# Math 1497 - Calc 2

## Lines & Planes

Here we consider a selection of problems

Ex 1 Find the eq$^n$ of the line through

$P(5,-3,-4)$ and parallel to $\vec{v} = \langle 2,-1,3\rangle$

Sol$^n$

$$x = 5 + 2t$$
$$y = -3 - t$$
$$z = -4 + 3t$$

Ex 2 Find the eq$^n$ of the plane through

$P(1,2,3)$ and parallel to $x - y + 3z = 4$

Sol$^n$: our plane will have the same $\vec{n} = \langle 1,-1,3\rangle$

so has

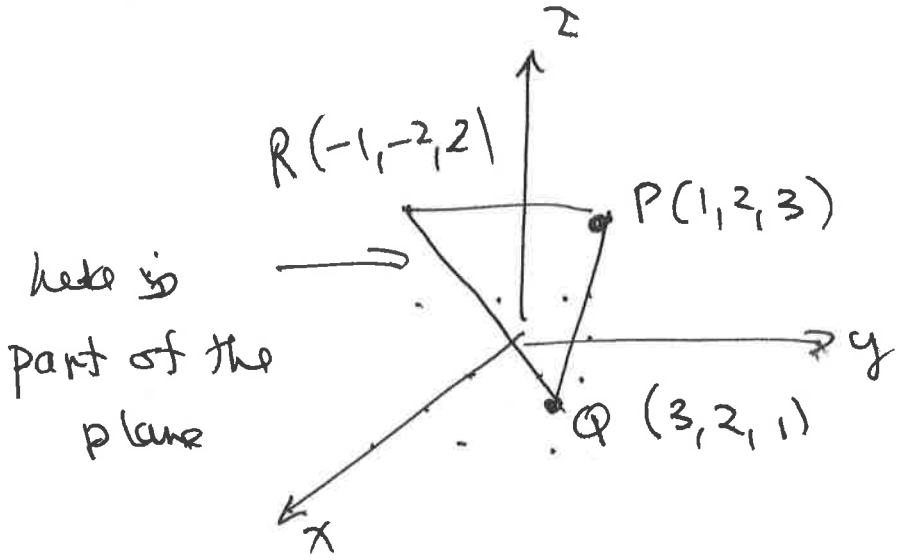
$$1(x-1) - 1(y-2) + 3(z-3) = 0$$

or $x - y + 3z = 8$

Ex3 Find the eq$^n$ of the plane through

$P(1,2,3)$ $Q(3,2,1)$ $R(-1,-2,2)$

Sol$^n$ First we plot the pts




In this problem, we will create 2 vectors and then cross them to get $\bar{n}$

$\bar{PQ} = \langle 2, 0, -2 \rangle$

$\bar{PR} = \langle -2, -4, -1 \rangle$

$$\bar{PQ} \times \bar{PR} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 0 & -2 \\ -2 & -4 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -2 \\ -4 & -1 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & -2 \\ -2 & -1 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & 0 \\ -2 & -4 \end{vmatrix} \vec{k}$$

$$= -8\vec{i} + 6\vec{j} - 8\vec{k}$$

$\hat{n} = \langle -8, 6, -8 \rangle$

For the plane we will use any pt so we will use $(1,2,3)$ so eq$^n$ of plane is

$$-8(x-1)+6(y-2)-8(z-3)=0$$

divide by $-2$

$$4(x-1)-3(y-2)+4(z-3)=0$$

or $$4x-3y+4z=10$$

check the other 2 pts lie on plane

$R(-1,-2,2)$ so $-4+6+8=10$ ✓

$Q(3,2,1)$ so $12-6+4=10$ ✓

Ex 4 Find the eq$^n$ of the plane that contains the lines

$$\begin{aligned} x &= 1-2t \\ y &= 4+t \\ z &= 6 \end{aligned}$$

&

$$\begin{aligned} x &= 2-3s \\ y &= 1+4s \\ z &= 2-s \end{aligned}$$

so the plane contains the vectors

$$\bar{u} = \langle -2, 1, 1 \rangle \text{ & } \bar{v} = \langle -3, 4, -1 \rangle$$

so $\hat{n} = \bar{u} \times \bar{v} = \begin{vmatrix} i & j & k \\ -2 & 1 & 1 \\ -3 & 4 & -1 \end{vmatrix} = \langle -5, -5, -5 \rangle$

as for a pt, as the 1st line is on the plane

choose $t = 0$ so $P(1, 4, 0)$

Plane $-5(x-1) - 5(y-4) - 5(z-0) = 0$

divide by $-5$ $\quad x - 1 + y - 4 + z = 0$

$$\text{or} \quad x + y + z = 5$$

check sub in line:

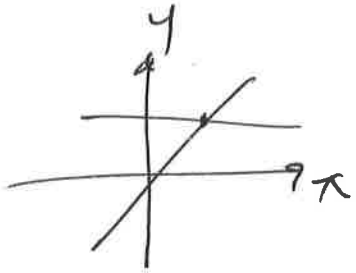
$$(1-2t) + (4+t) + t = 5 \; ?$$

$$1 - 2t + 4 + t + t = 5 \; \checkmark$$

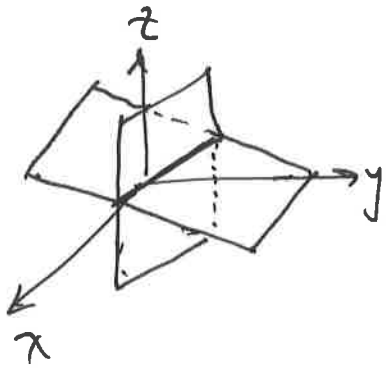
$$(2-3s) + (1+4s) + (2-s)$$

$$= 2 - 3s + 1 + 4s + 2 - s = 5 \; \checkmark$$

# Intersection of Planes




Just as lines can intersect so can planes. However, when they do, they intersect along a line




Ex: Find the intersection of

$3x+2y-z=7$ $P_1$

$x-4y+2z=0$ $P_2$

Sol" First solve $P_1$ for $z$ so $z=3x+2y-7$

Then sub into $P_2$ so

$x-4y+2(3x+2y-7)=0$

$x-4y+6x+4y-14=0$

so $7x=14$ so $x=2$ then sub

so $z=6+2y-7$

$\Rightarrow z-2y=-1$ or $2y-z=1$

if $y=t$, $z=-1-2t$ $x=2$ ← this is the line!