced budget. This implies that total wealth equals the capital stock $a_t = k_t$ and that the government budget constraint is

$$g + T_t = (R_t^* - R_t)\, k_t.$$

Agent Problems

Capitalist and Worker Behavior. Capitalists solve

$$\max_{\{C_t, a_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(C_t) \quad \text{s.t.} \quad C_t + a_{t+1} = R_t a_t \quad \text{and} \quad a_{t+1} \geq 0,$$

for some given initial wealth a_0. The associated Euler equation and transversality conditions,

$$U'(C_t) = \beta R_{t+1} U'(C_{t+1}) \qquad \text{and} \qquad \beta^t U'(C_t) a_{t+1} \to 0,$$

are necessary and sufficient for optimality.

Workers live hand to mouth, their consumption equals their disposable income

$$c_t = w_t^* + T_t = f(k_t) - f'(k_t) k_t + T_t,$$

which uses the fact that $F_n = F - F_k k$. Here $T_t \in \mathbb{R}$ represent government lump-sum transfers (when positive) or taxes (when negative) to workers.[8]

Planner's Problem

Planning Problem. Using the Euler equation to substitute out R_t, the planning problem can be written as[9]

$$\max_{\{c_t, C_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t (u(c_t) + \gamma U(C_t)), \tag{1a}$$

subject to

$$c_t + C_t + g + k_{t+1} = f(k_t) + (1 - \delta)k_t, \tag{1b}$$

$$\beta U'(C_t)(C_t + k_{t+1}) = U'(C_{t-1})k_t, \tag{1c}$$

$$\beta^t U'(C_t)k_{t+1} \to 0. \tag{1d}$$

Planner's Problem

The government maximizes a weighted sum of utilities with weight γ on capitalists. By varying γ one can trace out points on the constrained Pareto frontier and characterize their associated policies. We often focus on the case with no weight on capitalists, $\gamma = 0$, to ensure that desired redistribution runs from capitalists towards workers. Equation (1b) is the resource constraint. Equation (1c) combines the capitalists' first-order condition and budget constraint and (1d) imposes the transversality condition; together conditions (1c) and (1d) ensure the optimality of the capitalists' saving decision.

The necessary first-order conditions are

$$\mu_0 = 0, \tag{2a}$$

$$\lambda_t = u'(c_t), \tag{2b}$$

$$\mu_{t+1} = \mu_t \left(\frac{\sigma - 1}{\sigma \kappa_{t+1}} + 1 \right) + \frac{1}{\beta \sigma \kappa_{t+1} v_t} (1 - \gamma v_t), \tag{2c}$$

$$\frac{u'(c_{t+1})}{u'(c_t)} (f'(k_{t+1}) + 1 - \delta) = \frac{1}{\beta} + v_t (\mu_{t+1} - \mu_t), \tag{2d}$$

where $\kappa_t \equiv k_t / C_{t-1}$, $v_t \equiv U'(C_t)/u'(c_t)$ and the multipliers on constraints (1b) and (1c) are $\beta^t \lambda_t$ and $\beta^t \mu_t$, respectively.

Key Result

Theorem 1 (Judd, 1985)**.** *Suppose quantities and multipliers converge to an interior steady state, i.e.* c_t, C_t, k_{t+1} *converge to positive values, and* μ_t *converges. Then the tax on capital is zero in the limit:* $\mathcal{T}_t = 1 - R_t/R_t^* \to 0$.

The proof is immediate: from equation (2d) we obtain $R_t^* \to 1/\beta$, while the capitalists' Euler equation requires that $R_t \to 1/\beta$. The simplicity of the argument follows from strong assumptions placed on endogenous outcomes. This raises obvious concerns. By adopting assumptions that are close relatives of the conclusions, one may wonder if anything of use has been shown, rather than assumed. We elaborate on a similar point in Section 3.3.

Subsequent Reults

This result is robust

- Jones Manuelli Rossi 1997, Atkeson Chari, Kehoe 1999 , Chari and Kehoe 1999 relax assumptions, generalize

but can easily be broken

- Hubbard Judd 1986, Aiyagari 1995 Imhrohoroglu 1998 show how it can be broken in presence of idiosynchratic labor income risk and borrowing constraints
- Alvarez Burbidge Farell Palmer 1992, Erosa Gervais 2002, Garriga 2003 show not true in lifecycle framework
- Straub Werning (2015) show that convergence not general

How does this break in the lifecycle framework?

- Take equation (2d) for example. The Judd result relies on the fact the $c_t = c_{t+1} = c^\star$.
- This is not true in a lifecycle economy.
- Erosa Gervais show that if taxes can be age dependent, then Judd result holds
- If taxes can't be age dependent, then only holds if consumption is flat over the lifecycle.

