## Word Problems? No Problem!

## MA+:

MA+:

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$x A+H$

Say hello.

What makes word problems so difficult for students?
$x \mathrm{~A}+\dot{1}$

## Word Problems? No Problem!

```
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```

    www.sarahpowellphd.com
    Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?
A 100
B 33
C 25
D 20

Solve the problem

Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?
A 100
B 33
C 25
D 20

## Solve the problem. <br> What skills are necessary to solve this problem?



Key words tied to operations is an ineffective wordproblem strategy.
(Karp et al., 2019; Powell et al., 2022)

Using a meta-cognitive strategy improves word-problem performance.
(Freeman-Green et al., 2015; Krawec et al., 2012; Montague et al., 2011; Swanson et al., 2014)

A focus on schemas improves word-problem performance. (Alghamdi et al., 2020; Cook et al., 2020; Flores et al., 2016; Fuchs et al., 2021; Griffin et al., 2019; Jitendra et al., 2013; Lein et al., 2020; Peltier et al., 2020; Powell et al., 2022; Xin \& Xhang, 2009; Zheng et
al., 2013)

## WORD PROBLEMS

Research and Information



## 1. Keywords tied to operations



Carmelita had 8 pencils fewer than Jenny. If Jenny had 18 pencils, how many pencils did Carmelita have?

Carmelita had 8 pencils fewer than Jenny. If Carmelita had 18 pencils, how many pencils did Jenny have?



Description of Single-Step Word Problems $(n=132)$

|  | Occurre |  | $\begin{array}{r} \mathrm{Al}^{2} \\ \text { keyv } \end{array}$ |  | Sch <br> spe keyw | afic $\mathrm{rds}^{\mathrm{a}}$ |  |  | Keywor <br> to co solu | ) led ct ${ }^{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Schema | $n$ | \% | $n$ | \% | $n$ | \% | $n$ | \% | $n$ | \% |
| Total | 27 | 20.5 | 26 | 96.3 | 23 | 88.5 | 5 | 19.2 | 21 | 80.8 |
| Difference | 17 | 12.9 | 17 | 100.0 | 14 | 82.4 | 2 | 11.8 | 12 | 70.6 |
| Change | 11 | 8.3 | 7 | 63.6 | 5 | 71.4 | 5 | 71.4 | 2 | 28.6 |
| Equal groups | 29 | 22.0 | 26 | 89.7 | 22 | 84.6 | 18 | 69.2 | 8 | 30.8 |
| Comparison | 10 | 7.6 | 9 | 90.0 | 9 | 100.0 | 4 | 44.4 | 5 | 55.6 |
| Ratios or proportions | 29 | 22.0 | 23 | 79.3 | 9 | 39.1 | 9 | 39.1 | 6 | 26.1 |
| Product of measures | 9 | 6.8 | 9 | 100.0 | 8 | 88.9 | 1 | 11.1 | 5 | 55.6 |
| ${ }^{3}$ When a problem featured a keyword. |  |  |  |  |  |  |  |  |  |  |



Description of Multi-Step Word Problems ( $n=84$ )

| Schema | Occurrence of schema ${ }^{\text {a }}$ |  | Any keyword |  | Keyword(s) led to correct solution ${ }^{\text {b }}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $n$ | \% | $n$ | \% | $n$ | \% |
| Total | 40 | 47.6 | 39 | 97.5 | 3 | 7.7 |
| Difference | 11 | 13.1 | 11 | 100.0 | 1 | 9.1 |
| Change | 21 | 23.8 | 19 | 95.0 | 1 | 5.3 |
| Equal groups | 49 | 58.3 | 48 | 98.0 | 1 | 2.1 |
| Comparison | 7 | 8.3 | 7 | 100.0 | 0 | 0.0 |
| Ratios or proportions | 22 | 25.0 | 16 | 76.2 | 1 | 6.3 |
| Product of measures | 7 | 8.3 | 7 | 100.0 | 2 | 28.6 |

${ }^{\text {a }}$ Sum across schemas does not equal 100 because each word problem featured more than one schema.
${ }^{\text {b }}$ When a problem featured a keyword.

Keywords are important to identify and understand

Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords ("What does more than tell you about?")

But, do not tie a keyword to a specific operation!

