

# *Math for Life*

## **CONTENTS**

### *Topics*

1. Units and Conversions
2. Exponents and Powers of Ten
3. Reading and Reporting Numerical Data
4. Making Solutions
5. pH and Buffers
6. Rates, Reaction Rates, and  $Q_{10}$
7. Mapping Genes
8. Punnett Squares
9. Radioactive Dating

### *Reference Tables*

The Greek Alphabet/Symbols  
 Numerical Prefixes  
 Units  
 Constants  
 Useful Formulas  
 The Electromagnetic Spectrum  
 Microscopy Math  
 The Laws of Thermodynamics  
 A Biologist's Periodic Table  
 Biologically Important Elements  
 Molecular Weight and Formula Weight  
 Buffers  
 Functional Groups  
 Genetic Code  
 Biological Macromolecules  
 Biochemical Reactions  
 Evolutionary Time



Use the Remind Me button to link to corresponding explanations.

# Math for Life

Page 1

## TOPIC 1: UNITS AND CONVERSIONS

### UNITS 1.1: Why Use Units?

If you asked me to estimate the size of a parking spot and I said, “It’s about 4,” would you park there?

Clearly, it depends on whether I mean 4 feet, 4 square meters, or 4 car lengths. By itself “4” does not mean very much. So, by saying only “4” I have given you some information but it’s not terribly useful.

If you have made a measurement, you *want* to say what the units are; otherwise, why bother to make the measurement in the first place?



**The Point:** *Without units, the meaning of the number is lost.*

A numerical expression without units is considered incomplete or wrong because without units the number has lost much if not all of its meaning. (When you are working with dimensionless numbers, however, there are no units. Also, when you are working with **molecular weights**, the units are usually not indicated. In these cases the number’s meaning is preserved even though there are no units mentioned.)

You can use a knowledge of units to help you do math. For example, units can help you to set up equations correctly, and they can help you check your work.



**The Point:** *The units on the left hand side (LHS) of the equation must be the same as the units on the right hand side (RHS).*

Put another way, the units must balance. If you are checking your calculations and you find that the units do not balance, then you know right away that you’ve gone wrong somewhere.

Thus, an easy way to do a first check on your calculations is to make sure the units balance (note: this check alone will *not* tell you if the magnitude of the number is correct.)

You can do a first check only after you have written down every unit. Thus, rule 1 is:

*Write down every unit every time you write an expression or an equation.*

It may seem like a great bother, especially when you are not working on the final copy. But *do not give in to the urge to skip units*. Writing down units *every single time* you write an expression may seem tedious (and sometimes it definitely is!) but it *will* save you lots of time in the long run, because you will get your equations right the first time.



**The important idea:** when a unit in the denominator is the same as a unit in the numerator, they **cancel**.

**EXAMPLE:** Convert  $4.502 \times 10^3 \mu\text{L}$  to mL.

We know  $1 \text{ mL} = 10^3 \mu\text{L}$ , so  $\frac{1 \text{ mL}}{10^3 \mu\text{L}}$  equals one.

You can multiply any number by one without changing its magnitude. That's the information to use to convert units.

$$4.502 \times 10^3 \mu\text{L} \times \frac{\text{mL}}{10^3 \mu\text{L}} = ? \text{ mL}$$

Canceling:  $4.502 \times \cancel{10^3} \cancel{\mu\text{L}} \times \frac{\text{mL}}{\cancel{10^3} \cancel{\mu\text{L}}} = 4.502 \text{ mL}$

Check: On the LHS, the units are  $\mu\text{L}$ , on the RHS the units are mL. Milliliters do in fact equal milliliters; so, units balance, the equation is set up correctly.

Checking that units balance will help you check your work. The units on the LHS must match the units on the RHS; to see if they do, cancel units. It does not matter how long your expression is; however, if it is long, it can be helpful to re-write a "units only" version.

**EXAMPLE:**  $4.502 \times 10^3 \mu\text{L} \times 10^{-3} \frac{\text{mL}}{\mu\text{L}} \times 58.44 \frac{\text{g}}{\text{mol}} \times 27.95 \frac{\text{mol}}{\text{mL}} \times 10.80 \frac{\text{J}}{\text{g}} = 7.942 \text{ J}$

$$\cancel{\mu\text{L}} \times \frac{\cancel{\text{mL}}}{\cancel{\mu\text{L}}} \times \frac{\cancel{\text{g}}}{\cancel{\text{mol}}} \times \frac{\cancel{\text{mol}}}{\cancel{\text{mL}}} \times \frac{\text{J}}{\cancel{\text{g}}} = \text{J}$$

The units on the LHS (J) are the same as in the units on the RHS (also J). The units balance. The equation is set up correctly.

---

**CHECKING EQUATIONS SHORTCUT:**

**Always write down units.**

**Cancel and check that units balance.**

---

To convert among units, you need to pick an appropriate fraction by which to multiply the LHS. How do you do that?

**UNITS 1.2: Converting Among Units: A Five-Step Plan**

The key to getting conversions right is to balance units of measurement, using the following basic ideas:

When two measures are equal, one divided by the other has a magnitude of one.

**EXAMPLES:** 1.00 meter = 1.09 yards

so, 
$$\frac{1\text{m}}{1.09\text{yd}} = 1$$

or equivalently: 
$$\frac{1.09\text{yd}}{1.00\text{m}} = 1$$

A fraction like this is called a unit fraction (in this case, the word unit refers to the “1”).

Multiplying by a unit fraction is like multiplying by 1; that is, multiplying by a unit fraction does not change the magnitude of your measure, only its units. So, if you want to convert units without changing the magnitude, you can multiply by unit fractions.

**EXAMPLE:** Convert 10 meters to yards.

$$10.0\text{m} \times \frac{1.09\text{yd}}{1.00\text{m}} = 10.9\text{yd} \quad \text{These units balance.}$$

$$\frac{\cancel{m} \times \text{yd}}{\cancel{m}} = \text{yd} \qquad \text{Meters cancel.}$$

So, to convert units, use these key ideas:



1. You can multiply or divide by a unit fraction without changing the meaning of the equation.
2. You can cancel units.

The trick to conversions is to multiply the LHS of the equation by unit fractions as many times as needed to make the units match the RHS.

***Building the right unit fraction***

To start, write everything down. In fact, write down a place to put everything before you begin.

**Here is an example:**

Mitochondria are typically about 1 micrometer (1 micron) in diameter. How wide are they in picometers?

**Step one:** Write down the question as an equation. Be sure to write the units.

$$1 \mu\text{m} = ? \text{pm}$$

**Step two:** Count the number of different units that appear in the equation.

micrometers and picometers; two different units

**Step three:** Draw a place for as many fractions as you have units so that they can become unit fractions. Put them on the LHS.

$$1 \mu\text{m} \times ( \text{---} ) \times ( \text{---} ) = ? \text{pm}$$

**Step four:** Dedicate one fraction to each unit. Arrange the units in the fractions so that the cancellations you need can happen and so that correct new units will appear:

You need  $\mu\text{m}$  in the denominator to cancel the  $\mu\text{m}$  on the LHS, and you need  $\text{pm}$  in the numerator to balance the  $\text{pm}$  on the RHS.

$$1\mu\text{m} \times \left( \frac{\quad}{\mu\text{m}} \right) \times \left( \frac{\text{pm}}{\quad} \right) = ? \text{pm}$$

**Step five:** Make your fractions equal to 1. Do this by looking for relationships you know. You could do this in at least two ways.

[1] You know or can find out from consulting the table of [PREFIXES](#) listed in REFERENCES, that  $1 \mu\text{m} = 10^{-6} \text{m}$  and  $1 \text{pm} = 10^{-12} \text{m}$ .

You can use the information as follows to make useful unit fractions:

$$\text{If } 10^{-6} \text{m} = 1\mu\text{m} \text{ then, } \frac{10^{-6} \text{m}}{1\mu\text{m}} = 1.$$

and

$$\text{If } 10^{-12} \text{m} = 1\text{pm} \text{ then, } \frac{1\text{pm}}{10^{-12} \text{m}} = 1.$$

Now, fill in your equation:

$$1\mu\text{m} \times \frac{10^{-6} \text{m}}{1\mu\text{m}} \times \frac{1\text{pm}}{10^{-12} \text{m}} = ? \text{pm}$$

or, equivalently,

$$\mu\text{m} \times \frac{1\text{m}}{10^6 \mu\text{m}} \times \frac{10^{12} \text{pm}}{1\text{m}} = ? \text{pm}$$

Now, cancel both  $\mu\text{m}$  and  $\text{m}$ , solve, and you find that  $1 \mu\text{m} = 10^6 \text{pm}$ .

[2] If you happen to know that  $10^{-6} \mu\text{m} = 1 \text{pm}$ , you can combine your two unit fractions into a single unit fraction. Use one with  $\mu\text{m}$  in the denominator. That is, because

$$1\text{pm} = 10^{-6} \mu\text{m}, \quad \frac{1\text{pm}}{10^{-6} \mu\text{m}} = 1$$

This is what your equation looks like:

$$1\mu\text{m} \times \left( \frac{1\text{pm}}{10^{-6}\mu\text{m}} \right) = ?\text{pm}$$

Now, cancel microns, do the multiplication, and solve.

Once again  $1\mu\text{m} = 10^6\text{pm}$ .

Either way, you are simply multiplying by unit fractions (i.e., “1”) as many times as you need to. Here’s another set of examples. We’ll show the five steps for each example.

**EXAMPLE:** Convert  $5 \times 10^3$  Angstroms to micrometers.

1.  $5 \times 10^3 \text{ \AA} = ? \mu\text{m}$

2. There are two different units appearing,  $\text{\AA}$  and  $\mu\text{m}$ .

3.  $5 \times 10^3 \text{ \AA} \times \left( - \right) \times \left( - \right) = ? \mu\text{m}$

4.  $5 \times 10^3 \text{ \AA} \times \left( \frac{\mu\text{m}}{\text{\AA}} \right) = ? \mu\text{m}$

5.  $5 \times 10^3 \text{ \AA} \times \left( \frac{10^{-10} \text{ m}}{\text{\AA}} \right) \times \left( \frac{10^6 \mu\text{m}}{1 \text{ m}} \right) = ? \mu\text{m}; \underline{\underline{5 \times 10^3 \text{ \AA} = 5 \times 10^{-1} \mu\text{m}}}$

Convert  $5 \times 10^3$  Angstroms to millimeters.

1.  $5 \times 10^3 \text{ \AA} = ? \text{ mm}$

2. There are two different units appearing.

3.  $5 \times 10^3 \text{ \AA} \times \left( - \right) \times \left( - \right) = ? \text{ mm}$

4.  $5 \times 10^3 \text{ \AA} \times \left( \frac{\text{mm}}{\text{\AA}} \right) = ? \text{ mm}$

5.  $5 \times 10^3 \text{ \AA} \times \left( \frac{10^{-10} \text{ m}}{\text{\AA}} \right) \times \left( \frac{1 \text{ mm}}{10^{-6} \text{ m}} \right) = ? \text{ mm}; \underline{\underline{5 \times 10^3 \text{ \AA} = 5 \times 10^{-4} \text{ mm}}}$

Convert  $5 \times 10^3$  Angstroms to meters.

1.  $5 \times 10^3 \text{ \AA} = ? \text{ m}$

2. There are two different units appearing.

3.  $5 \times 10^3 \text{ \AA} \times \left(-\right) \times \left(-\right) = ? \text{ m}$

4.  $5 \times 10^3 \text{ \AA} \times \left(\frac{-}{\text{\AA}}\right) \times \left(\frac{\text{m}}{-}\right) = ? \text{ m}$

5.  $5 \times 10^3 \text{ \AA} \times \left(\frac{1\text{m}}{10^{10} \text{ \AA}}\right) = ? \text{ mm}; \underline{5 \times 10^3 \text{ \AA} = 5 \times 10^{-7} \text{ m}}$

Note: Nothing happened to the 5! If all you are doing is multiplying by powers of 10, the mantissa is equal to 1, and anything times 1 equals itself.



### UNITS 1.3: More Conversions Among Units

Use the following information when doing conversions:

1. You can multiply or divide by a unit fraction without changing the meaning of an equation.



2. You can cancel units.

The trick to conversions is to multiply the left hand side (LHS) of the equation by unit fractions as many times as needed to make the units match the right hand side (RHS).

To start, write everything down. Then write down a place to put everything before you begin.



**EXAMPLE:** How many moles of NaCl are there in 1.0 mg?

**Step one:** Write down the question as an equation.

$$1.0\text{mg NaCl} = ? \text{ mol}$$

**Step two:** Count the number of different units that appear in the equation.

milligrams and moles; two different units.

**Step three:** Draw a place for as many fractions as you have units so that they can become unit fractions. Put them on the LHS.

$$1.0\text{mg} \times \left( \frac{\quad}{\quad} \right) \times \left( \frac{\quad}{\quad} \right) = ? \text{ mol}$$

**Step four:** Dedicate one fraction to each unit. Arrange the units in the fractions so that the cancelations you need can happen and so that correct new units will appear.

$$1.0\text{mg} \times \left( \frac{\quad}{\text{mg}} \right) \times \left( \frac{\text{mol}}{\quad} \right) = ? \text{ mol}$$

**Step five:** Make your fractions equal to 1. To do this, you have to look at the units and determine which ones have known relationships. In this case, you know that **molecular weight** tells you the grams per mole of a molecule. So, you set the second fraction equal to 1 by putting in molecular weight:

$$1.0\text{mg} \times \left( \frac{\quad}{\text{mg}} \right) \times \left( \frac{1\text{mol}}{58.44\text{g}} \right) = ? \text{ mol}$$

You still have to deal with mg and g as well. Is there a relationship between mg and g? Yes.

$$1\text{g} = 10^3\text{mg}; \text{ therefore } \frac{1\text{g}}{10^3\text{mg}} = 1$$

So, make the first fraction equal to 1 by relating grams to milligrams.

$$1.0\text{mg} \times \left( \frac{1\text{g}}{10^3\text{mg}} \right) \times \left( \frac{1\text{mol}}{58.44\text{g}} \right) = ? \text{mol}$$

Now, solve:

$$1.0\text{mg} \times \left( \frac{1\text{g}}{10^3\text{mg}} \right) \times \left( \frac{1\text{mol}}{58.44\text{g}} \right) = 1.7 \times 10^{-5} \text{mol}$$

This example had two different **dimensions**—amount (moles) and mass (grams and milligrams). This technique works no matter how many different dimensions there are. As long as you multiply by 1, you are safe. For more examples, see the **SOLUTIONS** section.

### CONVERSIONS SHORTCUT

1. Write an equation.
2. Count units.
3. Insert that many fractions.
4. Make units cancelable.
5. Make fractions =1, then calculate.

## UNITS 1.4: Units Raised to Powers

What if you want to convert  $\text{m}^2$  to  $\text{mm}^2$ ? These units are both squared. Here is a handy technique for converting units that are raised to powers.



**The Point:** Use parentheses.

1. Put parentheses around the units, but not around the exponent, on the LHS of the equation.

$$5.0 (\text{m})^2 = ? \text{mm}^2 ?$$

2. Convert the units that are inside the parentheses.



$$5.0 \left( \text{m} \times \frac{10^3 \text{mm}}{\text{m}} \right)^2 = ? \text{mm}^2$$

3. Now carry out the calculations, raising *both* the number *and* the units within the parentheses to the appropriate power.

$$5.0 (\text{m})^2 = 5.0 \times (10^3 \text{mm})^2$$

$$5.0 \text{m}^2 = 5.0 \times 10^6 \text{mm}^2$$

---

**UNITS TO POWERS SHORTCUT**

1. Put parentheses around LHS units.
  2. Convert units.
  3. Calculate.
-

**UNITS – Try It Out**

**EXERCISE I:** Convert  $4.67 \times 10^4$  nm to [A] Angstroms, [B] microns, and [C] millimeters. Use the five-step method.



A.

1.  $4.67 \times 10^4$  nm = ? Å

2. two different units

3.  $4.67 \times 10^4$  nm  $\times$   $\left(\frac{\quad}{\quad}\right) \times \left(\frac{\quad}{\quad}\right) = ?$  Å

4.  $4.67 \times 10^4$  nm  $\times$   $\left(\frac{\quad}{\text{nm}}\right) \times \left(\frac{\text{Å}}{\quad}\right) = ?$  Å

5.  $4.67 \times 10^4$  nm  $\times$   $\left(\frac{10^{-9} \text{ m}}{\text{nm}}\right) \times \left(\frac{\text{Å}}{10^{-10} \text{ m}}\right) = ?$  Å;  $4.67 \times 10^4$  nm = Å

B.

1.  $4.67 \times 10^4$  nm = ? μm

2. two different units

3.  $4.67 \times 10^4$  nm  $\times$   $\left(\frac{\quad}{\quad}\right) \times \left(\frac{\quad}{\quad}\right) = ?$  μm

4.  $4.67 \times 10^4$  nm  $\times$   $\left(\frac{\quad}{\text{nm}}\right) \times \left(\frac{\mu\text{m}}{\quad}\right) = ?$  μm

5.  $4.67 \times 10^4$  nm  $\times$   $\left(\frac{\quad}{\text{nm}}\right) \times \left(\frac{\mu\text{m}}{\quad}\right) = ?$  μm;  $4.67 \times 10^4$  nm = μm

C.

1.  $4.67 \times 10^4 \text{ nm} = ? \text{ mm}$

2. two different units

3.  $4.67 \times 10^4 \text{ nm} \times \left( \frac{\quad}{\quad} \right) \times \left( \frac{\quad}{\quad} \right) = ? \text{ mm}$

4.  $4.67 \times 10^4 \text{ nm} \times \left( \frac{\quad}{\quad} \right) \times \left( \frac{\quad}{\quad} \right) = ? \text{ mm}$

5.  $4.67 \times 10^4 \text{ nm} \times \left( \frac{\quad}{\quad} \right) \times \left( \frac{\quad}{\quad} \right) = ? \text{ mm}; \underline{\underline{4.67 \times 10^4 \text{ nm} = \quad \text{mm}}}$

**EXERCISE II:** Convert 3 microns to [A] meters and [B] millimeters. Use the five-step method.



A.

1.  $3 \mu\text{m} = ? \text{ m}$

2. ? different units

3.  $3 \mu\text{m} \times \quad = ? \text{ m}$

4.  $3 \mu\text{m} \times \quad = ? \text{ m}$

5.  $3 \mu\text{m} \times \quad = ? \text{ m}; \underline{\underline{3 \mu\text{m} = \quad \text{m}}}$


For the next part, combine steps 3, 4, and 5 into one step.


B.


1.  $3 \mu\text{m} = ? \text{ mm}$

2.

3–5.  $3 \mu\text{m} \times \quad = ? \text{ mm}; \underline{\underline{3 \mu\text{m} = \quad \text{mm}}}$

**EXERCISE III:** Convert 800 cubic nanometers into cubic **Angstroms**. (You can use  $\times$  in place of  $*$  to show multiplication, as we do in this exercise.) 

1.  $800 \text{ nm}^3 = ? \text{ \AA}^3$
2.  $8.00 \times 10^2 (\text{nm})^3 = ? \text{ \AA}^3$
3.  $8.00 \times 10^2 \left( \text{nm} \times \left( - \right) \times \left( - \right) \right)^3 = ? \text{ \AA}^3$
4.  $8.00 \times 10^2 \left( \text{nm} \times \left( \frac{\text{nm}}{\text{nm}} \right) \times \left( \frac{\text{\AA}}{\text{nm}} \right) \right)^3 = ? \text{ \AA}^3$
5.  $8.00 \times 10^2 \left( \text{nm} \times \left( \frac{\text{nm}}{\text{nm}} \right) \times \left( - \right) \right)^3 = ? \text{ \AA}^3$ ;                      =                     . 

**EXERCISE IV:** Convert  $5.67 \times 10^{-4} \text{ mm}^2$  to  $\text{pm}^2$ . 

1.  $5.67 \times 10^{-4} \text{ mm}^2 = ? \text{ pm}^2$
2.  $5.67 \times 10^{-4} (\text{mm}^2) = ? \text{ pm}^2$
3.  $5.67 \times 10^{-4} \left( \text{mm} \times \left( - \right) \times \left( - \right) \right)^2 = ? \text{ pm}^2$
4.  $5.67 \times 10^{-4} \left( \text{mm} \times \left( - \right) \times \left( - \right) \right)^2 = ? \text{ pm}^2$
5.  $5.67 \times 10^{-4} ( \text{pm}^2 ) = ? \text{ pm}^2$ ;                      =                     .

**EXERCISE V:** Convert 10.0  $\mu\text{g}/\text{mL}$  of  $\text{CaCl}_2$  (molecular weight = 111) to [A]  $\text{mg}/\text{mL}$  and [B] molarity



A.

1.  $10.0 \frac{\mu\text{g}}{\text{mL}} = ? \frac{\text{mg}}{\text{mL}}$

2. three different units

3.  $10.0 \frac{\mu\text{g}}{\text{mL}} \times (-) \times (-) \times (-) = ? \frac{\text{mg}}{\text{mL}}$

4.  $10.0 \frac{\mu\text{g}}{\text{mL}} \times \left(\frac{\quad}{\mu\text{g}}\right) \times \left(\frac{\text{mL}}{\text{mL}}\right) \times \left(\frac{\text{mg}}{\quad}\right) = ? \frac{\text{mg}}{\text{mL}}$ ;  $10.0 \mu\text{g} / \text{mL} = \quad \text{mg} / \text{mL}$

B.

1.  $10.0 \frac{\mu\text{g}}{\text{mL}} = ? \frac{\text{mol}}{\text{L}}$

2. four different units

3.  $10.0 \frac{\mu\text{g}}{\text{mL}} \times (-) \times (-) \times (-) \times (-) = ? \frac{\text{mol}}{\text{L}}$

4.  $10.0 \frac{\mu\text{g}}{\text{mL}} \times (-) \times (-) \times (-) \times (-) = ? \frac{\text{mg}}{\text{mL}}$

5.  $10.0 \frac{\mu\text{g}}{\text{mL}} \times (-) \times (-) \times (-) \times (-) = ? \text{ m}$ ;  $10.0 \mu\text{g} / \text{mL} = \quad \text{M}$

**EXERCISE VI:** What is the molarity of 100% pure water (molecular weight 18.015 g/mol)?



$$100\% = 1 \frac{\text{g}}{\text{mL}} \quad ?$$

1.  $1 \frac{\text{g}}{\text{mg}} = ? \frac{\text{mol}}{\text{L}}$

2. four different units

3.  $1 \frac{\text{g}}{\text{mg}} = \left( - \right) \times \left( - \right) \times \left( - \right) \times \left( - \right) = ? \frac{\text{mol}}{\text{L}}$  Pure water = \_\_\_\_\_ M.