Conesa Kitao Krueger 2009

- Household born one of finite type $i \in 1, ..., M$
- Have finite life, age indexed by j, work until age j_r, retire after.
- Have idiosynchratic wage η
- Laobr supply endogenous
- Save assets a in risk free bond with return r.
- No aggregate risk
- Household problem thus indexed by (a, η, i, j)
- Technology Cobb-Douglas

$$C_t + K_t - (1 - \delta)K_t + G_t \leq ZK_t^{\alpha} N_t^{1-\alpha}$$

Government

- Runs balanced-budget social security system
- Faces exogenous consumption shock G_t
- Taxes income
- Taxes consumption linearly
- Taxes capital linearly

Income Tax

$$T(y, \kappa_0, \kappa_1, \kappa_2) = \kappa_0(y - (y^{-\kappa_1} + \kappa_2)^{-1/\kappa_1})$$

- κ_0 determines average tax rate
- κ_1 determines progressivity
- κ_2 balances budget

Question

- This is a realistic description of US economy/political system
- Question: What is the optimal tax on capital?

Equilibrium

- Competetive and stationary
- Definitions standard:
  - Firms/workers optimize
  - Prices wage satisfy FOCs of production function
  - Government budget constraints balance
  - Law of motion is consistent with expectations
- Stationary equilibrium:
  - Aggregate variables grow with population

Social Welfare

The remaining ingredient of our analysis is the social welfare function ranking different tax functions. We assume that the government wants to maximize the ex ante lifetime utility of an agent born into the stationary equilibrium implied by the chosen tax function. The government's objective is thus given by

$$SWF(\kappa_0, \kappa_1, \tau_k) = \int v_{(\kappa_0, \kappa_1, \tau_k)}(a = 0, \eta = \bar{\eta}, i, j = 1)\, d\Phi_{(\kappa_0, \kappa_1, \tau_k)}. \tag{26}$$

Given that *all* newborn households start with zero assets and average labor productivity, social welfare is simply equal to average expected lifetime utility across the two ability groups.[15]

Social Welfare Results

- Optimal capital tax rate of 36%
- Income tax rate of 23%, deduction of $7200

TABLE 2—CHANGES IN AGGREGATE VARIABLES IN THE OPTIMAL TAX SYSTEM

Variable	Change in percent
Average hours worked	−0.56
Total labor supply N	−0.11
Capital stock K	−6.64
Output Y	−2.51
Aggregate consumption C	−1.63
CEV	1.33

- Large gains from distribution of consumption across types. Assumption of ex-ante identical important!

Results

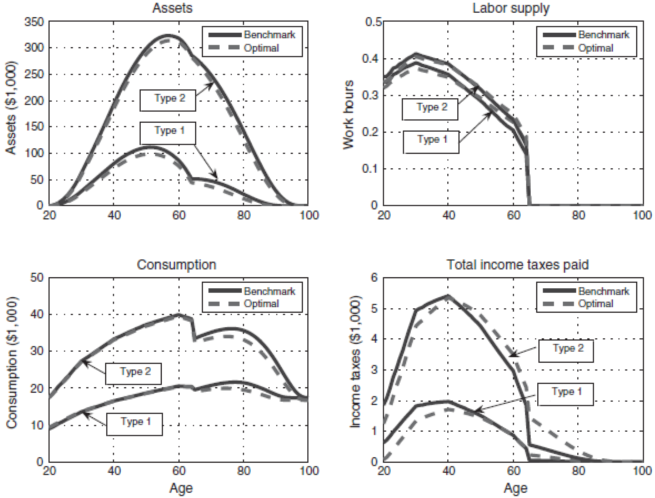

FIGURE 1. LIFE-CYCLE PROFILES OF ASSETS, LABOR SUPPLY, CONSUMPTION, AND TAXES

Results

- Non-trivial lifecycle hump-shaped profiles of labor supply key for welfare gains.

TABLE 4—SUMMARY OF QUANTITATIVE RESULTS

Model	End. lab.	BC	Type	Idio.	Life cycle	β	r	τ_k	τ_l	Prog.
M1	No	No	No	No	No	0.983	4.5	10	19	No
M2	No	No	No	No	Yes	1.001	3.2	−24	100	Yes
M3	No	Yes	Yes	Yes	Yes	1.001	4.3	−34	100	Yes
M4	Yes	No	No	No	No	0.979	4.7	20	17	No
M5	Yes	No	No	No	Yes	1.009	5.6	34	14	No
M6	Yes	No	Yes	No	Yes	1.009	5.2	32	18	Yes
M7	Yes	No	Yes	Yes	Yes	1.005	5.6	35	23	Yes
Bench	Yes	Yes	Yes	Yes	Yes	1.001	5.6	36	23	Yes

Results

What is important?

- Suppose K only used in production.
- A social planner will choose capital stock to satisfy

$$f'(K^\star) = \delta + n$$

  if no restrictions on government debt
- Labor supply - If labor supply is exogenous social optimal can be implemented via progressive labor income tax.
- How to otherwise break results: flat labor supply profile and/or age dependent income taxes

Results

To summarize, endogenous labor supply coupled with life-cycle model elements that generate a nonconstant age labor supply profile implies a robust role for positive capital income taxation, as long as the government cannot condition the tax code on age (and in the nonseparable case, even with age-dependent labor income taxes). *The capital income tax implicitly allows the government to tax leisure (labor) at different ages at different rates.* Since labor at different ages is supplied with different elasticities, the government makes use of the capital income tax for this reason.