## 2. Presenting problems by operation

1. Noah had 12 books. He got 5 more bo did Nooh have in all?
2. Edward had 5 toy cars cars did Edward have
3. Mariela collected 11 fe How many feathers dic
4. Bonnie found 8 rocks on he backyard. How many rocly

5. Zookeeper Als
qual number ot 567 bananas. How mc How many are left ove

VISION BLEMS
onkey at the zoo an nonkeys in the 200 and ach monkey get? And ach monkey get? And
elfz elf? k them up equally ox and how much

## Teach an attack strategy

Teach about schemas
$x A+1$

## WORD PROBLEMS

Attack Strategies

| SO LVE |
| :--- |
| Study the problem |
| Organize the information |
| Line up a plan |
| Verify the plan |
| Examine the answer |

$$
\begin{aligned}
& \text { UPS Check } \\
& \text { Understand }
\end{aligned}
$$

## R-CUBES

Read the problem
Circle key numbers
Underline the question
Box action words
Evaluate steps
Solve and check

## RIDE

Read the problem.
Identify the relevant information.
Determine the operation and unit for the answer.
Enter the correct numbers and calculate, then check the answer.

## RICE

## RIDGES

Read and record the problem. Illustrate your thinking. Compute.
Explain your thinking.
Read the problem.
I know statement.
Draw a picture.
Goal statement.
Equation development.
Solve the equation.

## SUPER

Slowly read the story problem twice.
Underline the question and circle the numbers you need.
picture it. Draw the scenario to show what is happening. Explain the problem with a number sentence.
Rewrite the answer in a sentence.

## STAR

## SHINES

Slowly and carefully read the problem. Highlight or underline key information. Identify the question by drawing a circle around it.
Now solve the problem. Show your work.
Examine your work for precision, accuracy, and clarity.
Share your answer by writing a sentence.

Stop and read the problem carefully.
Think about your plan and
the strategy you will use.
Act. Follow your plan and solve
the problem.
Review your answer.

## SOLVE

Study the problem.
Organize the facts.
Line up the plan.
Verify the plan with computation.
Examine the answer.

## UPS <br> Understand

Read and explain.

How will you solve the problem?

## R-CUBES

## Read the problem.

Circle key numbers.
Underline the question.
Box action words.
Evaluate steps.
Solve and check.

Solve
Set up and do the math!

## $\checkmark$ CHECK

Does your answer make sense?

Share your favorite attack strategy.

## Teach an attack strategy

Teach about schemas
$x A+1$

## Total

## Difference

## Change

# Equal Groups 

## Comparison

## Ratios/Proportions



Meanings of subtraction

## $\times A+H$

Count one set, count another set, put sets together, count sum


## Change

## Addition

Start with a set, add the other set, count sum


## Total

Parts put together into a total

Annette spotted 4 cardinals and 5 blue jays. How many birds did Annette see?

## Change

An amount that increases or decreases

Joan had \$4. Then they earned \$5 for cleaning their room. How much money does Joan have now?

## $3+9=$

If you are from Illinois: What's a Total story to show addition?
If you are from elsewhere: What's a Change/Join story to show addition?

Start with a set, take away from that set, count difference


Compare two sets, count difference


$$
5-3=2
$$

## Change

An amount that increases or decreases

Joshuan had 9 cookies. Then they ate 2 of the cookies. How many cookies does Joshuan have now?

## Difference

Greater and lesser amounts compared for a difference

Sisal hiked 9 miles. Taylor hiked 2 miles. How many more miles did Sisa hike? (How many fewer did Taylor hike?)

If you prefer winter: What's a Change/Separate story to show subtraction?
If you prefer summer: What's a Difference story to show subtraction?