## LINKS TO ANSWERS

[EXERCISE I](#)

[EXERCISE II](#)

[EXERCISE III](#)

[EXERCISE IV](#)

[EXERCISE V](#)

[EXERCISE VI](#)



## TRY IT OUT: ANSWERS

**EXERCISE I:** Convert  $4.67 \times 10^4$  nm to [A] Angstroms, [B] microns, and [C] millimeters.

$$\text{A. } 4.67 \times 10^4 \text{ nm} \times \left( \frac{10^{-9} \text{ m}}{\text{nm}} \right) \times \left( \frac{\text{\AA}}{10^{-10} \text{ m}} \right) = 4.67 \times 10^5 \text{ \AA}$$

$$\text{B. } 4.67 \times 10^4 \text{ nm} \times \left( \frac{10^{-9} \text{ m}}{\text{nm}} \right) \times \left( \frac{10^6 \mu\text{m}}{\text{m}} \right) = 4.67 \times 10^1 \mu\text{m}$$

$$\text{C. } 4.67 \times 10^4 \text{ nm} \times \left( \frac{\text{m}}{10^9 \text{ nm}} \right) \times \left( \frac{10^3 \text{ mm}}{\text{m}} \right) = 4.67 \times 10^{-2} \text{ mm}$$

**General Pattern:** If you look at these answers, you will see the pattern: if you are converting from smaller units to larger units, the magnitude of the number should decrease (you need fewer big things to take up the same space as the smaller things); if you are converting from larger units to smaller units, the magnitude of the number should increase (you need more smaller things to take up the same space as the bigger things).

**TRY IT OUT: ANSWERS**

**EXERCISE II:** Convert 3 microns to [A] meters and [B] millimeters.

$$\text{A. } 3\mu\text{m} \times \left( \frac{\text{m}}{10^6 \mu\text{m}} \right) = 3 \times 10^{-6} \text{ m}$$

$$\text{B. } 3\mu\text{m} \times \left( \frac{10^{-6} \text{ m}}{\mu\text{m}} \right) \times \left( \frac{10^3 \text{ mm}}{\text{m}} \right) = 3 \times 10^{-3} \text{ mm}$$

**TRY IT OUT: ANSWERS**

**EXERCISE III:** Convert  $800 \text{ nm}^3$  to cubic Angstroms.

$$800 \times \left( (\text{nm}) \times \left( \frac{10^{-9} \text{ m}}{\text{nm}} \right) \times \left( \frac{10^{10} \text{ \AA}}{\text{m}} \right) \right)^3 = 800 \times (10^1 \text{ \AA})^3 = 8.00 \times 10^5 \text{ \AA}^3$$

**TRY IT OUT: ANSWERS**

**EXERCISE IV:** Convert  $5.67 \times 10^{-4} \text{ mm}^2$  to  $\text{pm}^2$ .

$$5.67 \times 10^{-4} \times \left( (\text{mm}) \times \left( \frac{10^{-3} \text{ m}}{\text{mm}} \right) \times \left( \frac{10^{12} \text{ pm}}{\text{m}} \right) \right)^2 = 5.67 \times 10^{14} \text{ pm}^2$$

**TRY IT OUT: ANSWERS**

**EXERCISE V:** Convert 10.0  $\mu\text{g}/\text{mL}$  of  $\text{CaCl}_2$  (molecular weight = 111) to [A]  $\mu\text{g}/\text{mL}$  and [B] molarity.

$$\text{A. } 10.0 \frac{\mu\text{g}}{\text{mL}} \times \left( \frac{10^{-3} \text{ mg}}{\mu\text{g}} \right) = 1.00 \times 10^{-2} \frac{\text{mg}}{\text{mL}}$$

$$\text{B. } 10.0 \frac{\mu\text{g}}{\text{mL}} \times \left( \frac{10^{-6} \text{ g}}{\mu\text{g}} \right) \times \left( \frac{\text{mL}}{10^{-3} \text{ L}} \right) \times \left( \frac{\text{mol}}{111 \text{ g}} \right) = 9.01 \times 10^{-5} \text{ M} = 90.1 \mu\text{M}$$

**TRY IT OUT: ANSWERS**

**EXERCISE VI:** What is the molarity of 100% pure water? (molecular weight 18.015 g/mol)

$$1.00 \frac{\text{g}}{\text{mL}} \times \left( \frac{\text{mol}}{18.015 \text{ g}} \right) \times \left( \frac{\text{mL}}{10^{-3} \text{ L}} \right) = 55.5 \text{ M}$$

# *Math for Life*

## **TOPIC 2: EXPONENTS AND POWERS OF TEN**

### **EXPONENTS 2.1: Powers of Ten**

The powers of 10 are in a pattern. If you look at the pattern, you can see why  $10^0 = 1$ , and why the negative powers of 10 mean “reciprocal.”

For Example:                   ,                   , and so on.

You can also see why multiplying or dividing by 10 is as simple as moving the decimal point:

$$10^3 = 1000.$$

$$10^2 = 100.$$

$$10^1 = 10.$$

$$10^0 = 1.$$

$$10^{-1} = \frac{1}{10}.$$

$$10^{-2} = \frac{1}{100}.$$

$$10^{-3} = \frac{1}{1000}.$$

$$10^{-3} = 0.001 = \frac{1}{1000} = \frac{1}{10^3}$$

To divide by 10, all you have to do is move the decimal point one digit to the left; and to multiply by 10, all you have to do is move the decimal point one digit to the right.

This is a very useful shortcut if you are doing computations with powers of 10.

## EXPONENTS 2.2: Multiplying Numbers that Have Exponents

To multiply two numbers with the same base, add the exponents.

$$f^s \times f^t = f^{s+t}$$

Here  $f$  is called the base. The two numbers  $f^s$  and  $f^t$  have the same base. So the exponents  $s$  and  $t$  can be added.

**EXAMPLE:**  $12^4 \times 12^5 = 12^9$

**WHY:**  $(12 \times 12 \times 12 \times 12) \times (12 \times 12 \times 12 \times 12 \times 12) = 12^9$

The base, 12, is the same throughout, so we can just add the exponents  $4 + 5 = 9$ . The answer is  $12^9$ .

## EXPONENTS 2.3: Raising Exponents to a Power

To raise an expression that contains a power to a power, multiply the exponents.

$$(j^v)^w = j^{v \times w}$$

The exponents  $v$  and  $w$  are multiplied.

**EXAMPLE:**  $(3^2)^3 = 3^6$

**WHY:**  $(3 \times 3) \times (3 \times 3) \times (3 \times 3) = 3^6$

## EXPONENTS 2.4: Dividing Numbers that Have Exponents

To divide two numbers that have the same base, subtract the exponents.

$$\frac{a^m}{a^n} = a^m - a^n$$



The two numbers  $a^m$  and  $a^n$  have the same base,  $a$ . So the exponents  $m$  and  $n$  can be subtracted.

**EXAMPLE:**  $\frac{10^3}{10^2} = 10^{3-2} = 10^1 = 10$

**WHY:**  $\frac{10 \times 10 \times 10}{10 \times 10} = 10$

## EXPONENTS 2.5: Multiplying Numbers in Scientific Notation

When you are multiplying, order doesn't matter. You can choose a convenient order.

$$(a \times b) \times (c \times d) = (a \times c) \times (b \times d)$$

**EXAMPLE:** Multiply  $(4.1 \times 10^2) \times (3.2 \times 10^4)$ . Rearrange the order of the factors. Group the powers of 10 together.

$$(4.1 \times 3.2) \times (10^2 \times 10^4) = 13 \times 10^6$$

This answer could also be written as  $1.3 \times 10^7$ . However, in science, we're fond of powers of three, so there are **prefixes** that can substitute for powers that are multiples of three. If, for example, the above number were a length in nanometers, it could be written as  $1.3 \times 10^7$  nm or as 13 mm. This conversion is legitimate because  $1.3 \times 10^7$  nm =  $13 \times 10^6$  nm, and  $10^6$  nm = 1 mm, so  $1.3 \times 10^7$  nm = 13 mm. (See the discussion on **significant digits** in topic 3 to see why the answer is  $13 \times 10^6$  nm and not  $13.12 \times 10^6$  nm.)

In scientific notation, the number multiplying the power of 10 is called the mantissa. In  $5 \times 10^6$ , 5 is the mantissa. When there is no mantissa written, the mantissa is 1. So  $10^6$  has a mantissa of 1:  $10^6 = 1 \times 10^6$ .

**EXAMPLE:**  $\frac{5.678 \times 10^3}{10^6} = 5.678 \times 10^{-3}$

To see why, write  $10^6$  as  $1 \times 10^6$  and rearrange the factors:

**EXAMPLE:**  $\frac{5.678 \times 10^3}{1 \times 10^6} = \frac{5.678}{1} \times \frac{10^3}{10^6} = 5.678 \times 10^{-3}$

In scientific notation, when no power of 10 is written, the power of 10 that is meant is  $10^0$ . That's because  $10^0 = 1$ .

**EXAMPLE:**  $\frac{5.678 \times 10^3}{4.12} = 1.38 \times 10^3$

To see why, write 4.12 in scientific notation as  $4.12 \times 10^0$ . Then rearrange the factors and divide.

$$\frac{5.678 \times 10^3}{4.12 \times 10^0} = \frac{5.678}{4.12} \times \frac{10^3}{10^0} = 1.38 \times 10^3$$

A word about notation:

Another way of writing scientific notation uses a capital E for the  $\times 10$ . You may have seen this notation from a computer.

**EXAMPLES:**  $4.356 \times 10^7$  is sometimes written as 4.356E7.  
2.516E5 means  $2.516 \times 10^5$ .


The letter E is also sometimes written as a lower case e; this choice is unfortunate because e has two other meanings.

**EXPONENTS – Try It Out**

**EXERCISE I:** Solve the following equations; all answers should be in the form of a number (or variable) raised to an exponent:

**EXAMPLE:**  $14^4 \times 14^8 = 14^{12}$

A.

1.  $10^6 \times 10^4 =$  

2.  $10^3 \times 10^3 =$

3.  $10^{123} \times 10^0 =$

4.  $10^4 \times 10^4 =$

5.  $10^n \times 10^m =$

6.  $3^3 \times 3^2 =$


7.  $n^4 \times n^{12} =$

8.  $h^7 \times h^7 =$

9.  $a^n \times a^m =$

10.  $j^3 \times j^m =$

B.

1.  $(6^3)^3 =$  

2.  $(6^2)^3 =$

3.  $(10^4)^2 =$

4.  $(10^6)^3 =$

5.  $(2^2)^2 =$


6.  $(16^2)^{10} =$

7.  $(k^3)^4 =$

8.  $(m^6)^2 =$

9.  $(n^k)^1 =$

10.  $(s^m)^2 =$

C. 1.  $\frac{10^6}{10^1} =$  

2.  $\frac{10^2}{10^4} =$

3.  $\frac{10^{\frac{1}{2}}}{10^{\frac{1}{4}}} =$

4.  $\frac{10^m}{10^n} =$

5.  $\frac{m^{10}}{m^7} =$

6.  $\frac{m^t}{m^{(t-1)}} =$

7.  $\frac{(n+m)^5}{(n+m)^2} =$

8.  $\frac{j^{(n+4)}}{j^{(n-6)}} =$

9.  $\frac{e^{2\pi}}{e^\pi} =$

10.  $\frac{q^j}{q^h} =$

D.

1.  $(3 \times 10^6) \times (4 \times 10^4) =$

2.  $(5 \times 10^7) \times (7 \times 10^{-2}) =$

3.  $(1.6 \times 10^2) \times (4 \times 10^3) =$

4.  $(-4.6 \times 10^6) \times (-2.1) \times 10^4 =$

5.  $(1.11 \times 10^0) \times (6.00 \times 10^1) =$

**LINKS TO ANSWERS**[EXERCISE I A.](#)[EXERCISE I B.](#)[EXERCISE I C.](#)[EXERCISE I D.](#)

**TRY IT OUT: ANSWERS**

A.

1.  $10^{10}$

2.  $10^6$

3.  $10^{123}$

4.  $10^8$

5.  $10^{(n+m)}$

6.  $3^5$

7.  $n^{16}$

8.  $h^{14}$

9.  $a^{(n+m)}$

10.  $j^{(3+m)}$

**TRY IT OUT: ANSWERS**

B.

1.  $6^9$

2.  $6^6$

3.  $10^8$

4.  $10^{18}$

5.  $2^4$

6.  $16^{20}$

7.  $k^{12}$

8.  $m^{12}$

9.  $n^{kl}$

10.  $s^{2m}$

**TRY IT OUT: ANSWERS**

C.

1.  $10^5$

2.  $10^{-2}$

3.  $10^{\frac{1}{4}}$

4.  $10^{(m-n)}$

5.  $m^3$

6.  $m^1$  or  $m$

7.  $(n + m)^3$

8.  $j^{10}$

9.  $e^\pi$

10.  $q^{(j-h)}$



## TRY IT OUT: ANSWERS

D.

1.  $12 \times 10^{10}$

2.  $35 \times 10^5$

3.  $6.4 \times 10^5$

4.  $9.66 \times 10^{10}$

5.  $6.66 \times 10^1$

# *Math for Life*

## **TOPIC 3: READING AND REPORTING NUMERICAL DATA**

### **NUMERICAL DATA 3.1: Significant Digits; Honest Reporting of Measured Values**

Why report uncertainty? That is how you tell the reader how confident to be about the precision of the measurements.

Why use significant digits? They are the best way to report uncertainty about precision.

That is, significant digits tell your reader about the precision with which you measured. (Precision is different from accuracy; accuracy is how close your measure is to the true value, precision is how much resolution you had. Significant digits are the way you report precision; you need replication and statistics to report accuracy.)

If there is no error indicated explicitly, it means the last digit is precise within plus or minus one.

### **NUMERICAL DATA 3.2: Determining Uncertainty**

The precision of your measure is determined by the measuring device you use. A measurement with that device will have a certain number of significant digits. The number of significant digits depends on the precision of the device.

**EXAMPLE:** Suppose you are measuring temperature and have two devices. Measuring device 1 is a mercury thermometer, with black lines indicating increments of 0.1 degree Celsius ( $0.1^{\circ}\text{C}$ ).

Can you tell the difference between  $10^{\circ}\text{C}$  and  $11^{\circ}\text{C}$ ? Yes.

Can you tell the difference between  $10.5^{\circ}\text{C}$  and  $10.6^{\circ}\text{C}$ ? Yes.

Can you tell the difference between  $10.57^{\circ}\text{C}$  and  $10.56^{\circ}\text{C}$ ? No. Thus, it would be overstating the precision (lying) to report a temperature of  $10.56^{\circ}\text{C}$ .  $10.6^{\circ}\text{C}$  is the best you could say. This measurement has three significant digits.

Measuring device 2 is a digital thermocouple, with a readout that shows three decimal places.

Can you tell the difference between  $10.56^{\circ}\text{C}$  and  $10.57^{\circ}\text{C}$ ? Yes.

Can you tell the difference between  $10.565^{\circ}\text{C}$  and  $10.566^{\circ}\text{C}$ ? Yes.

Can you tell the difference between  $10.5656^{\circ}\text{C}$  and  $10.5657^{\circ}\text{C}$ ? No.  $10.566^{\circ}\text{C}$  is the best you could say. This measurement has five significant digits.

The thermocouple has higher precision than the mercury thermometer.

How many significant digits should you report?

1. First, figure out how precisely you can measure using each of your measuring tools.
2. Then, report the correct number of digits.

### **NUMERICAL DATA 3.3: Determining the Number of Significant Digits**

Which digits can be considered significant?

- Nonzero digits
- Zeros between nonzero digits
- Zeros to the right of the first nonzero digit

Which digits are not significant?

- Zeros to the left of the first nonzero digit.

**EXAMPLES:** Column A lists four measurements. Column B tells the number of significant digits of each of these values.

A	B
<u>Value</u>	<u>No. of significant digits</u>
1.0067 5	
0.0010067 5	
5.0 2	
453	3

So, when you report a value, you can use the above rules to determine how to accurately report the precision of your measurement.

If you have to manipulate your values—do conversions, for example—you need to know how multiplication and division affect the number of significant digits.

### NUMERICAL DATA 3.4: Multiplying and Dividing Significantly

When you are multiplying or dividing, the answer has the same number of significant figures as the measure with the *fewest significant figures*.

**EXAMPLE:**  $1.440 \times 10^{-6} \div 5.66609 = 2.540 \times 10^{-7}$

4	6	4	No. of significant digits
---	---	---	---------------------------

On the LHS, the two numbers being divided have 4 and 6 significant digits, respectively. The smaller number is 4, and so the answer has 4 significant digits.

### NUMERICAL DATA 3.5: Adding and Subtracting Significantly

When adding or subtracting, pay attention to decimal places, not significant figures. The answer should have the same number of decimal places as the number with the *fewest decimal places*.

**EXAMPLE:**  $200 + 4.56 = 205$

0	2	0	No. of decimal places
---	---	---	-----------------------

On the LHS, the two numbers being added have 0 and 2 decimal places, respectively. The smaller number is 0, and so the answer has 0 decimal places.

**EXAMPLE:**  $1.440 \times 10^{-2} - 5.6 \times 10^{-5} = 1.439 \times 10^{-2}$

5                      6                      5      No. of decimal places

Subtraction is treated the same way as addition.  $1.440 \times 10^{-2}$  can be determined to have 5 decimal places, (to see, rewrite it as 0.01440)  $-5.6 \times 10^{-5}$  has 6, so the answer has 5 decimal places.

### NUMERICAL DATA 3.6: Dealing with Exact Numbers

Some values have no uncertainty. For example, 1 liter is exactly 1,000 milliliters. These numbers do not affect the precision of your calculated value, so ignore them when determining uncertainty. That is, ignore them when counting significant digits, and ignore them when counting decimal places.

**EXAMPLE:**  $1.440 \times 10^{-2} \frac{\text{mg}}{\text{mL}} \times 10^3 \frac{\mu\text{g}}{\text{mg}} = 1.440 \times \frac{\mu\text{g}}{\text{mL}}$

4                      ignore                      4

In calculating the number of significant digits, we ignored the  $\frac{10^3 \mu\text{g}}{\text{mg}}$  because that is an exact value. The answer has 4 decimal places.

### NUMERICAL DATA 3.7: Why Use Scientific Notation?

What if a researcher reports a value of 7,000? How do you know if the number of significant digits is four, three, two, or one? Did this person use a measuring device that had its smallest divisions in the ones, tens, hundreds, or thousands place? You can't be 100 percent sure.

If, however, the person reporting 7,000 had used scientific notation, then the answer would be perfectly clear:

<u>Value</u>	<u>No. of significant digits</u>
$7 \times 10^3$	1
$7.0 \times 10^3$	2
$7.00 \times 10^3$	3
$7.000 \times 10^3$	4

That is a benefit of scientific notation. It is the only unambiguous way to report the precision of all of your measures.

## NUMERICAL DATA 3.8: Graphs

*What is a graph and what is it good for?*

A graph is *one way* of representing numerical data. A table is another. Tables are preferable if you have a few data points; a graph is preferable if you have many.

The point of graphing data is to *see* whether there is a relationship between the variables and whether that relationship can be described by an equation.

If there *is* a mathematical relationship between the variables, you can use an equation to describe the data, you can use the equation to predict the results of experiments that you have not done, and you can, in future, measure any one variable and immediately know the value of the other. So, describing a relationship using an equation can save you a lot of work.

Determining what the equation is requires a priori knowledge, and/or a curve-fitting algorithm.

Determining how well your equation describes your data requires statistics.

*Graph conventions*

Most graphs have two axes,  $x$  and  $y$ . If there is a third, it is designated  $z$ .

The origin is at  $(0,0)$ ; magnitudes increase with distance from the origin.

The variable on the  $x$  axis is the independent variable, that is, the variable that the researcher controls. Time is also usually graphed on the horizontal axis.

The variable on the  $y$  axis is the dependent variable, that is, the quantity that the researcher is measuring and that varies as a result of a variation in  $x$ . The value of  $y$  “depends” on the value of  $x$ .

### NUMERICAL DATA 3.9: Lines

The general equation for any line is  $y = mx + b$ ;  $m$  is the slope and  $b$  is the  $y$  intercept.

The  $x$  intercept =  $-\frac{b}{m}$ .

The  $x$  axis is the graph of the equation  $y = 0$ .

The  $y$  axis is the graph of the equation  $x = 0$ .

If the slope of the line is positive, the dependent variable is said to vary positively, or directly with the independent variable. If the slope of the line is negative, the dependent variable is said to vary negatively or inversely with the independent variable.

### NUMERICAL DATA 3.10: Transformations

Because lines are much easier to interpret than curves, data are sometimes transformed so that a line will be a good descriptor. To transform data means to perform an operation on each of the points.

Common transformations include:

- **Taking the log** of the data. This makes data that are very spread out at high values and closer together at lower values space out more evenly over their entire range.
- Taking the reciprocal of the data. This turns a certain kind of curve into a line.

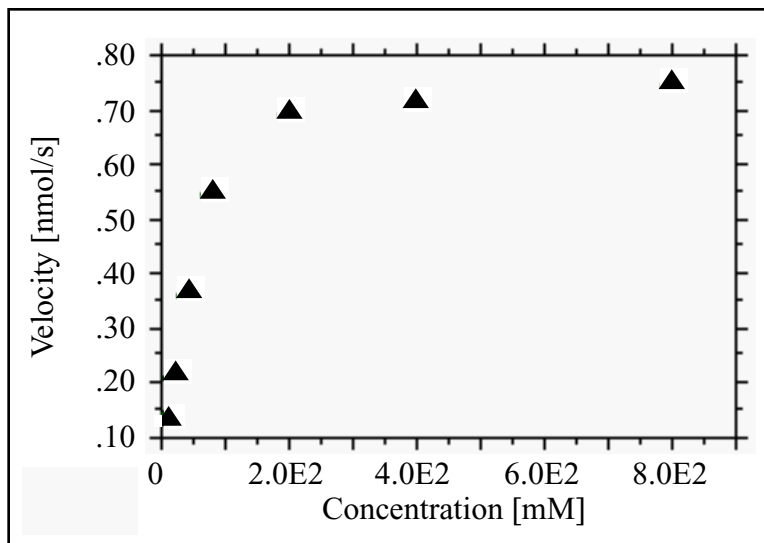
Either or both variables can be transformed. If the variable is transformed, its units must be transformed as well.

**EXAMPLE:**

The data:

Concentration of Reactant	Rate of Reaction
$1.0 \times 10^1$ mM 1.3	$\times 10^{-1}$ nmol/s
$2.0 \times 10^1$ mM 2.2	$\times 10^{-1}$ nmol/s
$4.0 \times 10^1$ mM 3.7	$\times 10^{-1}$ nmol/s
$8.0 \times 10^1$ mM 5.5	$\times 10^{-1}$ nmol/s
$2.0 \times 10^2$ mM 7.0	$\times 10^{-1}$ nmol/s
$4.0 \times 10^2$ mM 7.2	$\times 10^{-1}$ nmol/s
$8.0 \times 10^2$ mM 7.5	$\times 10^{-1}$ nmol/s

The graphical representation of the data:



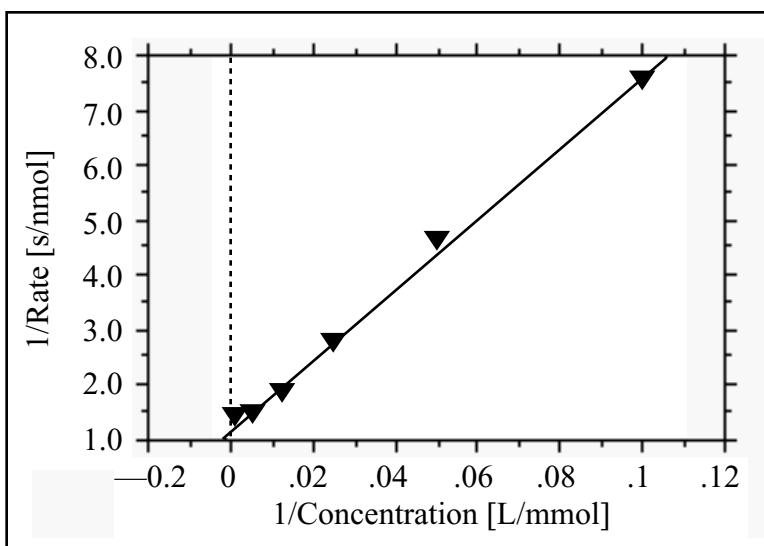
Even after looking at the graph, it is difficult to describe the relationship represented by these data.



Now look at the data transformed by taking the reciprocal of both variables:  
The transformed data:

(Concentration of Reactant) <sup>-1</sup>	(Rate of Reaction) <sup>-1</sup>
1.0 × 10 <sup>-1</sup> L/mmol 7.5	× 10 <sup>0</sup> s/nmol
5.0 × 10 <sup>-2</sup> L/mmol 4.6	× 10 <sup>0</sup> s/nmol
2.5 × 10 <sup>-2</sup> L/mmol 2.7	× 10 <sup>0</sup> s/nmol
1.3 × 10 <sup>-2</sup> L/mmol 1.8	× 10 <sup>0</sup> s/nmol
5.0 × 10 <sup>-3</sup> L/mmol 1.4	× 10 <sup>0</sup> s/nmol
2.5 × 10 <sup>-3</sup> L/mmol 1.4	× 10 <sup>0</sup> s/nmol
1.3 × 10 <sup>-3</sup> L/mmol 1.3	× 10 <sup>0</sup> s/nmol

The graphical representation of the transformed data:



Now, the data are linearly related, and the relationship is easily described by:

$$\frac{1}{\text{rate}} = 6.4 \times 10^1 \times \frac{1}{\text{concentration}} + 1.2$$

This is the equation for the line shown.

## READING AND REPORTING NUMERICAL DATA – Try It Out

**EXERCISE I:** How many significant digits should you use when reporting values measured in the following ways:



- A. 250 mL measured using a pipette marked in 10 mL increments
- B. 0.8 mL measured using a 1 mL pipette marked in 0.1 mL increments
- C. 1.5 grams measured using a digital balance that reports three decimal places
- D. An absorbance of 1.450 using a spectrophotometer that reports three decimal places
- E. A temperature of 350 K using a thermometer that reports one decimal place
- F. Half a meter measured using a meter stick divided into cm increments
- G. A weight of 56.6 kg measured using a digital scale that reports two decimal places

**EXERCISE II:** You have created a solution by pipetting 1.0 mL of water into a tube, then adding  $1.26 \times 10^{-3}$  mg of NaCl. How many significant digits should you use when reporting the concentration of this solution in mg/mL?



**EXERCISE III:** To determine the area of a rectangular plot of land, you measure the length of the two sides and find them to be 20.8 m and 9.1 m. How many significant digits should you use when you report the area of the plot?



**EXERCISE IV:** The following numbers were used in recent scientific journals. How many significant digits in each value? (Assume the authors know the rules for reporting values.)



- A. 100  $\mu\text{g}$  of a peptide
- B. 250 mM NaCl
- C.  $7.8 \times 10^2$  years
- D.  $58 \text{ kg ha}^{-1} \text{ yr}^{-1}$
- E. 303 K
- F.  $5 \times 10^6 \text{ mL}^{-1}$
- G. 2.57 hours
- H. 0.75 mN force on spider leg segment
- I.  $8 \times 10^6$  cells  $\text{L}^{-1}$
- J. 400 base pairs
- K. 4,562 citations

## LINKS TO ANSWERS

[EXERCISE I](#)

[EXERCISE II](#)

[EXERCISE III](#)

[EXERCISE IV](#)

## TRY IT OUT: ANSWERS

## EXERCISE I:

- A. The pipette is marked in 10 mL increments, so, you could tell the difference between 240, 250, and 260, but not between 249, 250, and 251. Written in scientific notation, then, you could honestly write  $2.5 \times 10^2$ , which has **two** significant digits.
- B. The pipette is marked in 0.1 mL increments, so, you could tell the difference between 0.7, 0.8, and 0.9, but not between 0.79, 0.80, and 0.81. Written in scientific notation, then, you could honestly write  $8 \times 10^{-1}$ , which has **one** significant digit.
- C. When you measured this, the balance read 1.500 g. So, you can tell the difference between 1.499g, 1.500g, and 1.501g. **Four** significant digits.
- D. When you measured this, the spectrophotometer read 1.450. **Four** significant digits.
- E. When you measured this, the thermometer read 350.0 K. Written in scientific notation, then, you could honestly write  $3.500 \times 10^2$ , which has **four** significant digits.
- F. When you measured, the object was closest to the line marked 500 cm. Written in scientific notation, then, you could honestly write  $5.00 \times 10^2$ , which has **three** significant digits.
- G. When you measured this, the balance read 56.60 kg. Written in scientific notation, then, you could honestly write  $5.660 \times 10^1$ , which has **four** significant digits.

## TRY IT OUT: ANSWERS

**EXERCISE II:** This is a division problem, so the answer has the same number of significant digits as the variable with the fewest significant digits. That's the 1.0 mL with two significant digits, so the answer has two significant digits.



## TRY IT OUT: ANSWERS

**EXERCISE III:** This is a multiplication problem, so the answer has the same number of significant digits as the variable with the fewest significant digits. That's the 9.1 m with two significant digits, so the answer has two significant digits.



**TRY IT OUT: ANSWERS**

**EXERCISE IV:**

A. 3

B. 3

C. 2

D. 2

E. 3

F. 1

G. 3

H. 2

I. 1

J. 3

K. 4



# *Math for Life*

## **TOPIC 4: MAKING SOLUTIONS**

### **SOLUTIONS 4.1: Making Solutions from Dry Chemicals**

How do you make particular volumes of solutions of particular **molarities**, starting with a bottle of a compound?

**EXAMPLE:** Suppose you want 100 mL of a 5.00 M stock solution of  $\text{CaCl}_2$ .

**Step one:** Figure out how many grams would go in one liter:

$$5.00 \text{ M} = 5.00 \text{ moles per liter}$$

Convert moles to grams using the formula weight:

Formula Weight of  $\text{CaCl}_2$  is 219.08 grams per mole.

**Step two:** Figure out what fraction of 1 liter you are making.

**Step three:** Use the same fraction of the 1095.4 g.

$$0.1 \times 1095.4 \text{ g} = 109.54 \text{ g}$$

So, to make 100 mL of a 5.00 M solution of  $\text{CaCl}_2$ , put 109.54 g into a container, then bring the solution up to 100 mL.

These steps can be simplified.

Written as one expression, the relationship looks like this:

$$5.00 \text{ mol} \times \frac{219.08 \text{ g}}{\text{mol}} \times \frac{10^{-1} \text{ L}}{\text{L}}$$

If you rewrite the L as the denominator of the first variable, and note that the second variable is molecular weight, this simplifies even further, providing a recipe for making solutions from dry chemicals.

---

**RECIPE SHORTCUT:**

**Final Molarity × Molecular weight × Final volume [L] = Grams to add**


---

Don't forget to bring the solution up to final volume.

## SOLUTIONS 4.2: Dealing with Hydrated Compounds

Some chemicals come with water molecules attached. For example, you can buy sodium phosphate as  $\text{NaH}_2\text{PO}_4 \cdot \text{H}_2\text{O}$  (sodium phosphate monobasic). The practical consequence of this is that for every mole of sodium phosphate you add to your solution, you are also adding a mole of water.

You can also buy sodium phosphate with 12 waters attached:  $\text{Na}_3\text{PO}_4 \cdot 12\text{H}_2\text{O}$  (sodium phosphate tribasic). For every mole of sodium phosphate you add, you are adding 12 moles of water.

One mole of water has a mass of 18.015 grams, and it has a volume of 18.015 mL. Twelve moles of water make up a volume of 216.18 mL. This can wreak havoc with your final concentrations. 

The easy way to deal with this potential problem is to use the following method when working with hydrated compounds.

---

**HYDRATED SHORTCUT 1**

- Use a graduated cylinder as your mixing vessel.
  - Fill the cylinder with about half the final volume of water.
  - Add the desired molar amounts of your compounds.
  - Bring the solution up to the final volume.
- 

With this method, the volume of any water you added as part of the hydrated compound is automatically taken into account. When you bring the solution up to the final volume, you will be adding just the right volume.

The other way to deal with the problem is to calculate exactly what volume of water you will be adding when you add the hydrated compounds, and subtract that from the final volume of water to add.

- To calculate the added volume of water, first determine the number of moles of the compound times the number of molecules of H<sub>2</sub>O in the compound. That is the number of moles of H<sub>2</sub>O you will be adding.
- The number of moles you are adding, times 18.015, will tell you the number of mLs of water you are adding.

**EXAMPLE:** I wish to make up 500 mLs of a 200mM solution of MgCl<sub>2</sub> using MgCl<sub>2</sub>•6H<sub>2</sub>O.

From the [Recipe Shortcut](#), I know that I need the following:

$$\text{Molarity} \times \text{Molecular weight} \times \text{Final volume [L]} = \text{Grams to add.}$$

The Molecular weight (listed on the jar as F.W.) is 203.3, which is the molecular weight of MgCl<sub>2</sub> (95.2) plus the molecular weight of 6H<sub>2</sub>O (108.1).

Using the Recipe Shortcut,

$$200 \times 10^{-3} \text{ mol/L} \times 203.3 \times 500 \times 10^{-3} \text{ L} = 20.33 \text{ grams to add}$$

How much H<sub>2</sub>O will that add?

$$20.33 \text{ grams of MgCl}_2 \cdot 6\text{H}_2\text{O} \div \text{MW} = 0.1000 \text{ moles of compound being added}$$

Each molecule of the compound has six molecules of H<sub>2</sub>O; each mole of compound has six moles of H<sub>2</sub>O.

$$0.1000 \text{ moles} \times 6 = 0.6000 \text{ moles of H}_2\text{O}$$

$$0.6000 \text{ moles} \times 18.02 \text{ g/mol} = 10.81 \text{ g}$$

$$10.81 \text{ g} \times 1\text{mL/g} = 10.81 \text{ mL of water.}$$

subtracting from the total,

$$500.0 \text{ mL} - 10.81 \text{ mL} = 489.2 \text{ mL of water}$$

The recipe is:

$$489.2 \text{ mLs of H}_2\text{O plus } 20.33\text{g of MgCl}_2 \cdot 6\text{H}_2\text{O}$$

The abbreviated version of all the above calculations is as follows:

$$\text{Molarity [M]} \times \text{final volume [L]} \times \text{number of H}_2\text{O's per molecule} \times 18.015 \text{ mL/mol} = \text{mLs of water contributed by hydrated compound}$$

We can express this relationship as the Hydrated Shortcut 2.

### HYDRATED SHORTCUT 2

$$M \left[ \frac{\text{mol}}{\text{L}} \right] \times \text{Vol[L]} \times \# \text{H}_2\text{O's} \times 18.015 \frac{\text{mL}}{\text{mol}} = \text{mL H}_2\text{O from Compound}$$

The easy way, however, is to add the compounds, then bring the solution up to the final volume.

### SOLUTIONS 4.3: Diluting Stocks to Particular Concentrations

If you know what you want the concentration to be and you want to figure out how much of your stock to add, you can use the following equation:

$$\frac{\text{What you want}}{\text{What you have}} \times \text{Final volume} = \text{Volume to add to mixture}$$

You may recognize this as a version of  $M_1V_1 = M_2V_2$

You use that formula to calculate everything but the water, then add enough water to bring the volume up to the desired volume.

Note: The units of what you want (the numerator) must be the same as the units of what you have, (the denominator).

**FOR EXAMPLE:**

I want 25 mL of the following solution:

0.50 M  $\text{CaCl}_2$   
1.0 M  $\text{MgSO}_4$

I have the following stock solutions:

5.0 M  $\text{CaCl}_2$   
2.5 M  $\text{MgSO}_4$

**Step one:** Figure out how much of the  $\text{CaCl}_2$  stock to add:

$$\frac{0.5\text{M}}{5.0\text{M}} \times 25\text{mL} = 2.5 \text{ mL of } \text{CaCl}_2 \text{ stock}$$

**Step two:** Figure out how much of the  $\text{MgSO}_4$  stock to add:

$$\frac{1.0\text{M}}{2.5\text{M}} \times 25\text{mL} = 10 \text{ mL of } \text{MgSO}_4 \text{ stock}$$

**Step three:** Bring the solution up to 25 mL:

$$25\text{mL} - (2.5 \text{ mL} + 10 \text{ mL}) = 12.5 \text{ mL of water to add}$$

**Step four:** Check your result.

For  $\text{CaCl}_2$ :

$$2.5\text{mL of } 5.0 \frac{\text{mol}}{10^3\text{mL}} \text{ put into } 25 \text{ mL total volume} \quad ?$$

$$\frac{2.5\text{mL} \times 5.0 \frac{\text{mol}}{10^3 \text{mL}}}{25\text{mL}} = 5.0 \times 10^{-4} \frac{\text{mol}}{\text{mL}} \times \frac{10^3 \text{mL}}{\text{L}} = 0.50 \text{ M}$$

And, for  $\text{MgSO}_4$ :

$$10\text{mL of } 2.5 \frac{\text{mol}}{10^3 \text{mL}} \text{ put into } 25 \text{ mL total volume}$$

$$\frac{10\text{mL} \times 2.5 \frac{\text{mol}}{10^3 \text{mL}}}{25\text{mL}} = 1.0 \times 10^{-3} \frac{\text{mol}}{\text{mL}} \times \frac{10^3 \text{mL}}{\text{L}} = 1.0 \text{ M}$$

Note: This is just a rearrangement of the dilution equation:

$$\text{That is, } \frac{\text{volume to add} \times \text{have}}{\text{final volume}} = \text{want, is a rearrangement of}$$

$$\frac{\text{want}}{\text{have}} \times \text{final volume} = \text{volume to add}$$

### DILUTION SHORTCUT

$$\frac{\text{what you Want}}{\text{what you Have}} \times \text{Final Volume} = \text{Amount to add or, } \frac{\text{W}}{\text{H}} \times \text{FV} = \text{A}$$

## SOLUTIONS 4.4: Calculating Concentrations from Recipes

Use the following words-to-symbols translation hints when calculating concentrations and dilutions:

1. "...of..." means multiply.
2. "Put ... into..." means divide by.

In other words, "*a of j*" means  $a \times j$  and "put *q* into *s*" means  $q \div s$ . Note also that whatever the actual order of the events, you should rephrase a "put... into ..." phrase so that you are putting the solid into the liquid.

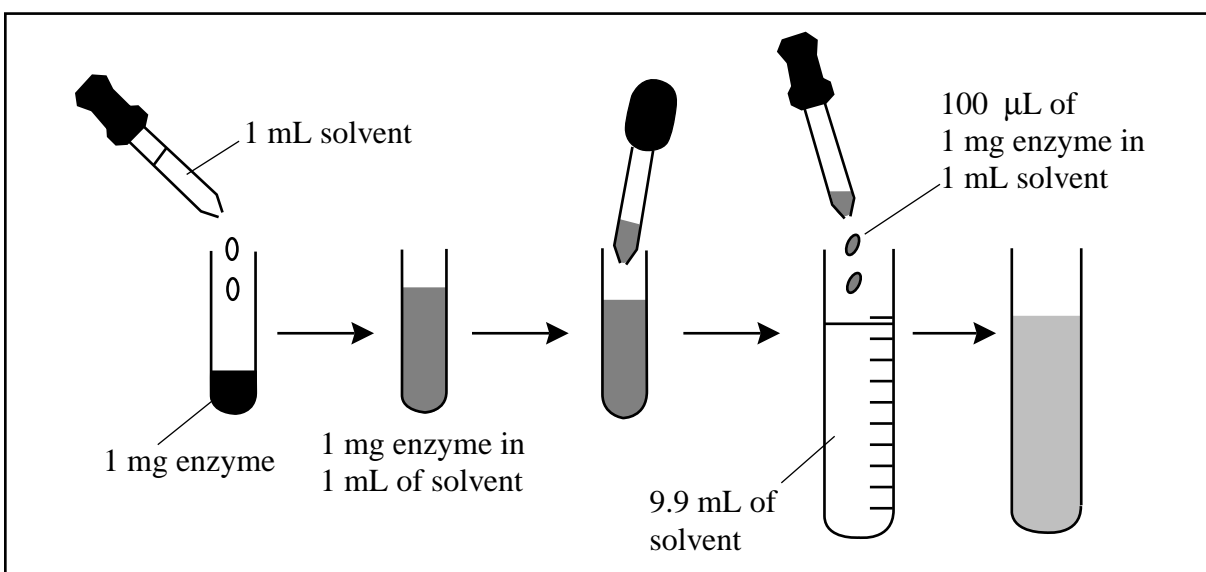
The trick to figuring out dilutions is to say, in words, what you have done. Then translate the words into algebraic expressions using the above words-to-symbols translation hints.

**EXAMPLE:** What is the concentration (in mg/mL) of enzyme in a given test tube?

**Step one:** Describe what happened.

I bought a bottle of enzyme. There was 1 mg of enzyme in the bottle. I added 1 mL of solvent to the bottle. Then, I took 100  $\mu\text{L}$  of that solution and added it to a test tube that had 9.9 mL of solvent in it.

**Step two:** Draw a picture.



**Step three:** Translate each sentence (the following assumes two significant digits).

Words: I bought a bottle of enzyme. There was 1.0 mg of enzyme in the bottle. I added 1.0 mL of solvent to the bottle.

Translation: I put 1.0 mg of enzyme into 1.0 mL (i.e.,  $1 \times 10^{-3}\text{L}$ ) of solvent.

Expression:

$$\frac{1.0 \text{ mg of enzyme}}{1.0 \text{ mL of solvent}} = \frac{1.0 \text{ mg}}{10^{-3} \text{ L}}$$

Words: Then, I took 100  $\mu\text{L}$  ( $1 \times 10^{-4}\text{L}$ ) of that solution.

Translation: I took  $1.0 \times 10^{-4} \mu\text{L}$  of that solution.

Expression:

$$1.0 \times 10^{-4} \text{ L} \times \frac{1 \text{ mg}}{10^{-3} \text{ L}} = 1.0 \times 10^{-1} \text{ mg of enzyme} = 1.0 \times 10^2 \mu\text{g enzyme}$$

Words: ... and added it to a test tube that had 9.9 mL of solvent in it.

Translation: I *put*  $1.0 \times 10^2 \mu\text{g}$  of enzyme *into*  $1.0 \times 10^1 \text{ mL}$  total volume.

Expression:

$$\text{converting } \frac{1.0 \times 10^2 \mu\text{g enzyme}}{1.0 \times 10^4 \mu\text{L solution}} \text{ to } \frac{\text{mg}}{\text{mL}} :$$

$$\frac{1.0 \times 10^2 \mu\text{g}}{1.0 \times 10^4 \mu\text{L}} \times \frac{10^3 \mu\text{L}}{\text{mL}} \times \frac{1 \text{ mg}}{10^3 \mu\text{g}} = \frac{1.0 \times 10^2 \text{ mg enzyme}}{1.0 \times 10^4 \text{ mL solution}}$$

$$\text{which simplifies to } \frac{1.0 \times 10^2 \text{ mg enzyme}}{1 \text{ mL solution}}$$

**Step four:** Thus, the answer is  $1.0 \times 10^{-2} \text{ mg/mL}$ .

From these steps we can write a shortcut that describes how to calculate concentrations from recipes.



**RECIPE TO CONCENTRATION SHORTCUT**

1. Draw a picture.
2. Describe the picture.
3. Translate each sentence. "Of" means multiply; "Put ... into" means divide by.
4. Calculate.

**SOLUTIONS 4.5: Converting Mass Per Volume to Molarity**

To convert from mass per volume to moles per liter (molarity), you need to know the relationship between the units. Use the [trick](#) for converting between units developed in Topic 1.