## Total

Difference

Change

| Schema and Definition | Equations and Graphic Organizers |  |  |  | Examples |  |  | Variations |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total (Combine; Part-partwhole) Parts combined for a sum | $P 1+P 2=T$ <br> (part + part $=$ total) |  |  |  | Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether? | Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have? |  | More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have? |
|  |  |  |  |  |  |  |  |  |
| Difference <br> (Compare) <br> Sets <br> compared <br> for a difference | $B-s=D$ <br> (bigger - smaller = difference) |  | $G-L=D$ <br> (greater - less $=$ difference) <br> (greater) |  | Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha? | Bigger/greater unknown: <br> Tabitha wrote 25 more words than sasha. If Sasha wrote 85 words, how many words did Tabitha write? | smaller/lesser unknown: <br> Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did sasha write? | (None) |
| Change <br> (Join; <br> Separate) <br> An amount <br> that <br> increases or <br> decreases | $S T+/-C=E$ <br> (start $+/-$ change $=$ end) |  |  |  | End (decrease) unknown: <br> Jorge had \$52. Then, he spent $\$ 29$ at the ballpark. How much money does Jorge have now? | Change (decrease) unknown: <br> Jorge had $\$ 52$ but spent some money when he went to the ballpark. Now, Jorge has $\$ 23$. How much did Jorge spend at the ballpark? | Start (increase) unknown: <br> Jorge has some money, and then he earned $\$ 16$ for babysitting. Now, Jorge has $\$ 68$. How much money did he have to start with? <br> start (decrease) unknown: <br> Jorge had some money. Then, he spent $\$ 29$ at the ballpark and has $\$ 23$ left. How much money did Jorge have before going to the ballpark? | Multiple changes: <br> Jorge had \$78. He stopped and bought a pair of shoes for $\$ 42$ and then he spent $\$ 12$ at the grocery. How much money does Jorge have now? |

Material collected from: Griffin \& Jitendra, 2009; Fuchs et al., 2014; Fuchs, Seethaler, et al., 2008; Fuchs et al., 2010; Jitendra, 2002; Kintsch \& Greeno, 1985; Van de Walle, Karp, \& Bay-Williams, 2013.


Total
Part-part-whole Combine

Parts put together into a total

Mandy saw 3 canoes and 8 kayaks. How many boats did Mandy see?

Mandy saw 11 boats. If 3 of the boats were canoes, how many were kayaks?

Mandy saw 11 boats. 8 of the boats were kayaks, how many were canoes?

## Total

"Are parts put together for a total?"

Total

## P1 + P2 $=T$




Total

## Share a Total problem.

$x \mathrm{~A}+\dot{1}$

## Difference



## Difference

Greater and lesser amounts compared for a difference

L'Tanya has 10 pencils. Vickie has 4 pencils. How many more pencils does L'Tanya have?

L'Tanya has 6 more pencils than Vickie. If Vickie has 4 pencils, how many does L'Tanya have?

Vickie has 6 fewer pencils than L'Tanya. L'Tanya has 10 pencils. How many pencils does Vickie have?

Difference

Greater amount

Lesser
amount

Total
"Are parts put together for a total?"

Difference
"Are amounts compared for a difference?"

## Difference

## $5-$



## Difference



Difference

## Share a Difference problem.

## Change

Additive Word Problems

| G. |  |
| :--- | :--- |
| A plant was 3 | $3 / 4$ inches tall at the beginning of |

A plant was 3 Martina has some money in her bank account June. By the end of July, the plant was $91 / 8$ inch- Then, she spent $\$ 135.69$ and has a balance of es tall. How many inches did the plant grow in $2-\$ 24.80$. How much money did Martina have to

1. Hui saved $\$ 70$ in January. In February, she spent
$\$ 64$ of the money she saved. She saved $\$ 92$ more
in March. How much has Hui saved by the end of
March?

## Change

An amount that increases or decreases

Annette had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Annette have now?

Annette had 6 notebooks. Then, she bought a few more notebooks. Now, Annette has 9 notebooks. How many notebooks did she buy?

Annette had some notebooks. Then, she bought 3

Change
amount

Start amount

## Change

An amount that increases or decreases

Jenny baked 20 cookies. Then, she ate 3 of the cookies. How many cookies does Jenny have now?

Jenny baked 20 cookies. Then, she ate some of the cookies. Now, she has 17 cookies. How many cookies did Jenny eat?

Jenny baked some cookies. She ate 3 of the cookies and

Change amount

Start amount has 17 cookies left. How many cookies did Jenny bake?

Total
"Are parts put together for a total?"

Difference
"Are amounts compared for a difference?"

Change
"Does an amount increase or decrease?"
$1 x A+H$

Change
St $+1-C=$


## Change

Additive Word Problems

| G. |  |
| :--- | :--- |
| A plant was 3 | $3 / 4$ inches tall at the beginning of |

A plant was 3 Martina has some money in her bank account June. By the end of July, the plant was $91 / 8$ inch- Then, she spent $\$ 135.69$ and has a balance of es tall. How many inches did the plant grow in $2-\$ 24.80$. How much money did Martina have to

1. Hui saved $\$ 70$ in January. In February, she spent
$\$ 64$ of the money she saved. She saved $\$ 92$ more
in March. How much has Hui saved by the end of
March?