**EXAMPLE:** Convert mg/mL to M.

Set up a units-only equation.

$$\frac{\text{mg}}{\text{mL}} \times \frac{\text{mL}}{\text{mg}} \times \frac{\text{mL}}{\text{L}} \times \frac{\text{mol}}{\text{L}} = \frac{\text{mol}}{\text{L}}$$

You know the relationship between mL and L:  $10^3 \text{ mL} = 1 \text{ L}$

Is there a relationship between mg and mol? Yes. Molecular weight  $\frac{\text{g}}{\text{mol}}$ .

Let's say the last example concerned an enzyme of molecular weight,  $8.98 \times 10^4 \frac{\text{g}}{\text{mol}}$ ; the conversion looks like this:

$$\frac{1.0 \times 10^{-2} \text{ mg}}{\text{mL}} \times \frac{1 \text{ mol}}{8.98 \times 10^4 \text{ g}} \times \frac{1 \text{ g}}{10^3 \text{ mg}} \times \frac{10^3 \text{ mL}}{1 \text{ L}} = 1.1 \times 10^{-7} \frac{\text{mol}}{\text{L}}$$

Thus, for this enzyme  $1.0 \times 10^{-2} \text{ mg/mL}$  is  $1.1 \times 10^{-7} \text{ M}$ .

The general equation looks like this:

$$\frac{\text{mg}}{\text{mL}} \times \frac{\text{mol}}{\text{g}} \times \frac{1 \text{ g}}{10^3 \text{ mg}} \times \frac{10^3 \text{ mL}}{1 \text{ L}} = \frac{\text{mol}}{\text{L}}$$

Now, look at the general equation above. The final two fractions on the LHS multiply to 1, so they have no effect on the magnitude of the number. Thus, as long as you have set up your equations *with the correct units*, the general equation can be simplified and written as a shortcut.

---

### MASS PER VOLUME TO MOLARITY SHORTCUT

$$\frac{\text{mg}}{\text{mL}} \div \text{MW} = \text{M}$$

---

(Here M.W. stands for molecular weight.) In fact, as long as the prefix is the same on both the mass and the volume, you can use this trick, because you will always be multiplying by 1 when you do the conversion.

But, don't forget: If the prefixes are *not* the same, you *cannot* use the above shortcut; you must start at the beginning.

### SOLUTIONS 4.6: Percents

Percent means "out of 100"; it is not a unit. Percents are dimensionless numbers. 37% of  $x$  means 37 hundredths of  $x$  or  $0.37 \times x$ .

The amount of an ingredient in a solution is sometimes described as a per cent of the total solution. A percent can be thought of as the amount of one quantity *per* another. If the same units appear in the numerator and denominator, percents mean what you'd expect:

$$\frac{5\text{L}}{20\text{L}} = 25\%$$

$$100\% = 1\text{g}/1\text{g}$$

$$1\% = 10 \text{ mL}/\text{L}$$

Percent mass *per* volume is based on a convention and it works as follows:

---

**MASS PER VOLUME**

**100% means 1 g/mL**

**10% means 100 mg/mL**

**1% means 10 mg/mL**

---

This convention is based on the mass/volume of pure water: one mL of water has a mass of 1 gram, hence  $1\text{g/mL} = 100\%$ .

**SOLUTIONS 4.7: Dilution Ratios**

Recipes for solutions sometimes contain directions for diluting a stock solution according to a certain ratio. To read these directions, you need to know the following: A dilution in the ratio  $1:x$  means add 1 volume of concentrate to  $(x - 1)$  volumes of diluent to create a total volume equal to  $x$ . It makes sense: In the final solution, what you added will be  $1/x$ th of the total.

1:100	means one part concentrate, 99 parts diluent
1:14	means one part concentrate, 13 parts diluent
1:2	means one part concentrate, 1 part diluent
1:1	means straight concentrate

*Note:* This is the original convention; however, it is not always followed. If you see 1:9, chances are the author means 1:10, and if you see 1:1, chances are the author means 1:2. You have to use your judgment in interpreting such ratios. Because of this confusion, it is always better to report concentrations rather than recipes.

---

**DILUTION DEFINITION**

**1:x means 1 part concentrate in  $(x - 1)$  parts diluent**

---

## SOLUTIONS – TRY IT OUT

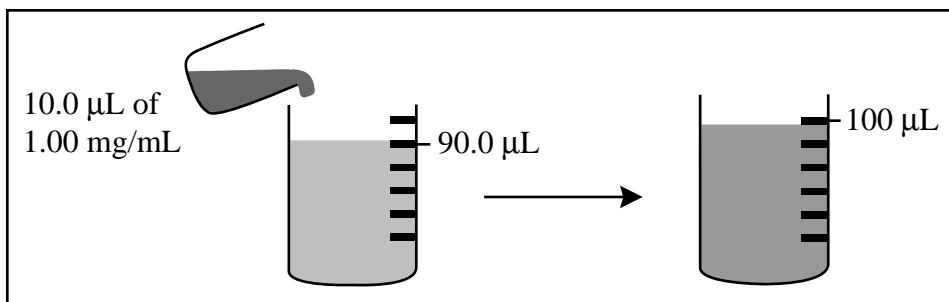
### EXERCISE I

Note: as you work through this exercise, you will be practicing progressively more of the technique, so it is important to work through the entire exercise.

You are preparing to use an antibody against the protein E-cadherin to find out where it is located in a cell. You do not know what concentration will be just enough but not too much, so you are going to prepare five solutions of different concentrations. Five mL of the antibody already in solution arrives from the company. The antibody is at a concentration of 1 mg/mL. What is the concentration of the following solutions?

A. Solution A is 10.0  $\mu\text{L}$  of antibody plus 90.0  $\mu\text{L}$  of solvent buffer. What is the concentration in mg/mL of antibody in solution A?

1. Draw a picture.



2. Describe the picture.

*Put 10.0  $\mu\text{L}$  of 1.00 mg/mL into 100  $\mu\text{L}$  total.*

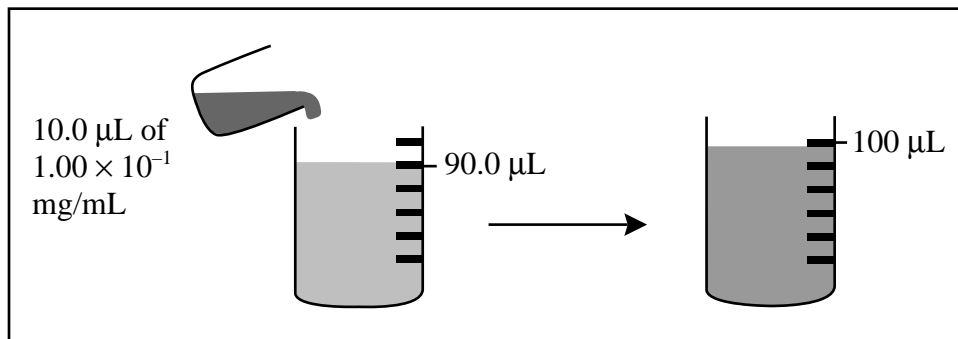
3. Translate.

$$\left(10.0 \mu\text{L} \times 1.00 \frac{\text{mg}}{\text{mL}}\right) \div 100 \mu\text{L}$$

4. Calculate mg/mL.
5. What information would you need to calculate the molarity of this solution?



- B. Solution B is 10.0  $\mu\text{L}$  of solution A plus 90.0  $\mu\text{L}$  of solvent buffer. What is the concentration in mg/mL of antibody in solution B?



1. Draw a picture.
2. Describe the picture.

*Put 10.0  $\mu\text{L}$  of  $1.00 \times 10^{-1}$  mg/mL into 100  $\mu\text{L}$  total.*

3. Translate.
  4. Calculate mg/mL.
- C. Solution C is 20.0  $\mu\text{L}$  of solution B plus 80.0  $\mu\text{L}$  of solvent buffer. What is the concentration in mg/mL of antibody in solution C?

1. Draw a picture.
2. Describe the picture.
3. Translate.
4. Calculate mg/mL.

D. Solution D is 50.0  $\mu\text{L}$  of solution C plus 275  $\mu\text{L}$  of solvent buffer. What is the concentration in mg/mL of antibody in solution D?

1. Draw a picture.
2. Describe the picture.
3. Translate.
4. Calculate mg/mL.




E. Solution E should be 100  $\mu\text{L}$  of antibody solution at a concentration of  $1.50 \times 10^{-3}$  mg/mL. How would you make solution E using solution B and solvent?

1. Draw a picture.
2. Describe the picture.
3. Calculate the amount of solution B to add.
4. Calculate the amount of solvent to add.

## EXERCISE II

The following is a recipe for agar:

3.00g NaCl (F.W. 58.44 g/mol)  
17.0g agar  
2.50 g peptone  
5.00 mg cholesterol (F.W. 386.7 g/mol)  
Bring to a final volume of  $1.00 \times 10^3$  mL

- A. What is the concentration of NaCl (in mg/mL?) 
- B. What is the concentration of NaCl (in mM?) 
- C. What is the molarity of cholesterol in this solution? 

- D. How much water would you add to 1.00 mL of this solution to make the molarity of cholesterol  $6.45 \mu\text{M}$ ?



### EXERCISE III

The result of a Formosan Banded Krait *Bungarus multicinctus* bite is paralysis caused by a toxin known as  $\alpha$ -bungarotoxin.  $\alpha$ -bungarotoxin can be purchased in lots of 1.00 mg of crystalline venom. You wish to observe the effect of this toxin on cultured muscle cells at a concentration of  $1.24 \times 10^{-5} \text{ M}$ .

- A. Create a 1.00 mM **stock solution** of toxin, then make further **dilutions** of the stock to make a test solution of the appropriate concentration. Assume this protein has a molecular weight of 6421. All of your numbers should have three **significant digits**.

1. Draw a picture.

2. Calculate how much solvent to add to make the stock solution.



3. Calculate the amounts of stock and solvent to mix to make  $1.00 \times 10^1 \text{ mL}$  of  $1.24 \times 10^{-5} \text{ M}$  solution; call this solution A.



Amount of stock =

Amount of solvent =




- B. Solution A killed all your cultured cells, so you decide to dilute it. You add 3.00 mL of solvent to 1.00 mL of solution A. What is the molarity of this new dilution, solution B?



1. Draw a picture.

2. Describe the picture.

3. Translate.

4. Calculate molarity.
  5. Convert to mg/mL.
  6. Shortcut: You are diluting solution A 1:4; therefore, the final molarity of solution B will be 1/4 the molarity of the solution A. Recalculate the molarity this way. 
  7. How many mL of solution B do you have?
- C. Using solution B, calculate how much solution and solvent to mix to make 25.0 mL of a solution C that is  $6.20 \times 10^{-7}$ M. 
1. What volume do you have and what volume do you want?
  2. Calculate the volume of solution to add.
  3. Calculate the volume of solvent to add.
  4. Do you have enough of solution B to make this final dilution?
  5. What is the maximum amount of  $6.20 \times 10^{-7}$ M solution that you could make from the solution B you already have? 
  6. What is the maximum amount of  $6.20 \times 10^{-7}$ M solution that you could make if you used the entire amount of venom that you purchased?
  7. What reminder can be drawn from this exercise?



## LINKS TO ANSWERS

**EXERCISE I** – The antibody solutions

- A.
- B.
- C.
- D.
- E.

**EXERCISE II**

**EXERCISE III** – The Formosan Banded Krait toxin

- A.
- B.
- C.

## TRY IT OUT: SOLUTIONS

I.A. Solution A is 10.0  $\mu\text{L}$  of antibody plus 90.0  $\mu\text{L}$  of solvent buffer.

4. Calculate mg/mL.

$$1.00 \times 10^{-1} \frac{\text{mg}}{\text{mL}}$$

5. What information would you need to calculate the molarity of this solution?

**TRY IT OUT: SOLUTIONS**

I.B. Solution B is 10.0  $\mu\text{L}$  of solution A plus 90.0  $\mu\text{L}$  of solvent buffer.

3. Translate.

$$\frac{(10.0\mu\text{L})(1.00 \times 10^{-1} \frac{\text{mg}}{\text{mL}})}{100\mu\text{L}}$$

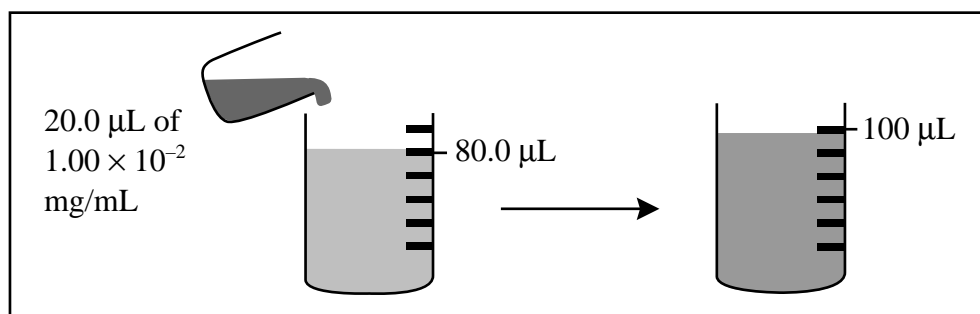
4. Calculate mg/mL.

$$1.00 \times 10^{-2} \frac{\text{mg}}{\text{mL}}$$

## TRY IT OUT: SOLUTIONS

I.C. Solution C is 20.0  $\mu\text{L}$  of solution B plus 80.0  $\mu\text{L}$  of solvent buffer.

1. Draw a picture



2. Describe the picture.

*Put 20.0  $\mu\text{L}$  of  $1.00 \times 10^{-2}$  mg/mL into 100  $\mu\text{L}$  total.*

3. Translate.

$$\frac{(20.0\mu\text{L})(1.00 \times 10^{-3} \frac{\text{mg}}{\text{mL}})}{100\mu\text{L}}$$

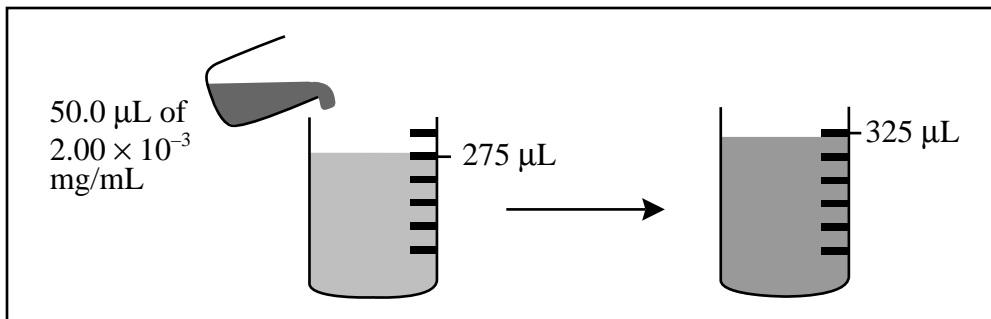
4. Calculate mg/mL.

$$2.00 \times 10^{-3} \frac{\text{mg}}{\text{mL}}$$

**TRY IT OUT: SOLUTIONS**

I.D. Solution D: 50.0  $\mu\text{L}$  of solution C plus 275  $\mu\text{L}$  of solvent buffer?

1. Draw a picture.



2. Describe the picture.

*Put 50.0  $\mu\text{L}$  of  $2.00 \times 10^{-3} \text{ mg/mL}$  into 325  $\mu\text{L}$  total*

3. Translate.

$$\frac{(50.0\mu\text{L})(2.00 \times 10^{-3} \frac{\text{mg}}{\text{mL}})}{3.25 \times 10^2 \mu\text{L}}$$

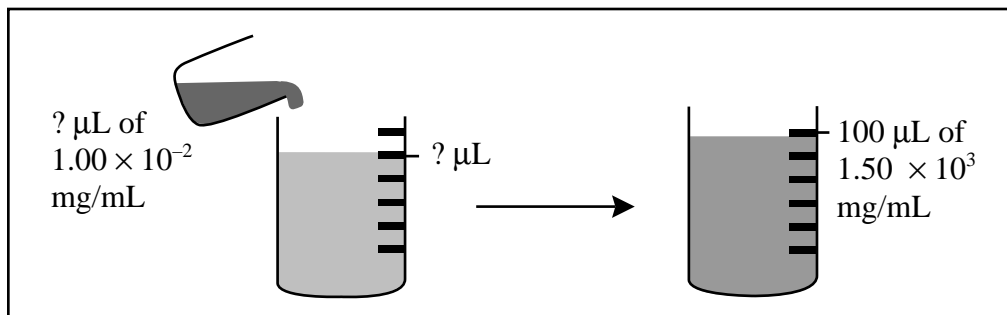
4. Calculate mg/mL.

$$3.08 \times 10^{-4} \frac{\text{mg}}{\text{mL}}$$

### TRY IT OUT: SOLUTIONS

I.E. Solution E should be 100 μL of antibody solution at a concentration of  $1.50 \times 10^{-3}$  mg/mL. How would you make Solution E using Solution B and solvent?

1. Draw a picture.



2. Describe the picture.

I want  $1.50 \times 10^{-3}$  mg/mL.

I have  $1.00 \times 10^{-2}$  mg/mL.

I want a final volume (FV) of 100 μL.

3. Calculate the amount of solution B to add.

$$\frac{W}{H} \times FV = A \qquad \frac{1.50 \times 10^{-3} \frac{\text{mg}}{\text{mL}}}{1.00 \times 10^{-2} \frac{\text{mg}}{\text{mL}}} \times 100 \mu\text{L} = 1.50 \times 10^1 \mu\text{L} \quad ?$$

4. Calculate the amount of solvent to add.

$$100 \mu\text{L} - 1.50 \times 10^1 \mu\text{L} = 85 \mu\text{L} \quad (\text{see the section on adding and subtracting with significant digits})$$

## TRY IT OUT: SOLUTIONS

### II. The agar recipe

A. What is the concentration of NaCl in mg/mL?

$$\frac{3.00\text{g} \times \frac{10^3 \text{mg}}{\text{g}}}{1.00 \times 10^3 \text{mL}} = 3.00 \frac{\text{mg}}{\text{mL}}$$

B. What is the concentration of NaCl in mM?

$$3.00 \frac{\text{mg}}{\text{mL}} \times \frac{1 \text{mol}}{58.44 \text{g}} \times \frac{1 \text{g}}{10^3 \text{mg}} \times \frac{10^3 \text{mL}}{\text{L}} \times \frac{10^3 \text{mmol}}{\text{mol}} = 5.13 \times 10^1 \text{mM}$$

C. What is the molarity of cholesterol in this solution?

$$5.00 \frac{\text{mg}}{\text{L}} \times \frac{1 \text{mol}}{386.7 \text{g}} \times \frac{1 \text{g}}{10^3 \text{mg}} = 1.29 \times 10^{-5} \text{M}$$

D. How much water would you add to 1.00 mL of this solution to make the molarity of cholesterol 6.45  $\mu\text{M}$ ?

$$\left( \frac{W}{H} \times FV = A \right) \text{ then } \left( \frac{A \times H}{W} = FV \right) \text{ and } (FV - A = \text{water to add}) \quad ?$$

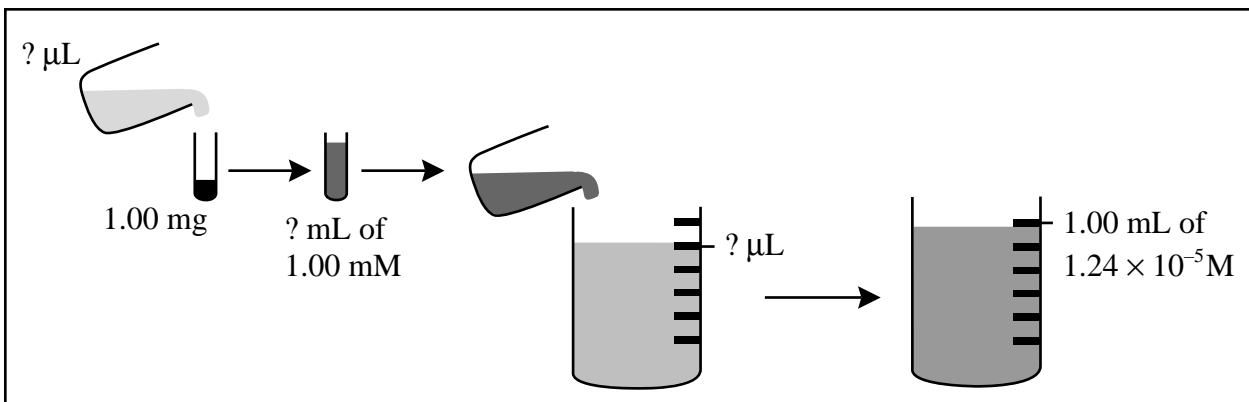
$$\frac{1.00 \text{ mL} \times 1.29 \times 10^{-5} \text{M}}{6.45 \times 10^{-6} \text{M}} = 2.00 \text{ mL}; 2.00 \text{ mL} - 1.00 \text{ mL} = 1.00 \text{ mL}$$

## TRY IT OUT: SOLUTIONS

### III. The Formosan Banded Krait

A.

1. Draw a picture.



2. Calculate how much solvent to add to make the stock solution.

$$\frac{1.00 \text{ mg}}{?L} = 10^{-3}M \text{ rearranges to } \frac{1.00 \text{ mg}}{10^{-3}M} = ?L$$

$$\frac{1.00 \text{ mg}}{10^{-3} \frac{\text{mol}}{L}} \times \frac{1 \text{ mol}}{6421 \text{ g}} \times \frac{1 \text{ g}}{10^3 \text{ mg}} = 1.56 \times 10^{-4}L$$

3. Calculate the amounts of stock and solvent to mix to make  $1.00 \times 10^1$  mL of  $1.24 \times 10^{-5}M$  solution (Solution A).

$$\frac{W}{H} \times FV = A \quad \frac{1.24 \times 10^{-5}M}{1.00 \times 10^{-3}M} \times 1.00 \times 10^1 \text{ mL} = 1.24 \times 10^{-1} \text{ mL}$$



$$\text{Amount of stock} = 1.24 \times 10^{-1} \text{ mL}$$

$$\text{Amount of solvent} = 10.0 \text{ mL} - 1.24 \times 10^{-1} \text{ mL} = 9.9 \text{ mL}$$

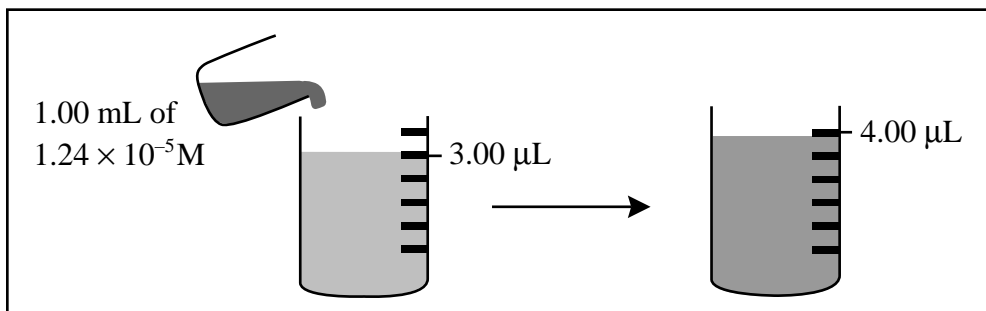


## TRY IT OUT: SOLUTIONS

## III.B. Venom dilution (A → B)

B.

1. Draw a picture



2. Describe the picture.

*Put 1.00 mL of 1.24 × 10<sup>-5</sup> M into 4.00 mL total.*

3. Translate.

$$\frac{1.00\text{mL} \times 1.24 \times 10^{-5}\text{M}}{4.00\text{mL}}$$

4. Calculate molarity.

$$3.10 \times 10^{-6}\text{M}$$

5. Convert to mg/mL.

$$3.10 \times 10^{-6} \frac{\text{mol}}{\text{L}} \times \frac{10^{-3}\text{L}}{\text{mL}} \times \frac{6421\text{g}}{\text{mol}} \times \frac{10^3\text{mg}}{\text{g}} = 1.99 \times 10^{-2} \frac{\text{mg}}{\text{mL}}$$

6. Shortcut: You are diluting solution A 1:4; therefore, the molarity of solution B will be 1/4 the molarity of solution A. Recalculate the molarity this way.

$$\frac{1.24 \times 10^{-5} \text{ M}}{4} = 3.10 \times 10^{-6} \text{ M}$$

7. How many mL of solution B do you have?

$$1.00 \text{ mL} + 3.00 \text{ mL} = 4.00 \text{ mL}$$

## TRY IT OUT: SOLUTIONS

III.C. Using solution B, calculate how much solution and solvent to mix to make 25.0 mL of a solution C that is  $6.20 \times 10^{-7}$  M.

C.

1. What do you have and what do you want?

I want  $6.20 \times 10^{-7}$  M.

I have  $3.10 \times 10^{-6}$  M.

I want a final volume of 25.0 mL.

2. Calculate the volume of solution to add.

$$\frac{6.20 \times 10^{-7} \text{ M}}{3.10 \times 10^{-6} \text{ M}} \times 25.0 \text{ mL} = 5.00 \text{ mL}$$

3. Calculate the volume of solvent to add.

$$25.0 \text{ mL} - 5.00 \text{ mL} = 20.0 \text{ mL}$$

4. Do you have enough of solution B to make this final dilution?

No.

5. What is the maximum amount of  $6.20 \times 10^{-7}$  M solution that you could make from the solution B you already have?

$$\frac{A \times H}{W} = FV$$

$$\frac{4.00 \text{ mL} \times 3.10 \times 10^{-6} \text{ M}}{6.20 \times 10^{-7} \text{ M}} = 20.0 \text{ mL} \quad ?$$

or, alternatively:

$$\frac{5.00 \text{ mL}}{25.0 \text{ mL}} = \frac{4.00 \text{ mL}}{x \text{ mL}}$$

$$x = \frac{4.00 \times 25.0}{5.00} = 20.0 \text{ mL}$$

6. What is the maximum amount of  $6.20 \times 10^{-7}$  M solution that you could make if you used the entire amount of venom that you purchased?

$$\frac{1\text{mg}}{6.20 \times 10^{-7} \frac{\text{mol}}{\text{L}}} \times \frac{10^{-3}\text{g}}{\text{mg}} \times \frac{1\text{mol}}{6421\text{g}} = 2.51 \times 10^2 \text{mL}$$

7. What reminder can you draw from this exercise?

Do *all* the calculations *before* you begin to mix reagents.

# *Math for Life*

## **TOPIC 5: pH AND BUFFERS**

### **pH AND BUFFERS 5.1: Definition and Measurement of pH**

A molecule can be described as an acid or a base:

- An acid is a molecule that can donate (give up) a proton (a hydrogen ion,  $H^+$ ).
- A base is a molecule that can donate a hydroxide ion ( $OH^-$ ).
- The pH scale quantifies acidity.
- The pH scale runs from 1 to 14, with  $pH = 7$ , the pH of pure water, considered neutral.
- Lower pH means more acid; higher pH means more basic (that is, more alkaline).

pH is the negative of the log of the hydrogen ion concentration (concentration must be in units of moles per liter, i.e. M).

$$pH = -\log[H^+]$$

In fact, “p” in front of any substance means “the negative of the log of the concentration of” that substance.

For example,  $pCa = -\log[Ca^{2+}]$

Because of that “negative”:

- The *lower* the pH, the *higher* the concentration of  $H^+$ .
- The *higher* the pH, the *lower* the concentration of  $H^+$ .

Because of that “log”:

- A change in pH of 1 means a  $10^1\times$  or  $10\times$  change in  $[H^+]$ .

- A change in pH of 2 means a  $10^2\times$  or  $100\times$  change in  $[H^+]$ .
- A change in pH of 3 means a  $10^3\times$  or  $1000\times$  change in  $[H^+]$ .

and so on, where  $\times$  means “times.”

pH is not usually calculated; it is usually measured directly.

To measure pH you use indicator dyes, litmus paper, or a pH meter. Here is a brief description of how they work:

1. Indicator dyes change color when pH changes. They are calibrated so that you can look at the color of the solution with the dye in it, and, by comparing that color to a chart, read off the pH value. This pH measuring method is the one that most pet stores sell for measuring the acidity and alkalinity of fish tanks. Many biological media contain indicator dyes, like phenol red, so that the user can tell at a glance whether the pH is correct.
2. Litmus paper is coated with an indicator dye that changes color when pH changes. It is calibrated so that a certain color results when you put the paper into your solution. By comparing that color to the chart provided, you can read off the pH value.
3. A pH meter works by measuring the voltage across a thin glass membrane that conducts electricity. On one side of the membrane is a known concentration of  $H_3O^+$ ; on the other side is the solution of unknown pH. The voltage across that membrane is proportional to the pH; the meter converts that voltage to a measure of pH by comparing the voltage to a reference electrode.

To keep pH stable, use buffers.

## **pH AND BUFFERS 5.2: Definition and Action of Buffers**

Buffers are chemicals that prevent pH from changing easily.

A buffer is a weak acid that is chosen such that a key value (see the discussion on  $K$  below) is approximately equal to the pH of interest. It prevents changes in pH by substituting changes in the relative concentrations of the weak acid and its conjugate

base, (the conjugate base of an acid is the base that is formed when the acid gives up its proton).

Compare the following scenarios:

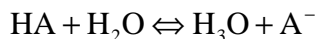
1. You add NaOH to a solution. Result: the pH goes up. This is because the NaOH dissociates into  $\text{Na}^+$  and  $\text{OH}^-$ . The  $\text{OH}^-$  combines with the  $\text{H}^+$ 's already in the solution, thus making water. Because the  $\text{H}^+$  concentration went down, the pH goes up.
2. You add NaOH to a solution with a buffer in it: Result: The acid component of the buffer gives up  $\text{H}^+$  ions to combine with the  $\text{OH}^-$  ions from the NaOH and the pH does not change. What *does* change is the relative amount of acid and base in the solution.

So, a buffer works by replacing a change in  $[\text{H}^+]$  with a change in relative amounts of the acid and its conjugate base.

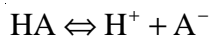
- When you add acid to a buffered solution, the pH stays the same, the amount of buffer goes down, and the amount of its conjugate base goes up.
- When you add base to a buffered solution, the pH stays the same, the amount of buffer goes up, and the amount of its conjugate base goes down.

### pH AND BUFFERS 5.3: Calculating pH and Buffer Strength

To understand the equations that describe buffers, you have to think about acids and bases. Every acid base reaction looks like this generic formula:



This reaction is sometimes written as



where HA is an acid and  $\text{H}_2\text{O}$  is the base it reacts with to form  $\text{H}_3\text{O}^+$  (or  $\text{H}^+$ ), the conjugate acid of  $\text{H}_2\text{O}$ , and  $\text{A}^-$  is the conjugate base of HA.

This generic reaction is quantified by  $K$ , the dissociation constant (which is the acid–base equivalent of an equilibrium constant):

$$K = \frac{[\text{H}_3\text{O}^+][\text{A}^-]}{[\text{HA}][\text{H}_2\text{O}]}$$

$K$  is a dimensionless quantity that measures the proton (i.e.,  $\text{H}^+$ ) affinity of the  $\text{HA}/\text{A}^-$  pair relative to the proton affinity of the  $\text{H}_3\text{O}^+/\text{H}_2\text{O}$  pair; that is, it tells you whether you are more likely to have  $\text{HA}$  or  $\text{H}_3\text{O}^+$ . Another way to say this is that  $K$  tells you whether the generic reaction is more likely to go to the left or to the right.

If  $K$  is greater than 1.0, the reaction goes to the right, and  $\text{HA}$  is a strong acid. If  $K$  is less than 1.0, the reaction goes to the left, and  $\text{HA}$  is a weak acid.

Because most biologically important solutions are relatively dilute—that is, they are mostly water—the concentration of  $\text{H}_2\text{O}$  is, for all practical purposes, constant at 55.5M.

Rearranging the equation defining  $K$  gives

$$K[\text{H}_2\text{O}] = \frac{[\text{H}_3\text{O}^+][\text{A}^-]}{[\text{HA}]}$$

which leads to the defining of  $K_a$ , which is  $K[\text{H}_2\text{O}]$  as

$$K_a = \frac{[\text{H}_3\text{O}^+][\text{A}^-]}{[\text{HA}]} = \frac{[\text{H}^+][\text{A}^-]}{[\text{HA}]}$$

Be careful: Sometimes the subscript is not used, causing confusion.

### pH AND BUFFERS 5.4: Why pH = 7 Is Neutral

$\text{H}_2\text{O}$  is an acid (albeit a weak one), so it has a dissociation constant:

$$K_a = \frac{[\text{H}^+][\text{OH}^-]}{[\text{H}_2\text{O}]} = \frac{[\text{H}^+][\text{OH}^-]}{55.5\text{M}}$$



which leads to defining  $K_w$  as

$$K_w = K_a[\text{H}_2\text{O}] = [\text{H}^+][\text{OH}^-]$$

At 25°C,  $K_w = 10^{-14} \text{ M}^2$ . In pure water the concentration of  $\text{H}^+$  must equal the concentration of  $\text{OH}^-$  (because they are present in equimolar amounts in  $\text{H}_2\text{O}$ ), so  $[\text{H}^+]$  must equal the square root of  $K_w$ , that is,  $10^{-7} \text{ M}$ . If  $[\text{H}^+] = 10^{-7} \text{ M}$ , then  $\text{pH} = 7$ . Hence, the pH of pure water is 7, and that is defined as neutral.

- A solution with  $\text{pH} > 7$  is basic.
- A solution with  $\text{pH} < 7$  is acidic.

### pH AND BUFFERS 5.5: pH and the Concentration of Acids and Bases (The Henderson–Hasselbalch Equation)

Rearranging the definition of  $K_a$  shows the relationship between pH and the concentration of an acid and its conjugate base.

$$K_a = \frac{[\text{H}^+][\text{A}^-]}{[\text{HA}]}$$

becomes

$$[\text{H}^+] = K_a \left( \frac{[\text{HA}]}{[\text{A}^-]} \right)$$

If you now combine this equation with the definition of pH (by taking the negative of the log of both sides), you get:

$$-\log[\text{H}^+] = -\log K_a + \left( -\log \left( \frac{[\text{HA}]}{[\text{A}^-]} \right) \right)$$

$$-\log[\text{H}^+] = -\log K_a + \log \left( \frac{[\text{A}^-]}{[\text{HA}]} \right)$$

Now, if you substitute  $p$  for  $-\log$  you get:

$$\text{pH} = \text{p}K_a + \log\left(\frac{[\text{A}^-]}{[\text{HA}]}\right)$$

This is the Henderson–Hasselbalch equation. It tells you the relationship between the concentration of the acid,  $[\text{HA}]$ , the concentration of the conjugate base,  $[\text{A}^-]$ , and pH.

The Henderson–Hasselbalch equation also tells you that if  $[\text{A}^-] = [\text{HA}]$ , then  $\text{pH} = \text{p}K_a$  (because the log of 1 is 0).

### pH AND BUFFERS 5.6: Choosing a Buffer

Because the pH of a solution can go down or up, you want your buffer to be able to respond equally well to an increase or a decrease in  $[\text{H}^+]$ . Because of the way buffers work, you want the concentration of your buffer (which can donate  $\text{H}^+$  thus preventing a rise in pH) to be the same as the concentration of your buffer's conjugate base (which can pick up  $\text{H}^+$ , thus preventing a lowering of pH). The Henderson–Hasselbalch equation tells you that when those two concentrations are equal,  $\text{p}K_a = \text{pH}$ .

So, when picking a buffer, choose an acid with  $\text{p}K_a$  equal to the desired pH. This is not always possible; the rule of thumb is to pick an acid with a  $\text{p}K_a$  within the range  $\text{pH} \pm 1$ . By using this guideline, you ensure that the buffer and its conjugate base will be present in approximately equimolar amounts in your solution.

*Note:* Biological solutions frequently become acidic over time. So, when choosing a buffer to use in a biological application, try to select a buffer with a  $\text{p}K_a$  that is slightly lower than the desired pH. That way, as the pH of the solution begins to change, the buffer is still working over the range at which it is best at buffering.

The [Buffers](#) section in the Reference topics lists the  $\text{p}K_a$  of some acids that are commonly used as buffers. Chemical company catalogues also list the  $\text{p}K_a$  of acids they sell.

Some acids are “polyprotic;” that is, they can donate more than one proton. That means that there is more than one  $\text{p}K_a$ . When this is the case,  $\text{p}K$  will have a subscripted number (e.g.,  $\text{p}K_2$ ) telling you which proton it donates in the pH range of interest.

## pH AND BUFFERS 5.7: Deciding How Much Buffer to Add to a Solution

There are no firm rules for deciding how much buffer to add to a solution, but there are some facts to consider.

The concentrations of various buffers in humans are as follows:

$$[\text{HCO}_3^-] = 24 \text{ mM}$$

$$[\text{HPO}_4^{2-}] = 1 \text{ mM}$$

$$[\text{SO}_4^{2-}] = 0.5 \text{ mM}$$

For solutions to be used in biological applications, buffer concentrations are usually between 10 mM and 100 mM. Some catalogues indicate correct concentration for the buffers they sell.

The higher the concentration of buffer, the more you should be concerned about the effect the buffer may have on the ionic strength and/or the osmolarity of your solution.

## pH AND BUFFERS – Try It Out

**EXERCISE I:** 

A. What is the molarity of hydrogen ions for a system with the following pH?

1. 2.00

2. 3.00

3. 7.00

4. 7.50

5. 10.50

6. 10.80

B. What is the pH of the system whose concentration of  $H^+$  is as follows?

1.  $1.0 \times 10^{-6}$  M

2.  $1.0 \times 10^{-3}$  M

3.  $1.0 \times 10^{-2}$  M

4.  $5.0 \times 10^{-13}$  M

5.  $2.5 \times 10^{-11}$  M

6.  $8.5 \times 10^{-11}$  M

## EXERCISE II:

Most biological systems are at a pH close to 7.0; human blood, for example, is carefully buffered at pH 7.4 by a carbonic acid—bicarbonate buffering system. Biofluids *must* be carefully buffered because most biological reactions can only happen at the correct pH. Are you likely to find the following agents buffering a solution in a biology lab? (Give a yes or no answer for each.)

Citric acid ( $pK_a = 3.13$ )

MOPS ( $pK_a = 7.15$ )

Piperidine ( $pK_a = 11.12$ )

HEPES ( $pK_a = 7.31$ )

TRIZMA ( $pK_a = 7.8$ )

## LINKS TO ANSWERS

### EXERCISE I

A.

B.

### EXERCISE II

## TRY IT OUT: ANSWERS

### EXERCISE I:

A.

1. 0.010 M
2. 0.0010 M or 1 mM
3.  $1.0 \times 10^{-7}$  M
4.  $3.2 \times 10^{-8}$  M
5.  $3.2 \times 10^{-11}$  M
6.  $1.6 \times 10^{-11}$  M

### EXERCISE I:

B.

1. 6.00
2. 3.00
3. 2.00
4. 12.30
5. 10.60
6. 10.070

## TRY IT OUT: ANSWERS

### EXERCISE II:

Citric acid – no

MOPS – yes

Piperidine – no

HEPES – yes

TRIZMA – yes

## *Math for Life*

### **TOPIC 6: RATES, REACTION RATES, AND $Q_{10}$**

#### **RATES 6.1: What Is a Rate and How Do You Measure It?**

Usually, the word “rate” refers to the amount of time it takes for something to happen. It has dimensions of something over time (whatever is being measured). So, kilometers per hour is a rate with dimensions  $L T^{-1}$ . Kilograms of chocolate per semester is another rate, with dimensions  $M T^{-1}$ . What rates have in common is the  $T^{-1}$ . (Aside: the difference between a rate and a velocity is that rate is a scalar while velocity is a vector; that is, rate has magnitude, velocity has magnitude and direction).

More generally, when we compare any two quantities that have different units—40 miles in 1 hour (comparing distance and time) or 500 miles on 20 gallons (comparing distance and capacity—we have used a rate.

A rate can be written as a fraction:

or expressed in words; 5km *per* hour, 3 mm *per* day.

To determine a rate, graph the measured variable on the vertical axis and time on the horizontal axis. Then determine the slope of the line. What you are doing when you find the slope is measuring the change in measured variable, over the change in time. (This is sometimes called “rise over run.”) The change in  $y$  over the change in time **is** the rate.

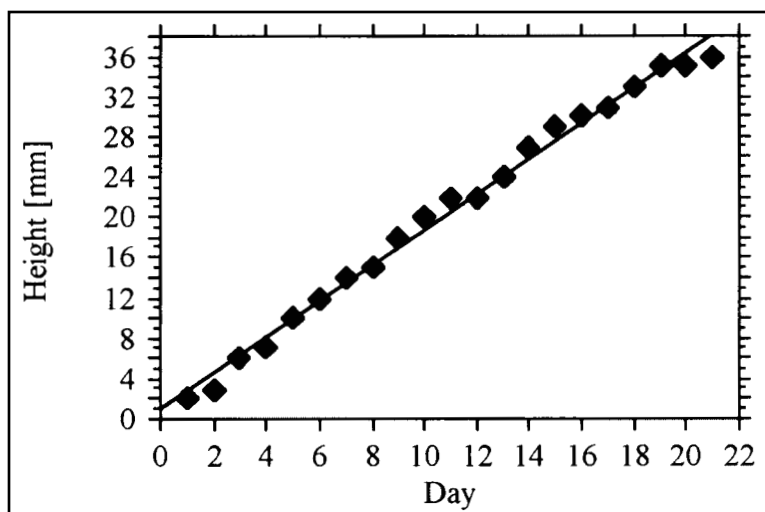
**EXAMPLE:** You measure the height of a plant every day for three weeks. Now, you want to know the rate of growth.



The data:

Day	Height [mm]
1	2
2	3
3	6
4	7
5	10
6	12
7	14
8	15
9	18
10	20
11	22
12	22
13	24
14	27
15	29
16	30
17	31
18	33
19	35
20	35
21	36

The graph:



The *slope* of the line,  $\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$  or  $\left(\frac{\Delta y}{\Delta x}\right)$  or the “rise” over the “run,” is  $1.8 \text{ mm day}^{-1}$  so, the growth *rate* is 1.8 mm per day.



**The Point:** *If time is graphed on the x axis, the slope of the line is the rate.*

## RATES 6.2: Inconstant Rates

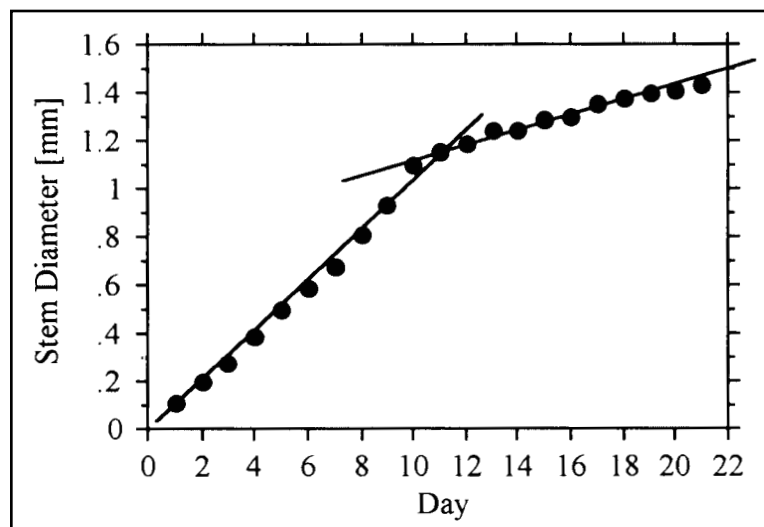
Frequently, rates are not constant over time. Many processes slow down or speed up as time passes. If that happens, then the curve that describes your data will not be a line. If the curve is not a line (that is, the slope of the curve is not constant), you need to measure the slope at the time point of interest. One way to do that is to find a function that describes the data, then take the derivative of the function; *the derivative gives the slope, which is the rate*. The other way to find the slope at the time point of interest is to draw a line tangent to the curve at that point, and find the slope of that tangent line.

**EXAMPLE:** You measure the diameter of a plant stem every day for three weeks. Now, you want to know the rate of growth.

The data:

Day	Height [mm]
1	2
2	3
3	6
4	7
5	10
6	12
7	14
8	15
9	18
10	20
11	22
12	22
13	24
14	27
15	29
16	30
17	31
18	33
19	35
20	35
21	36

The graph:



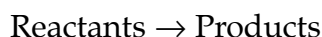
In this example, the diameter of the plant grows quickly for the first 9 days, but then it slows down. This is represented by two lines—one for the diameters measured on days 1 through 9, and a second for the diameters measured on days 10 through 21. The slope of the first line is  $0.10 \text{ mm day}^{-1}$ , the slope of the second line is  $0.030 \text{ mm day}^{-1}$ . So, the growth rate for days 1 to 9 is 0.10 mm per day; the growth rate for days 10 through 21 is 0.030 mm per day.