Change

## Share a Change problem.



## Schema Check!

## Change

Pablo goes to a stamp show where he can share, buy, and sell stamps.

## 26. Part A

The first day, Pablo starts with 744 stamps. He buys 27 stamps from his friend. He then sells 139 stamps.

What is the total number of stamps that Pablo has after the first day of the stamp show?

## Difference

The graph below shows the number of pounds of plastic the Keller family recycled for five months.

Recycled Plastic


Each $\square$ means 20 pounds.

Based on the graph, how many more pounds of plastic did the family recycle in July than in April?

## Total

Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.

10. Part A

What is the total number of packages Mr. Conley delivered on Monday and Tuesday?
(4) 300
(8) 340
© 350
(2) 360

## Total

## Difference

## Change

# Equal Groups 

## Comparison

## Ratios/Proportions



Show the groups, show the amount for each group, count product

$3 \times 2=6$

Show the groups, show the amount for each group, count product

$$
3 \times 2=6
$$

## Comparison

Show a set, then multiply the set

$3 \times 2=6$

## Equal Groups

Groups multiplied by number in each group for a product

Callie has 2 boxes of crayons. There are 12 crayons in each box. How many crayons does Callie have altogether?

## Comparison

Set multiplied by a number of times for a product

Carmen had 12 stickers. Phoenix had 2 times as many stickers as Carmen. How many stickers did Phoenix have?

## $2 \times 5=$

If you wear glasses: What's an Equal Groups story to show multiplication?
If you don't wear glasses: What's a Comparison story to show multiplication?
$x \mathrm{~A}+\cdots$

Show the dividend, divide equally among divisor, count quotient


Show the dividend, make groups of the divisor, count groups

$$
8 \div 2=4
$$

## Equal Groups

Groups multiplied by number in each group for a product

Taylor has 12 pencils. She wants to share them equally among her 2 friends. How many pencils will each friend receive?

Taylor has 12 pencils. She put them into pencil pockets with 6 pencils each. How many pencil pockets did Taylor use?

## $12 \div 4=$

If you like comedies: What's a Partitive story to show division?
If you like dramas: What's a Quotative story to show division?

## Total

## Difference

## Change

# Equal Groups 

## Comparison

## Ratios/Proportions



Multiplicative Word Problems

| J. | K. |
| :--- | :--- |
| Lola baked 6 pies. For each pie, Lola used 5 apples. |  |
| How many apples did Lola use? |  | \(\begin{aligned} \& Jane bought 112 light bulbs. The light bulbs come <br>

\& in packs of 4. How many packs of light bulbs did\end{aligned}\) Jane buy?

Lachary has 3 feet of string. He makes braclets
Zachary has 3 feet of string. He makes braclets,
and each bracelet needs $51 / 4$ inches of string. How
many bracelets could Zachary make?

Groups multiplied by number in each group for a product

Mandy has 2 boxes. There are 6 muffins in each box. How many muffins does Mandy have?

Mandy has 12 muffins. They want to place them equally into 2 boxes. How many muffins will Mandy place in each box?

Mandy has 12 muffins. They put them into boxes with 6 muffins each. How many boxes did Mandy use?

Equal Groups
"Are there groups with an equal number in each group?"

## Equal Groups

$$
\begin{aligned}
& G R \times N=P \\
& G R \times E=P
\end{aligned}
$$

$\times A+1 \dot{1}$

Multiplicative Word Problems

| J. | K. |
| :--- | :--- |
| Lola baked 6 pies. For each pie, Lola used 5 apples. |  |
| How many apples did Lola use? |  | \(\begin{aligned} \& Jane bought 112 light bulbs. The light bulbs come <br>

\& in packs of 4. How many packs of light bulbs did\end{aligned}\) Jane buy?

Lachary has 3 feet of string. He makes braclets
Zachary has 3 feet of string. He makes braclets,
and each bracelet needs $51 / 4$ inches of string. How
many bracelets could Zachary make?

Equal Groups

## Share an Equal Groups problem.

$x A+1$

## Comparison

| M. | N. |
| :--- | :--- |
| Enrique has 2 times as many pencils as Ava. Ava | Susan has 7 times as many books as Mo. Mo has 18 |

Enrique has 2 times as many pencils as Ava. Ava
has 6 pencils. How many pencils does Enrique have?