### RATES 6.3: Reaction Rates

A reaction rate is the amount of time it takes for a chemical reaction to happen. Reaction rates are usually measured in dimensions, mass, or amount per volume per time; the units of a reaction rate are usually: moles per liter per second or M/s.

Like any other rate, you determine a reaction rate by graphing concentration on the  $y$  axis and time on the  $x$  axis, then take the slope. The slope of the curve describing those data is the reaction rate.

Reactions can be summarized as a very simple equation that says “reactants become products”:




Products must be appearing at the same rate that reactants are disappearing (otherwise matter would be disappearing into thin air or appearing out of thin air). So, if you are interested in a reaction rate, you can measure *either* the concentration of reactants (which will be disappearing at the reaction rate) *or* the concentration of products (which will be appearing at the reaction rate). In practice, it can be better to measure something that is appearing; that is, it is better to measure the concentration of the product. Once you have the data, you graph the concentration of the product versus time so that you can take the slope, which is the reaction rate.

### RATES 6.4: Temperature and Reaction Rate

As indicated [above](#), rates sometimes vary, and this is certainly true of reaction rates. The rate of the reaction can vary over time. Reaction rates can also vary with temperature.

To see if the reaction rate of interest varies with temperature, you have to make the same observations—that is, you have to measure concentration of product at several different times—and you have to make those same measurements *again* at a different temperature, then *again* at a third temperature, and so on.

Let's say you measure concentration over time at six different temperatures. At the end of all that measuring, you will have to make six different graphs, each representing how concentration ( $y$  axis) changed with time ( $x$  axis) at one given temperature.

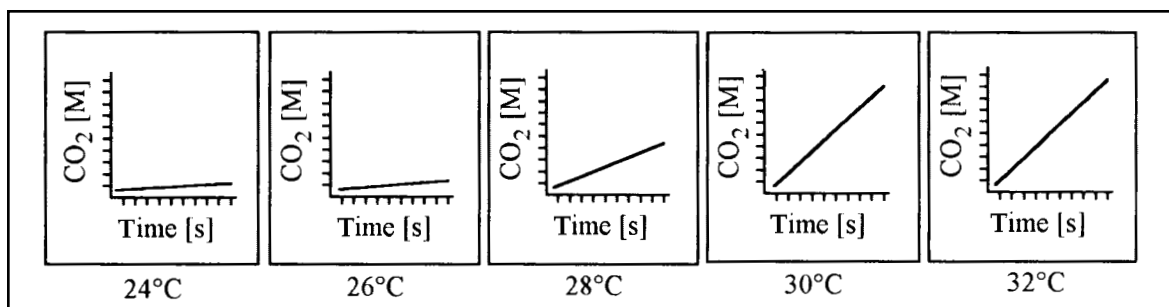
But you aren't done yet. What you want to know is if there is a relationship between reaction rate and temperature. To see if there is a relationship between two variables, you graph them and look at the visual information. 

**EXAMPLE:** Suppose you want to know if the rate of photosynthesis is temperature dependent. This example starts with five separate graphs of concentration versus time. We want to have one graph of reaction rate versus temperature. To get that one graph, we'll take the slopes (rates) from each of the five graphs, then make a new graph with the rates (slopes) on the  $y$  axis and the temperature on the  $x$  axis. Then, we'll look at this [new graph of rate versus temperature](#) to see if there is a relationship between temperature and rate.

**Step one:** You decide to measure the rate of the reaction by monitoring the production of  $\text{CO}_2$ .

**Step two:** You measure  $\text{CO}_2$  concentration every three minutes for 30 minutes at  $24^\circ\text{C}$ ,  $26^\circ\text{C}$ ,  $28^\circ\text{C}$ ,  $30^\circ\text{C}$ , and  $32^\circ\text{C}$ .

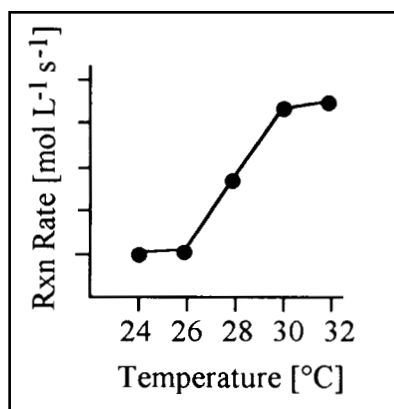
**Step three:** You graph your data on five separate graphs of concentration versus time:



**Step four:** You calculate the five reaction rates by measuring the slopes of the five graphs.

Temperature [°C]	Rate (slope) [M/s]
24 Slow	
26 Slow	
28 Medium	
30 Fast	
32 Fast	

Step five: You graph rate versus temperature.



Voila! Yes, the rate of photorespiration is temperature dependent. At lower temperatures, the photorespiration rate is lower; at higher temperatures the photorespiration rate is higher.

### RATES 6.5: $Q_{10}$

$Q_{10}$  is a number that sums up the effect of temperature on the rate of a reaction.

$Q_{10}$  tells you how much faster a reaction will go if the temperature is raised 10°C (that's where the 10 comes from).

$Q_{10}$  is a quotient (that's where the  $Q$  comes from) that is calculated as follows:

$$Q_{10} = \frac{R_T}{R_{T-10}}$$

where  $R_T$  is the rate at some temperature  $T$  and  $R_{T-10}$  is the rate at 10°C below  $T$ .

**EXAMPLE:** If a reaction rate at 26°C is 20 moles per liter per second, and the reaction rate at 36°C is 50 moles per liter per second,

$$Q_{10} = \frac{50}{20} = 2.5.$$

For more examples, see [Figure 40.8](#) in [Life](#).

**RATES, REACTION RATES, AND  $Q_{10}$  – TRY IT OUT**

**EXERCISE I:** You have done a 12-year study of a coral reef. You have collected these data every January:

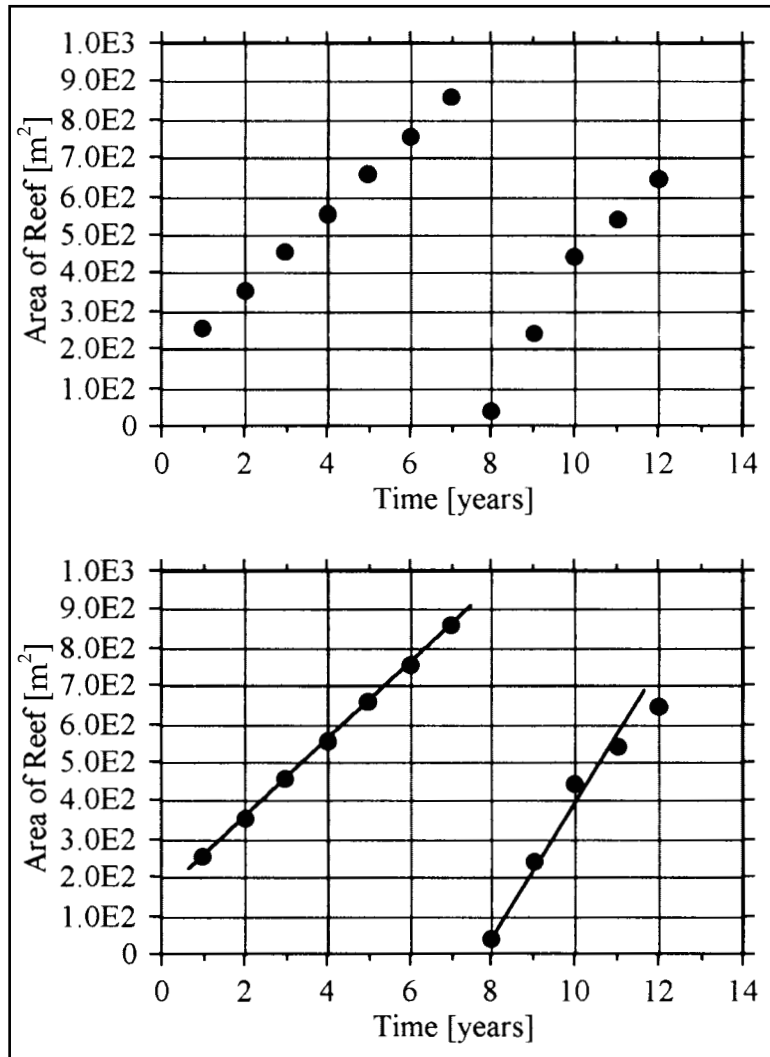
Year of study	Area of reef [ $\text{m}^2$ ]
1	$2.58 \times 10^2$
2	$3.57 \times 10^2$
3	$4.55 \times 10^2$
4	$5.57 \times 10^2$
5	$6.58 \times 10^2$
6	$7.56 \times 10^2$
7	$8.58 \times 10^2$
8	$4.00 \times 10^1$
9	$2.43 \times 10^2$
10	$4.42 \times 10^2$
11	$5.40 \times 10^2$
12	$6.43 \times 10^2$

In August of year 7 a hurricane moved through, destroying much of the reef. After the hurricane, while you are SCUBA diving on the reef to take your data, you get the feeling that the reef is growing faster than before.

To determine whether your intuition is supported by the data, you decide to look at growth rate.



You graph the data, and you look at the rates. Here is the graph, first, with the data, then with the lines you think describe the data:



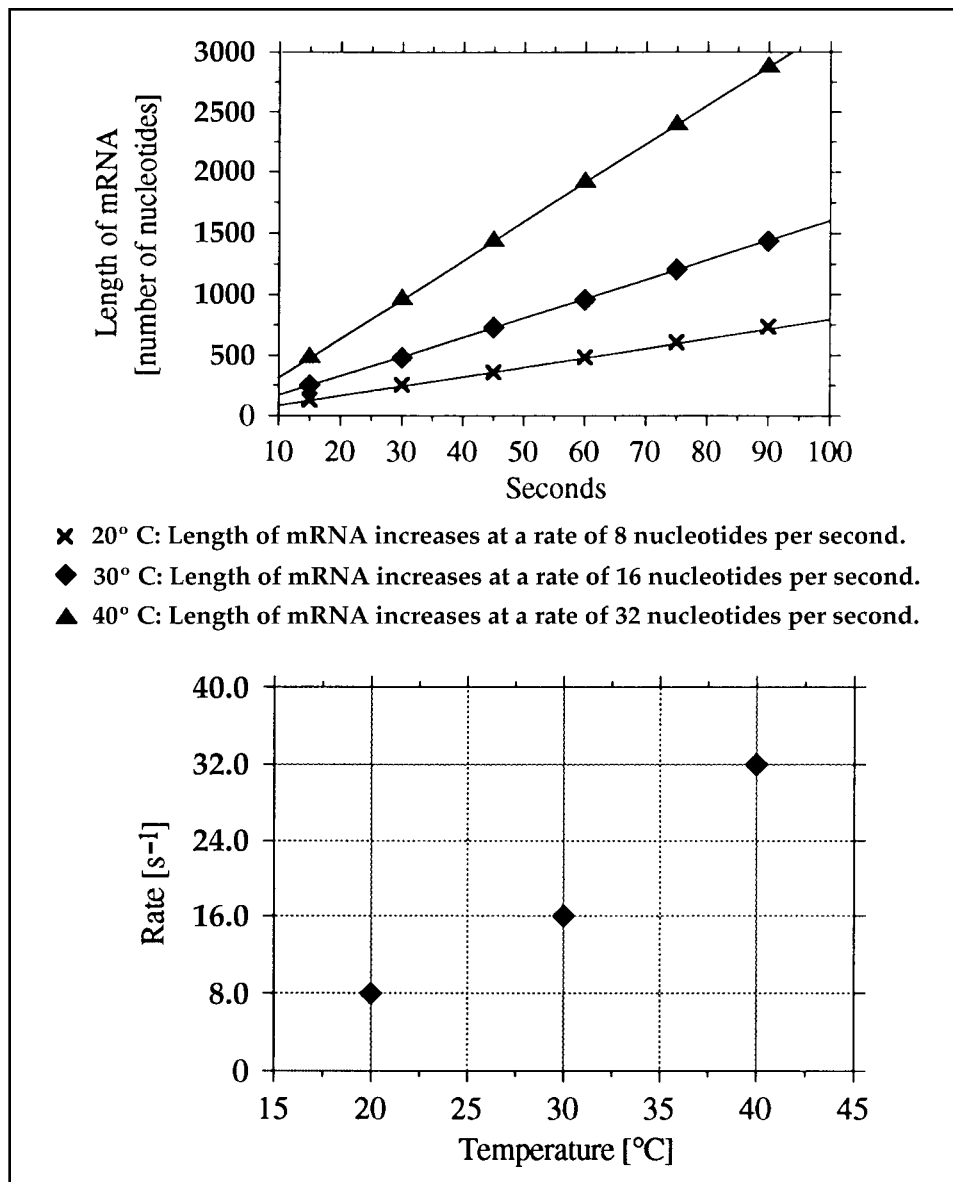
- A. Approximately what is the growth rate before the hurricane?
- B. Approximately what is the growth rate after the hurricane?
- C. Was your intuition correct?





**EXERCISE II:** You are studying the rate at which Polymerase II transcribes DNA into mRNA. You do the following:

1. Collect mRNA length data at three different temperatures.
2. Graph length versus time at each of the three temperatures.
3. Graph the reaction rate versus temperature.

The data looks like this:



What is the  $Q_{10}$  for this reaction? 

**EXERCISE III:** What steps do you need to take to get a measure of  $Q_{10}$ ? 

## LINKS TO ANSWERS

[EXERCISE I](#)

[EXERCISE II](#)

[EXERCISE III](#)

## TRY IT OUT: ANSWERS

### EXERCISE I

- A. About  $1.00 \times 10^2$  m<sup>2</sup> per year
- B. About  $1.50 \times 10^2$  m<sup>2</sup> per year
- C. Yes

## TRY IT OUT: ANSWERS

### EXERCISE II

$$Q_{10} = \frac{R_T}{R_{T-10}} = \frac{32\text{s}^{-1}\text{C}^{-1}}{16\text{s}^{-1}\text{C}^{-1}} = \frac{16\text{s}^{-1}\text{C}^{-1}}{8\text{s}^{-1}\text{C}^{-1}} = 2$$

## TRY IT OUT: ANSWERS

### EXERCISE III

1. Measure the amount of product at a number of different times at a number of temperatures that differ by a multiple of  $10^{\circ}\text{C}$ .
2. Graph amount or concentration of product versus time at each of the different temperatures and draw lines through the data.
3. Calculate the reaction rate at each of the different temperatures by taking the slope of each of the lines.
4. Divide the rate at one temperature by the rate at  $10^{\circ}\text{C}$  below that temperature.

# *Math for Life*

## **TOPIC 7: MAPPING GENES**

### **MAPPING GENES 7.1: Turning Crossover Frequencies into Maps**

Making a genetic map comprises three processes:

1. Counting the number of crossover events between two genes
2. Converting crossover events into distances
3. Figuring out where the genes are situated relative to each other

Suppose two genes are described. You want to make a genetic map. We study each of these three processes in turn in order to make the genetic map.

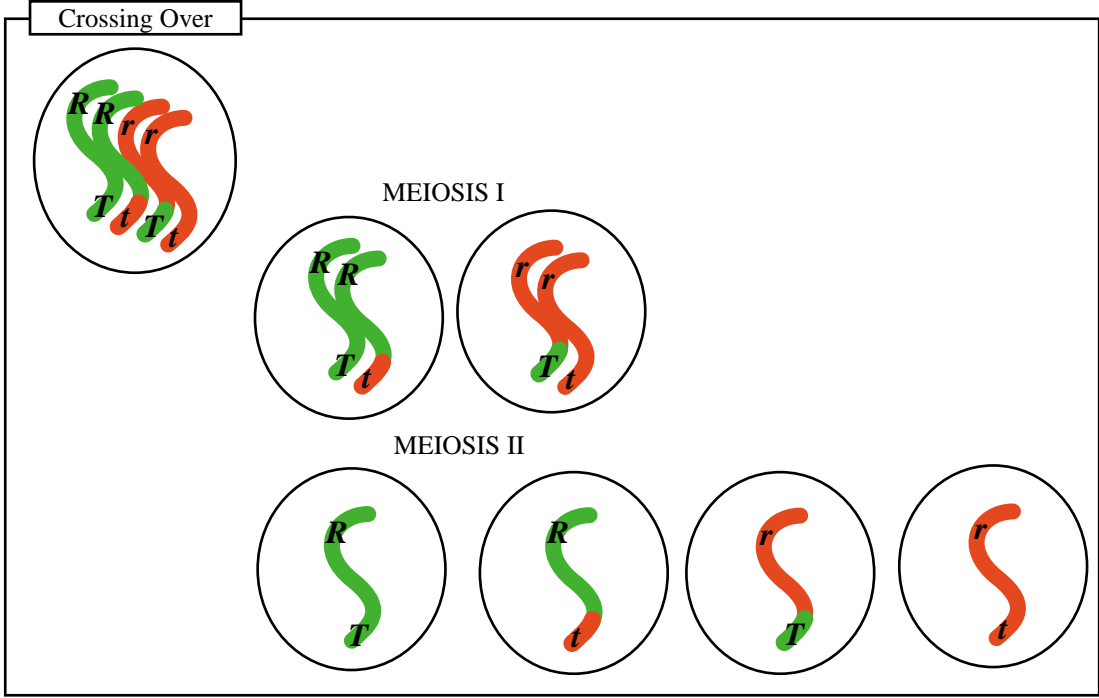
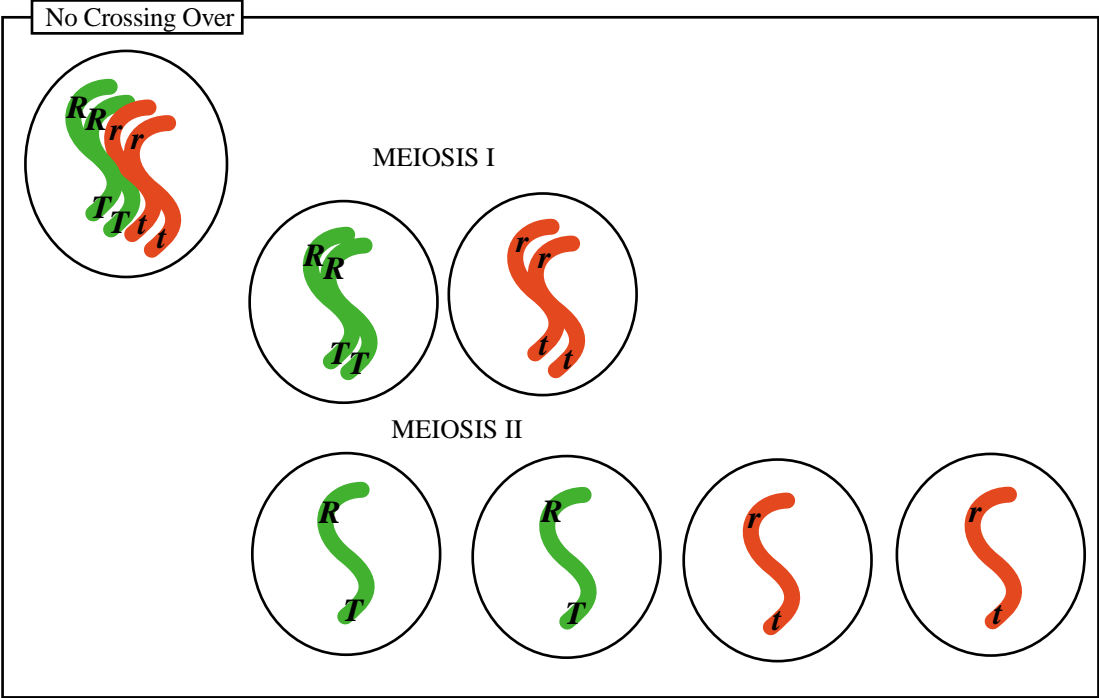
**Process 1:** How many crossover events were there?

You are trying to figure out the positions of two genes relative to one another. The two alleles of one gene, gene *R*, are referred to as *R* and *r*. The two alleles of the other gene, gene *T*, are referred to as *T* and *t*.

You cross an *RRTT* to an *rrtt*, and all the progeny will be *RrTt*. 

Now, you cross the *RrTt* to an *rrtt* (a backcross).

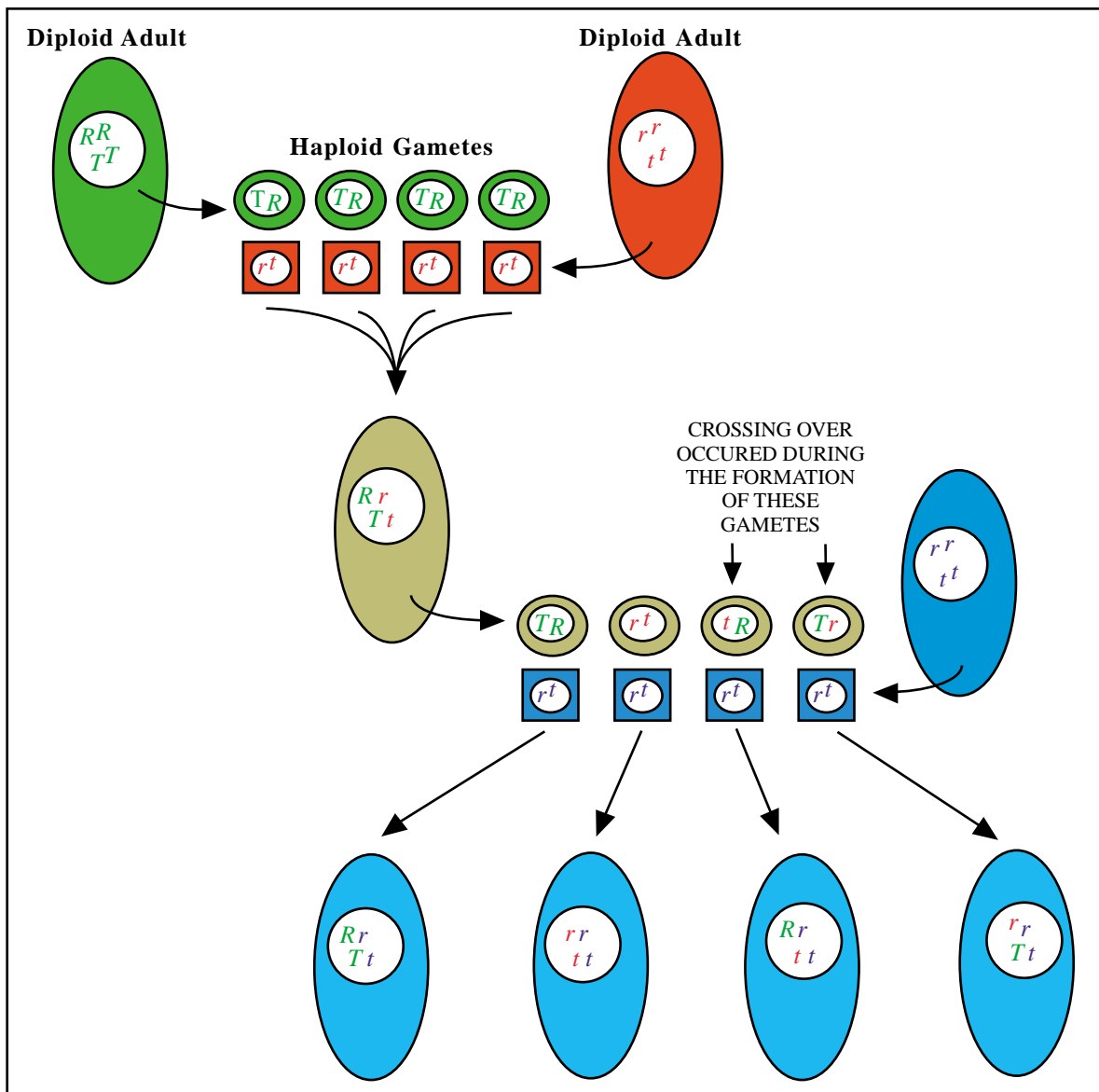
Was there crossing over? The following diagrams show the situations of no crossing over and of crossing over.





If *no* crossing occurred over during gametogenesis, then the heterozygote has made gametes that are either  $RT$  or  $rt$ , while the homozygote has made only  $rt$  gametes. Therefore, the genotypes that result when a gamete from the heterozygote meets a gamete from the homozygote (if crossing over has not happened) are  $RrTt$  and  $rrtt$ .

If crossing over *did* occur during gametogenesis, then the heterozygote also made some gametes that are either  $Rt$  or  $rT$ . The homozygote still made  $rt$  gametes only. So, the genotypes that result when these gametes meet (if crossing over has occurred) are  $Rrtt$  and  $rrTt$ , in addition to  $RrTt$  and  $rrtt$ .



To calculate the frequency of crossovers, you add the number of *Rrtt*'s plus the number of *rrTt*'s then divide by the total number of offspring.

**EXAMPLE:**

Genotype	Number of offspring of that genotype
<i>RrTt</i>	39
<i>rrtt</i>	48
<i>Rrtt</i>	12
<i>rrTt</i>	16

The number of offspring in which crossing over occurred is the number of *Rrtt*'s plus the number of *rrTt*'s, or 12 + 16 = 28.

The total number of offspring is 39+48+12+16 = 115.

The frequency of crossing over is

$$\frac{28 \text{ events}}{115 \text{ offspring}} = 0.24 \frac{\text{events}}{\text{offspring}}$$

**Process 2:** How far apart are the two genes?

It may seem strange, but to figure out the distances between pairs of genes, you use frequency data to determine length. The units in the computation are also unusual.

The good news is that the procedure is exactly the same as converting between any two dimensions. You set up your conversion equation then solve it.



In this situation, the frequency you measure is the number of recombination events per number of offspring, and the units of length are Morgans [M], or, more usually, centimorgans [cM].

Here is the [conversions shortcut](#), again, as a reminder.

**CONVERSIONS SHORTCUT**

- 1. Write an equation.**
- 2. Count units.**
- 3. Insert that many fractions.**
- 4. Make units cancelable.**
- 5. Make fractions =1, then calculate.**

So, to convert number of recombination events per offspring to cM, you follow the same directions:

1.  $\frac{\text{number of events}}{\text{offspring}} = ?cM$
2. three different units
3.  $\frac{\text{number of events}}{\text{offspring}} \times \left(\frac{\quad}{\quad}\right) \times \left(\frac{\quad}{\quad}\right) \times \left(\frac{\quad}{\quad}\right) = ?cM$
4.  $\frac{\text{number of events}}{\text{offspring}} \times \left(\frac{\quad}{\text{number of events}}\right) \times \left(\frac{\text{offspring}}{\quad}\right) \times \left(\frac{cM}{\quad}\right) = ?cM$

Here is where it gets tricky. How do you make these fractions equal to one? What is the relationship between the units?

The relationship between the units is given in the definition of the centimorgan:

$$1 \text{ cM} = 1 \text{ event per } 100 \text{ of offspring} = \frac{1 \text{ event}}{100 \text{ offspring}}$$

Rearranging that definition we find that  $= \frac{1 \text{ cM} \times 100 \text{ offspring}}{1 \text{ event}} = 1$

That is the relationship among the units. Returning to step 5 of the conversion shortcut, we write:

$$5. \frac{\text{number of events}}{\text{offspring}} \times \frac{1 \text{ cM} \times 100 \text{ offspring}}{\text{event}} = ? \text{ cM}$$

As you can see, all you are doing is multiplying the frequency by  $10^2$  while converting units. It's actually quite simple once you know the rule.

**EXAMPLE:** Consider the same genes *R* and *T* as in the example above. If there were 28 crossover events out of 115 offspring, how many cM apart are the two genes?

$$\frac{28 \text{ events}}{115 \text{ offspring}} \times \frac{1 \text{ cM} \times 100 \text{ offspring}}{\text{event}} = 24 \text{ cM}$$

The *T* gene is 24 cM away from the *R* gene.

**Process 3:** Where are the alleles located?

This question comes up when you are mapping more than two genes.

To figure gene positions relative to each other, you figure the distances between each pair (which means doing three backcrosses), then look to see what order is consistent with all three distances.

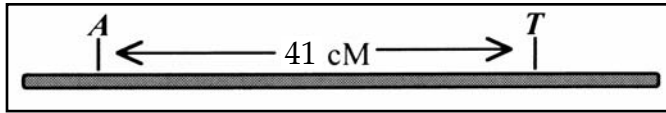
Let's add gene *A*, with its two alleles, *A* and *a*.

Here are the results of the backcrosses:

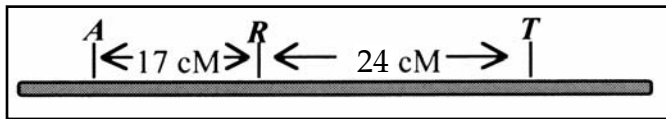
Genes	Crossovers	Total number of offspring	Crossover frequency	cM
<i>R</i> to <i>T</i>	28	115	0.24	24
<i>A</i> to <i>T</i>	55	134	0.41	41
<i>R</i> to <i>A</i>	26	151	0.17	17

From this chart, you can tell the distances between each pair of genes.

To go from the chart to a map takes two steps:



**Step one:** Start the map using the two genes that are separated by the *greatest* distance.



**Step two:** Put the third gene between the other two, the correct distance from either (both) of the first two.

That is your genetic map.

## MAPPING GENES – TRY IT OUT

**EXERCISE I:** The genes *ATP7B*, *BRCA2*, and *RB1* are found on human chromosome 13. Map them using these data (the data are completely imaginary, the map will be relatively accurate):



Genes	Events	Total number of offspring
<i>ATP7B</i> and <i>BRCA2</i>	5	24
<i>ATP7B</i> and <i>RB1</i>	3	32
<i>BRCA2</i> and <i>RB1</i>	2	17

**EXERCISE II:** Map the imaginary genes *Hlr*, *Oria*, *Prvs*, and *Sdva*

Genes	Events	Total number of offspring
<i>Hlr</i> and <i>Oria</i>	17	119
<i>Hlr</i> and <i>Prvs</i>	40	121
<i>Hlr</i> and <i>Sdva</i>	8	130
<i>Oria</i> and <i>Prvs</i>	26	138
<i>Oria</i> and <i>Sdva</i>	25	125
<i>Prvs</i> and <i>Sdva</i>	55	141

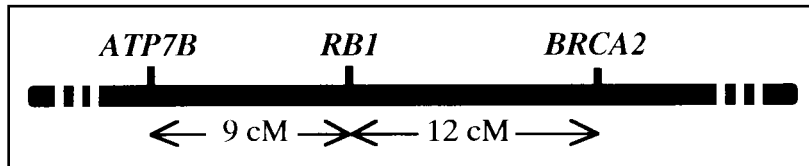
## LINKS TO ANSWERS

[EXERCISE I](#)

[EXERCISE II](#)

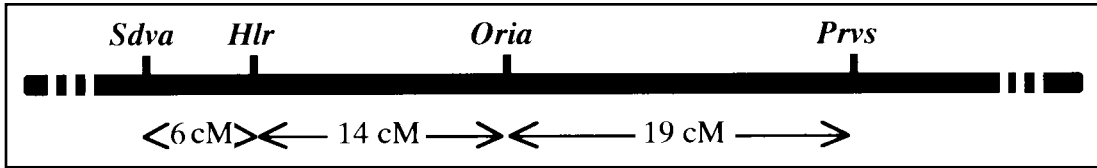
**TRY IT OUT: ANSWERS**

**EXERCISE I:** The genes *ATPTB*, *BRCA2*, and *RB1* are found on human chromosome 13. Map them using these data.



**TRY IT OUT: ANSWERS**

**EXERCISE II:** Map the imaginary genes *Hlr*, *Oria*, *Prvs*, and *Sdva*.





# *Math for Life*

## **TOPIC 8: PUNNETT SQUARES**

### **PUNNETT SQUARES 8.1: Definition**

A Punnett square is a device to help you predict the possible genotypes of the offspring if you know the genotypes of the parents.

Because it tells you how many of each phenotype to expect if all the genes are assorting independently, a Punnett square gives you a standard with which to compare the results of your cross so that you can determine whether or not your genes are, in fact, assorting independently.

First, you have to look at the genotype of each parent, predict what will happen when gametes are formed, then predict what could happen when two gametes combine.

### **PUNNETT SQUARES 8.2: Crossing Over and Genetic Variation**

During meiosis I, when there are four copies of each chromosome, the four copies are held close together by a structure called the synaptonemal complex; the whole structure is called a tetrad.

Within the tetrad, one copy of each homolog can trade stretches of DNA with one copy of the other homolog by the process of crossing over.

Because each of the two homologs was donated by a different parent (of the organism in whose gonad this meiosis is taking place), the two homologs are not necessarily identical. So, when crossing over happens, each of the participating homologs can end up different from when it started.

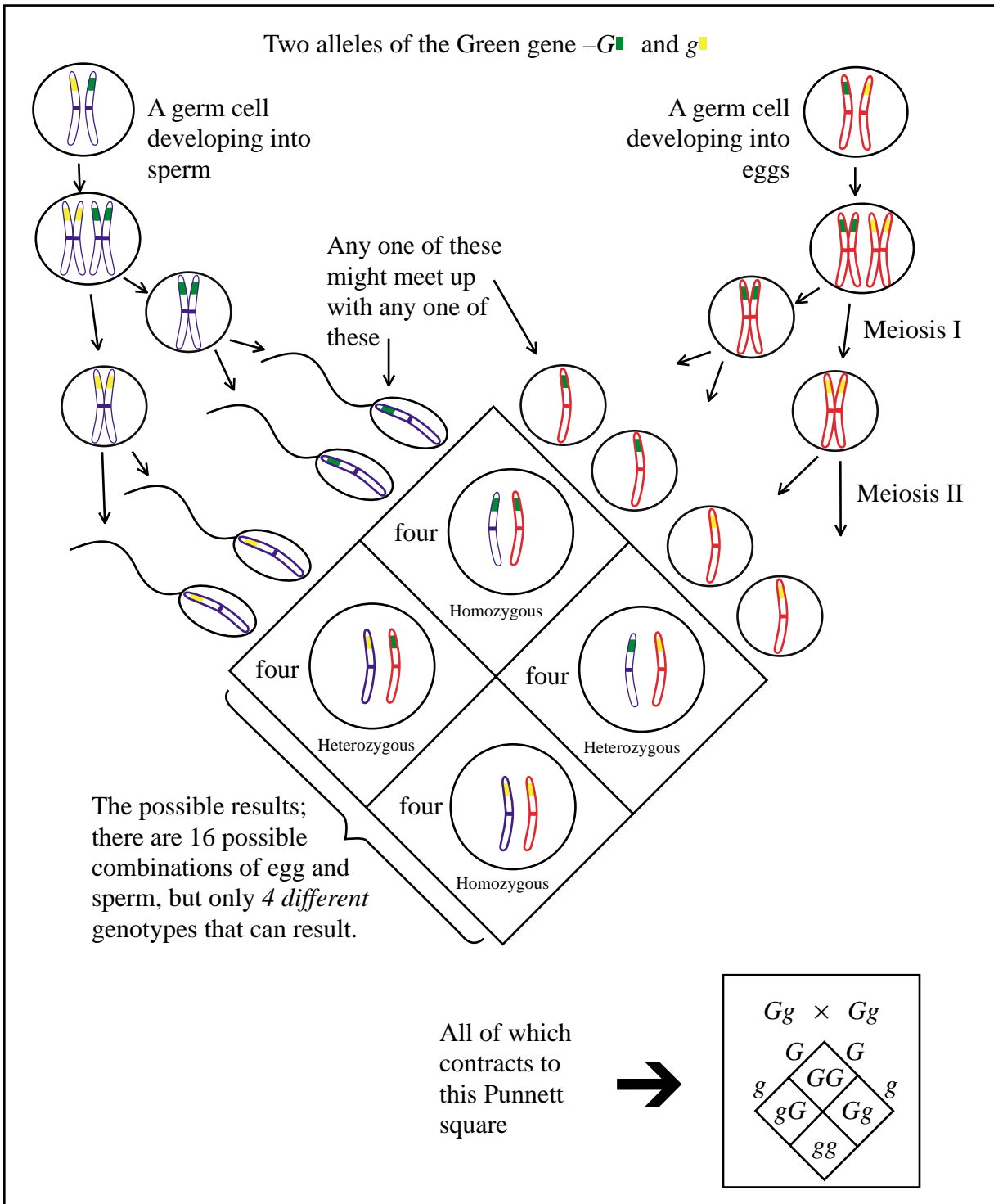
That means that the gametes that result from this meiosis can all end up with slightly different versions of each chromosome.

Thus meiosis, in addition to reducing chromosome number from two to one, also shuffles the genetic information. This is one very important mechanism by which genetic variation is generated.

By the way, this is why offspring are not identical to their parents or to their siblings. This is also why siblings can be more similar genetically to each other than they are to either of their parents: offspring are genetically related exactly 50% to each parent (half of their genes come from dad, half from mom), but are genetically related to each other 50% *on average*, meaning they can be over (or under) 50% related to each other.

### **PUNNETT SQUARES 8.3: Using Punnett Squares for Possible Genotypes**

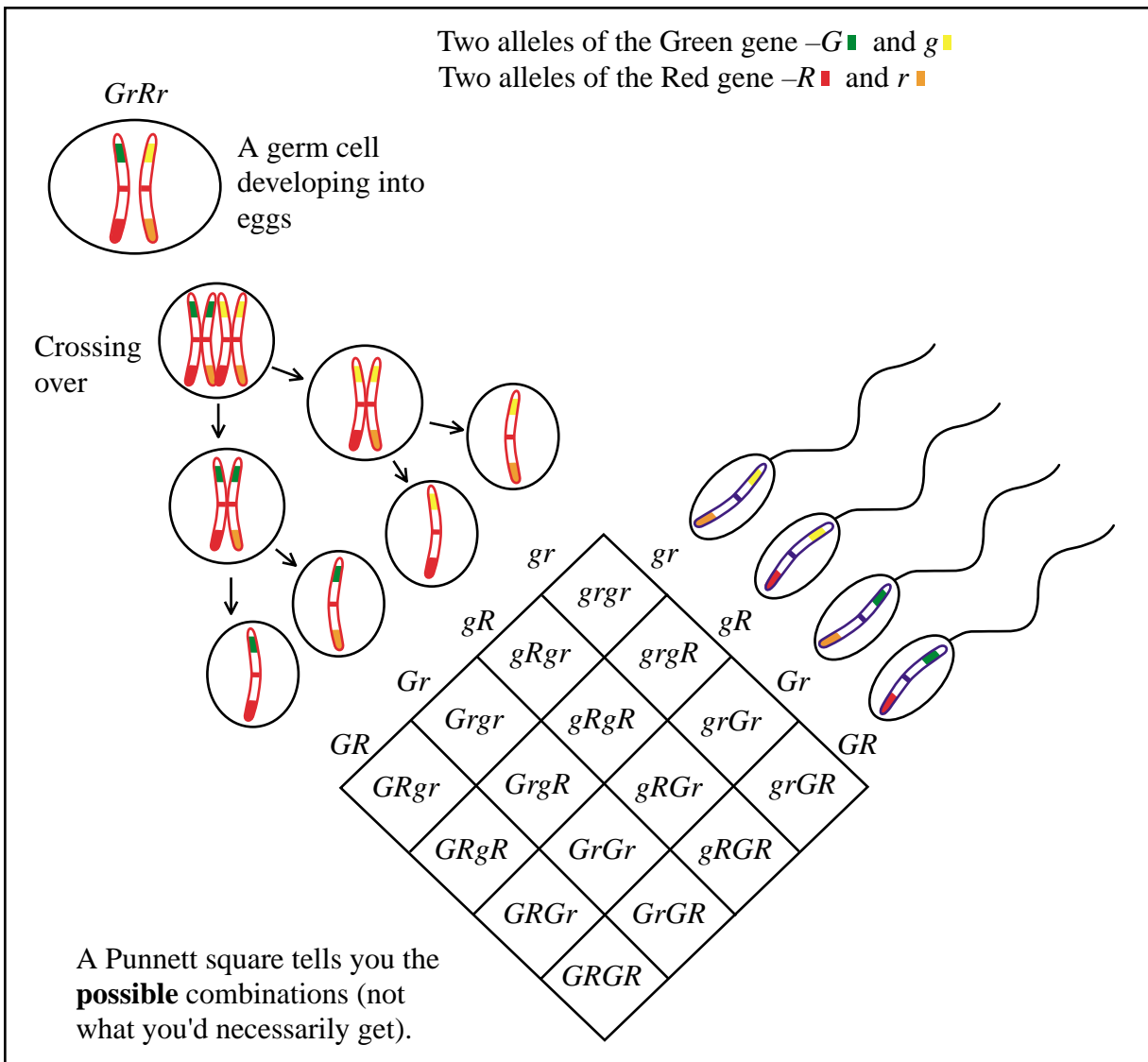
**EXAMPLE I:** How many possible genotypes can be generated from mating a male to a female, both of which are heterozygous at the *green* locus?



If you are trying to figure out the possible genotype at a single locus (one gene, two alleles), then each parent has  $2^1$  or 2 alleles (two versions of that gene) to offer, so the possible combinations are  $2 \times 2 = 4$ . 4 is  $2^2$ , and if you draw a rectangle that is 2 by 2, it will be a square. Hence, the “square” in a Punnett square.

**EXAMPLE II:** What if you are interested in more than one gene?

If you are trying to figure out the possible genotype at two loci (two genes, two alleles) then each parent has  $2^2$  or 4 possible alleles to offer, so the possible combinations are  $4 \times 4 = 16$ . 16 is  $4^2$ , and if you draw a rectangle that is 4 by 4, it will be a square. Hence, you will have another Punnett square.



If you are trying to figure out the possible genotype at three loci (three genes, two alleles) then each parent has  $2^3$  or 8 alleles to offer, so the possible combinations are  $8 \times 8 = 64$ .

The rule: The number of gametes with *different* genotypes that an individual can produce equals  $H^L$  where  $L$  is the number of loci under investigation, and  $H = 1$  or  $2$ , depending on whether those loci are homozygous ( $H = 1$ ) or heterozygous ( $H = 2$ ). The number of combinations possible in the offspring equals the number the mother can produce times the number the father can produce.

The diagrams in examples I and II illustrate some of the points made above:

1. Germ cells are  $2N$ , then undergo two meiotic divisions to produce  $1N$  gametes (both illustrations).
2. Meiosis I separates homologues; meiosis II separates sisters (both illustrations).
3. During meiosis I, one copy of each homolog can participate in a crossing over event; the result is that the chromosomes found in the resulting gametes can be different from each other and from the chromosomes of the germ cell (second illustration).

## PUNNETT SQUARES 8.4: Using Punnett Squares to Determine Linkage

If the genes under investigation are assorting independently (that is, if they are on different chromosomes), then, as illustrated by the Punnett square, the possible genotypes will be present in particular ratios relative to each other.