## Comparison

Set multiplied by a number of times for a product

Joan ran 6 minutes. L'Tanya ran 4 times longer than Joan. How many minutes did l'Tanya run?

## Equal Groups

"Are there groups with an equal number in each group?"

Comparison
"Is a set compared a number of times?"

## Comparison


(set)

## Comparison

| M. | N. |
| :--- | :--- |
| Enrique has 2 times as many pencils as Ava. Ava | Susan has 7 times as many books as Mo. Mo has 18 |

Enrique has 2 times as many pencils as Ava. Ava
has 6 pencils. How many pencils does Enrique have?

## Comparison

## Share a Comparison problem.

## Ratios/Proportions



## Ratios/Proportions

Description of relationships among quantities

Melissa baked cookies and brownies. The ratio of cookies to brownies was $3: 5$. If she baked 25 brownies, how many cookies did she bake?

Emma typed 56 words in 2 minutes. At this rate, how many words could Emma type in 7 minutes?

## Equal Groups

"Are there groups with an equal number in each group?"

Comparison
"Is a set compared a number of times?"
Ratios/Proportions
"Are there relationships among quantities

$x A+H$

## Ratios/Proportions



## Ratios/Proportions



## Ratios/Proportions

Share a Ratios or Proportions problem.


## Schema Check!

## Equal Groups

Mr. Kowolski ordered 35 boxes of granola bars. Each box contained 24 granola bars.

What is the total number of granola bars Mr. Kowolski ordered?

## Ratios/Proportions

A company makes 625 cell phone cases each day. How many cell phone cases does the company make in 31 days?

## Comparison

Danielle's full-grown dog weighs 10 times as much as her puppy. The puppy weighs 9 pounds.

Enter the number of pounds the full-grown dog weighs.

## Total

## Difference

## Change

# Equal Groups 

## Comparison

## Ratios/Proportions



Ratio/
Proportion

$$
x x
$$



Ratio/
Proportion

$$
x x
$$



Ratio/
Proportion

$$
x x
$$



Ratio/
Proportion

$$
x x
$$



Ratio/
Proportion

$$
x x
$$



Ratio/
Proportion

$$
x x
$$

## Multi-Step Problems

```
R.
Leslie had 3 pizzas. Each pizza was cut into 8
slices. Leslie ate 2 slices. How many slices were left?
Mr. Kahn gave away 8 blue balloons and 6 red slices. Leslie ate 2 slices. How many slices were left? balloons. He gave away 3 times the number of white balloons as red balloons. What was the total number of balloons Mr. Kahn gave away?
```

An egg farm packages 264 total cartons of eggs each month. The farm has 3 different sizes of cartons The small carton hold 8 eggs, and $1 / 6$ of the total cartons are small.
The medium carton holds 12 eggs, and $2 / 3$ of the total cartons are medium
The large carton holds 18 eggs, and the rest of the total cartons are large.
Determine how many each size of carton is needed each month. Then determine how many eggs are needed to fill the 264 cartons.



## https://intensiveintervention.org

National Center on
INTENSIVE INTERVENTION
at American Institutes for Research

| Intensive | Tools | Implementation | Intervention | Information |
| :--- | :--- | :--- | :--- | :--- |
| Intervention ₹ | Charts v | Support - | Materials ~ | For... • |

## Intensive Intervention in <br> Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction This content is designed to support faculty and professional development providers with instructing p


MODULE 5: INTENSIVE MATHEMATICS INTERVENTION: INSTRUCTIONAL STRATEGIES service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

> Intensive instruction was recently identified as a high-leverage practice in special educations , and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

> NCII, through a collaboration with the University of Connecticut and the National Center on Leadership in Intensive Intervention and with support from the CEEDAR Center『, developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the Features of Explicit Instruction Course and so we suggest that users complete both courses.



## Instructional Routines for <br> Mathematics Intervention



The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 2-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples - each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

## TH

https://spedsupportstage.tea.texas.gov/resource-library/instructional-routines-mathematics-intervention
https://ies.ed.gov/ncee/wwc/PracticeGuide/26


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$x A+H$