### EXAMPLE

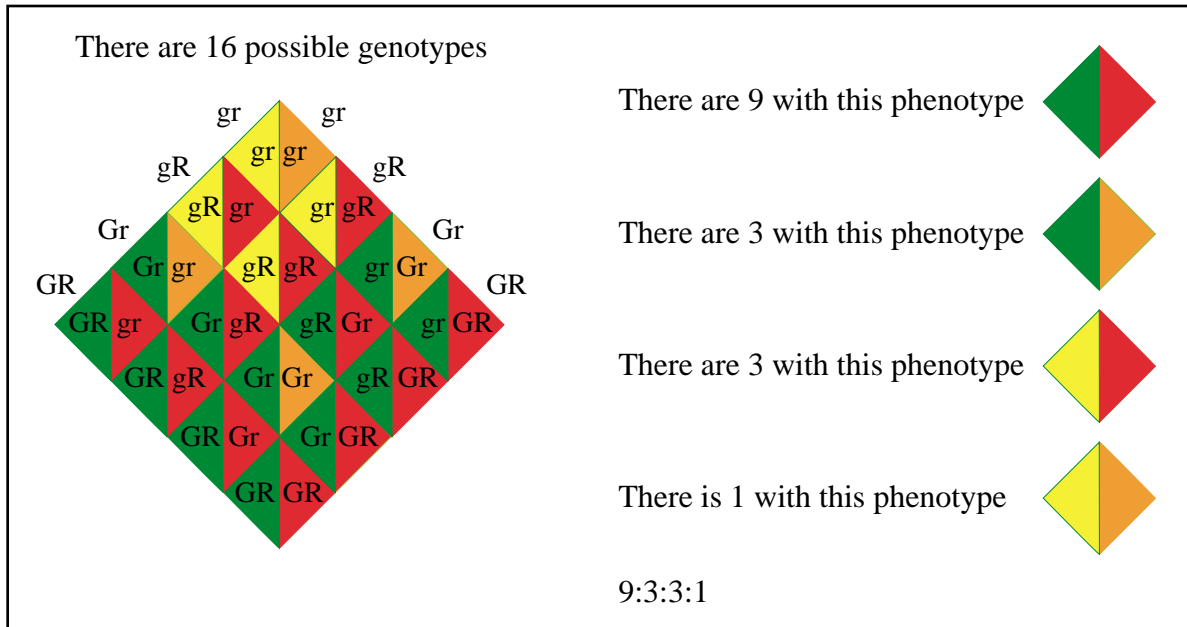
If you are looking at two genes, and they are on different chromosomes there will be 16 possible combinations.

These result in 9 different genotypes (assuming that, for example,  $GgRr = gGrR = GgrR = gGRr$ , which is usually the case).

If you are referring to loci where  $G$  and  $R$  are dominant and  $g$  and  $r$  are recessive, then those 9 different genotypes can result in only 4 different phenotypes.

And those 4 different phenotypes will be represented in the ratio 9:3:3:1.

To see why, look at the Punnett square:



So, if you do a cross, and find the phenotypes present in a ratio of 9:3:3:1, then the two genes were independently assorted.

You may have to do some statistics to determine whether the ratio is 9:3:3:1 or not; nonetheless, the Punnett square gives you a *prediction* with which to compare your actual results.

**PUNNETT SQUARES – TRY IT OUT**

**EXERCISE I:** You are crossing two individual plants that are heterozygous at the Leafy locus; the gene found there determines how many leaves the plant will have. The dominant allele makes the plant that has it very leafy; the recessive allele leads to few leaves.

- A. Draw the Punnett square using  $L$  for the dominant allele and  $l$  for the recessive allele.
- B. What ratio of genotypes do you expect?
- C. What ratio of phenotypes, leafy and/or not leafy, do you expect?

**EXERCISE II:** You are crossing two sea urchins, one of which is purple because it is homozygous dominant at the locus for its purple color ( $PP$ ); the other is pink because it is homozygous recessive at the purple locus ( $pp$ ).

- A. Draw the Punnett square representing this cross.
- B. What ratio of genotypes do you expect?
- C. What ratio of phenotypes, purple and/or pink, do you expect?

**EXERCISE III:** You are crossing two plants, one of which is heterozygous at the tasty roots locus ( $Tt$ ); the other is homozygous recessive at the tasty roots locus ( $tt$ ).

- A. Draw the Punnett square representing this cross.
- B. What ratio of genotypes do you expect?
- C. What ratio of phenotypes, tasty and/or not tasty, do you expect?

**EXERCISE IV:** You are crossing flies that are heterozygous at the *White* locus ( $Ww$ ); their eyes are red. They are also heterozygous at the *Curly* locus ( $Cc$ ); they have normal wings. Flies that are homozygous recessive at both of those loci have white eyes and curly wings.

- A. Draw the Punnett square representing this cross.
- B. How many genotypes do you expect?
- C. What will the ratio of phenotypes be if the loci are independent?
- D. Here are the data from that cross:

Phenotype	Number of offspring
Red eyes, normal wings	130
White eyes, normal wings	40
White eyes, curly wings	15
Red eyes, curly wings	43

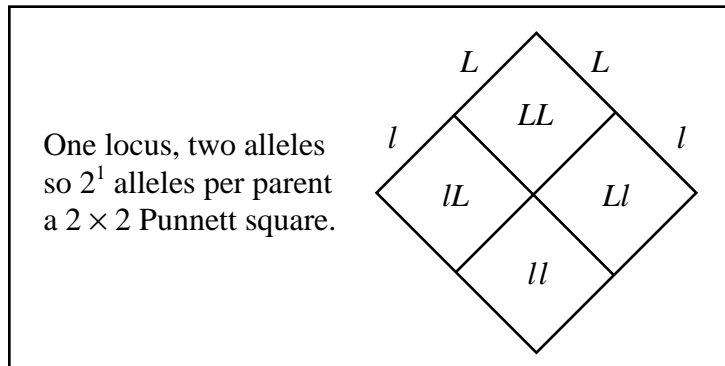
Is the *White* gene on the same chromosome as the *Curly* gene?



**TRY IT OUT: ANSWERS**

**EXERCISE I:**

A.



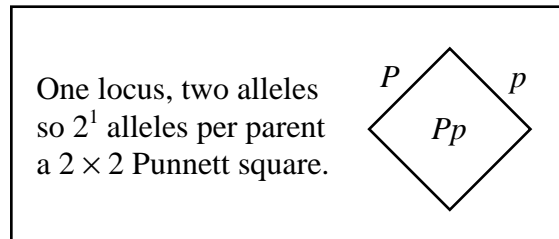
B.  $LL:Ll:lL:ll = 1:1:1:1$ . If  $Ll = lL$ , then  $LL:Ll:ll = 1:2:1$

C. Leafy : not leafy = 3:1 because  $LL$ ,  $Ll$ , and  $lL$  will be leafy,  $ll$  will be not leafy.

**TRY IT OUT: ANSWERS**

**EXERCISE II:**

A.

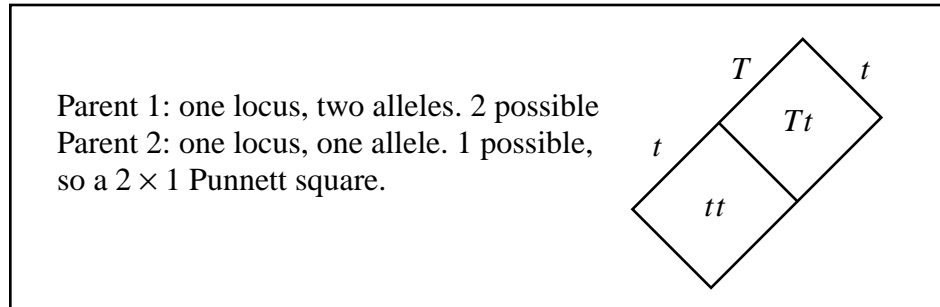


- B. This one isn't fair—it's a trick question. All of the offspring will be  $Pp$ , so there isn't really a ratio. You could say that the ratio is 0:1:0, but that description isn't commonly used.
- C. All the offspring will be purple.

**TRY IT OUT: ANSWERS**

**EXERCISE III:**

A. This one isn't really a square.



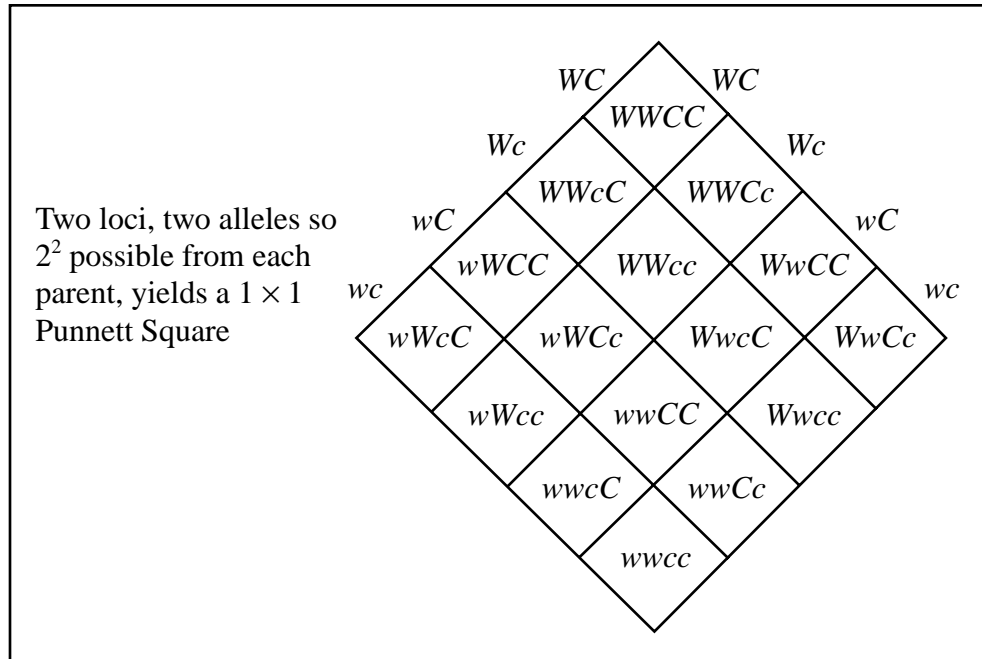
B.  $Tt:tt = 1:1$

C. Tasty : not tasty = 1:1

**TRY IT OUT: ANSWERS**

**EXERCISE IV:**

A.



- B. If  $Cc = cC$  and  $Ww = wW$  there can be 9; if not, there can be 16.
- C. 9:3:3:1
- D. No. The ratio of phenotypes is approximately 9:3:3:1, so the two genes are assorting independently.

# *Math for Life*

## **TOPIC 9: RADIOACTIVE DATING**

### **DATING 9.1: How to Keep Time**

If you want to know how old something is, you need an appropriately scaled device for measuring the passage of time.

**EXAMPLE:** How old are you?

To answer that question, you use a calendar that keeps track of the passage of years; you subtract the year you were born from the current year to find out how many years have passed since you were born; that's your age.

**EXAMPLE:** How fresh is the milk in my refrigerator?

To answer that question, you use a calendar that keeps track of the passage of days; you subtract the last date of freshness from today's date to determine how many days (or weeks) have passed since your milk was last fresh.

**EXAMPLE:** How long until we get there?

To answer that question you usually use a clock that keeps track of the passage of minutes; you subtract the time it is now from the estimated time of arrival to predict the minutes remaining.

The above examples use familiar time pieces: calendars and clocks. To make those measuring devices, humans divided time into equal-sized, repeating intervals of time, (based ultimately on the rotation of Earth about its axis and the revolution of Earth around the sun).

Note: That's how you make *any* measuring device: You take the [dimension](#) under consideration (e.g., length, time, mass) and divide it up into equal-sized portions.

The size of the portion is *defined* as a standard against which you can measure whatever you want. Having a standard that everybody agrees on allows you to say “the pod is 5 units long and will float for 8 time units” and everyone will know what sizes you mean. (Historical note: The meter, a **unit** of length, was defined in 1790 as  $10^{-7}$  the earth’s quadrant passing through Paris. In 1960 it was redefined as 1,650,763.73 wavelengths of the radiation of krypton 86.)

So, we use meter sticks to measure lengths in units of millimeters, centimeters and meters; we use clocks to measure time in units of seconds, minutes, and hours; we use calendars to measure time in units of days, weeks, months, and years; we use *radioactive decay* to measure time in units of thousands of years up to billions of years.

## DATING 9.2: Radioactive Decay

For the first 4.5 billion years of Earth’s existence, there were no calendars. However, a certain number of radioactive isotopes of elements were present when the earth began, and radioactive decay keeps regular time. Here is how:

Each radioisotope loses its radioactivity at a specific, well-defined rate. That is, the radioactivity decays at a rate characteristic for that element.

**EXAMPLE:** It takes  $4.46 \times 10^9$  years for 50 percent of an amount of  $^{238}\text{U}$  to decay.

**EXAMPLE:** It takes  $5.715 \times 10^3$  years for 50 percent of an amount of  $^{14}\text{C}$  to decay.

Those characteristic rates (units are: number per time, or  $\text{T}^{-1}$ ) can be used to convert the number of radioisotopes (which is measurable) into units of time.

To use radioactive decay to measure time duration, that is, to use it as a calendar, all you need to know is the **conversion factor**. The conversion factor is the characteristic rate of decay.

### **What is the half-life of a radioisotope?**

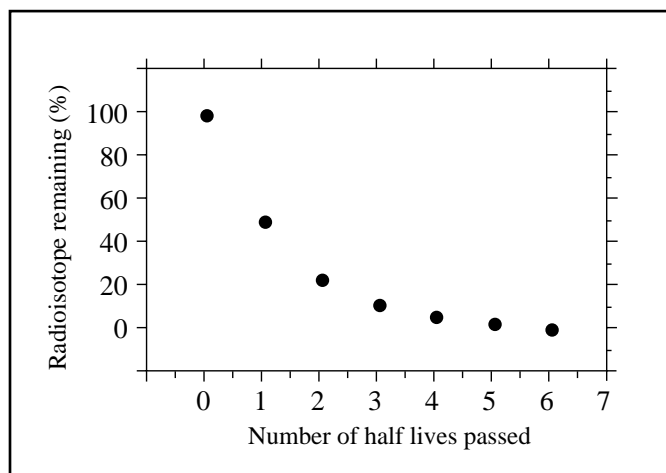
The half-life, frequently abbreviated  $t_{1/2}$ , is the amount of time it takes for 50 percent of the radioisotope to decay. That means that the rate of decay depends on the amount of radioisotope present.

**EXAMPLE:**  $4.5 \times 10^9$  years is said to be the **half-life** of  $^{238}\text{U}$ , because that’s how long it takes for half of it (50 percent) to decay.

**Radioactive decay as a calendar**

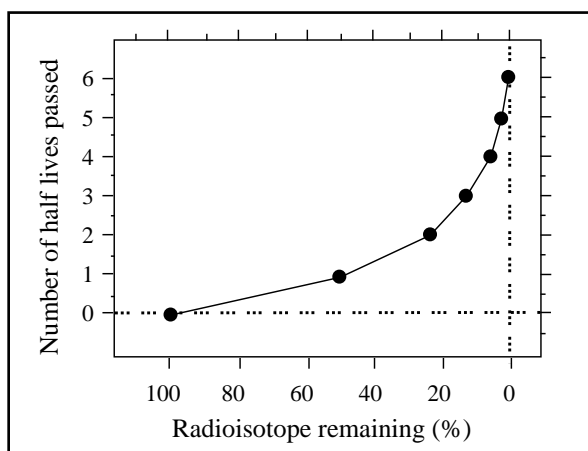
Because the decay rate depends on the amount of radioisotope present, the rate of radioactive decay is *not linear*; that is, if you start out with 1.0 kg of  $^{238}\text{U}$ , after the passage of  $4.5 \times 10^9$  years, there will be 0.50 kg of  $^{238}\text{U}$  and 0.50 kg of  $^{206}\text{Pb}$ . After another  $4.5 \times 10^9$  years pass, another 50 percent of the  $^{238}\text{U}$  will have decayed, so there will be 0.25 kg of  $^{238}\text{U}$  and 0.75 kg of  $^{206}\text{Pb}$ .

A graph of radioactive decay looks like this:



Note: Although theoretically this curve never reaches zero, in reality, the last radioactive atom from the original amount will decay eventually. When it does, (and when depends on how many atoms there were to begin with), there will be no radioisotope left.

So, if you want to measure the age of an item using radioactivity as a clock, you can look at your item and determine what percentage of a relevant radioisotope is gone, then calculate how much time it must have taken for that amount to decay. It is easier to see how that works if we interchange the axes of the above graph:



The information is exactly the same in the two graphs shown; the orientation of the axes has simply been changed. You can use this graph as a clock. Find the per centage of radioisotope remaining on the horizontal axis and read the number of half-lives passed from the vertical axis. That provides one method of dating a fossil.

To find the half-life of a particular radioisotope, you can refer to the *CRC Handbook of Physics and Chemistry*; the half-lives of some radioisotopes used by life scientists are listed here:


Radioisotope	Half-Life
<sup>238</sup> Uranium ( <sup>238</sup> U) 4.46	×10 <sup>9</sup> years
<sup>40</sup> Potassium ( <sup>40</sup> K) 1.26	×10 <sup>9</sup> years
<sup>235</sup> Uranium ( <sup>235</sup> U) 7.04	×10 <sup>8</sup> years
<sup>239</sup> Plutonium ( <sup>239</sup> Pu) 2.44	×10 <sup>4</sup> years
<sup>14</sup> Carbon ( <sup>14</sup> C)	5715 years
<sup>3</sup> Hydrogen ( <sup>3</sup> H, also called tritium, T)	12.32 years
<sup>35</sup> Sulfur ( <sup>35</sup> S)	87.2 days
<sup>32</sup> Phosphorus ( <sup>32</sup> P) 14.28	days
<sup>131</sup> Iodine ( <sup>131</sup> I)	8.040 days

To determine how much of your radioisotope has decayed, you need to know how much was there to begin with. This answer depends on what radioisotope you are interested in. And each isotope provides a different kind of calendar. If you are interested in determining the age of a fossil (i.e., something that was a living organism sometime within the last 30,000 years), <sup>14</sup>C dating is commonly used.



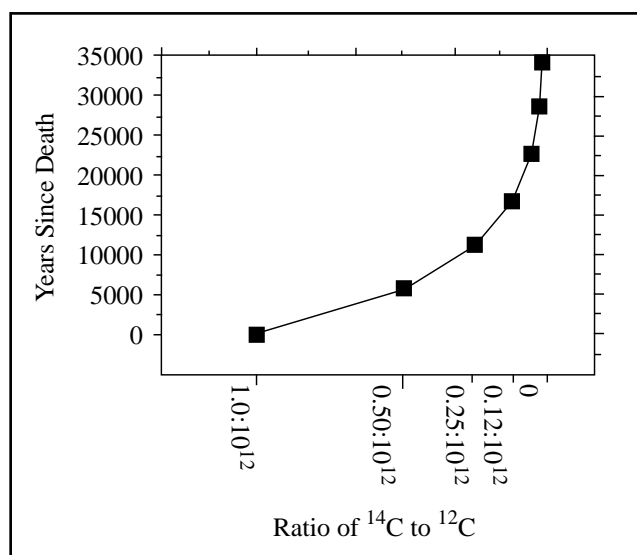
### DATING 9.3: $^{14}\text{C}$ Decay as a Calendar

The  $^{14}\text{C}$  that was present when Earth was created has long since decayed. However, new  $^{14}\text{C}$  is produced in the atmosphere at the same rate it decays on Earth, so, there is always the same amount on this planet; that is, for every  $10^{12}$  atoms of  $^{12}\text{C}$  there is one atom of  $^{14}\text{C}$ . Put another way, for every mole of  $^{12}\text{C}$  atoms, there are about  $6 \times 10^{11}$  atoms of  $^{14}\text{C}$ .

Because living organisms constantly exchange carbon with the environment,  the proportion in any living organism is also one  $^{14}\text{C}$  for every  $10^{12}$   $^{12}\text{C}$  isotopes.

Once an organism dies, however, it no longer exchanges carbon with its environment. So, starting from the moment of death, the proportion of  $^{14}\text{C}$  relative to  $^{12}\text{C}$  inside the organism starts to decrease. And it starts to decrease at a characteristic decay rate, that is, the half-life of 5,715 years.

So, if we recreate the graph above, filling in the appropriate numbers for  $^{14}\text{C}$ , we get the following relationship that allows us to convert the ratio of  $^{14}\text{C}$  to  $^{12}\text{C}$  in a fossil into time since death of the organism:



To determine how long an organism has been dead using this graph, all you need do is measure the ratio of  $^{14}\text{C}$  to  $^{12}\text{C}$ , then see how that corresponds on the graph to the number of years that has passed; that is the age of the fossil.

An alternative to using the graph is to use an equation that relates the percentage of radioisotope remaining to time passed. The equation is:

$$\text{percentage of radioisotope remaining} = \frac{{}^{14}\text{C}}{{}^{12}\text{C}} \times 10^{12} = \frac{100}{2^x}$$

where  $x$  is the number of half-lives that have passed. (Note: this is the equation of the curve in the graph.)

Rearranging the above equation gives:

$$2^x = \frac{100}{\text{Percentage of radioisotope remaining}}$$

Taking the  $\log_2$  of both sides gives:

$$x = \log_2(100) - \log_2(\% \text{ radioisotope remaining})$$

$$x = \log_2(100) - \log_2(\% \text{ radioisotope remaining})$$

And,  $x$  times the half life in years tells you the number of years that have passed since the organism was alive.

---

### **<sup>14</sup>C FOSSIL DATING SHORTCUT**

- 1. Multiply the current ratio of <sup>14</sup>C to <sup>12</sup>C by 10<sup>12</sup>.**
  - 2. Multiply that quotient by 100 to calculate % remaining.**
  - 3. Take the log<sub>2</sub> of that percentage.**
  - 4. Subtract that number from 6.644.**
  - 5. Multiply the difference by 5715.**
- 

You cannot use <sup>14</sup>C to date anything older than 30,000 years because after about 5 half lives ( $3 \times 10^4$  years) there is too little <sup>14</sup>C to measure accurately.

## RADIOACTIVE DATING – TRY IT OUT

**EXERCISE I:** You find a fossil and discover that the ratio of  $^{14}\text{C}$  to  $^{12}\text{C}$  is  $1:12.5 \times 10^{12}$ . How old is the fossil?

**EXERCISE II:** You find a bone and measure the ratio of  $^{14}\text{C}$  to  $^{12}\text{C}$  and find it to be  $1:4 \times 10^{12}$ . How old is the fossil?

## LINKS TO ANSWERS

[EXERCISE I](#)

[EXERCISE II](#)

**TRY IT OUT: ANSWERS**

**EXERCISE I:** You find a fossil and discover that the ratio of  $^{14}\text{C}$  to  $^{12}\text{C}$  is  $1:12.5 \times 10^{12}$ . How old is the fossil?

**Step one:**  $\frac{1}{12.5 \times 10^{12}} \times 10^{12} = \frac{10^{12}}{12.5 \times 10^{12}} = 0.08$

**Step two:** 8.0% is left

**Step three:**  $\log_2(8.0) = 3.0$  (because  $2^3 = 8$ ).

**Step four:**  $x = 6.644 - 3.0 = 3.6$

**Step five:**  $3.6 \times 5715 = 2.1 \times 10^5$  years old.

Check: Does this look right? Well, 3.6 is between 3 and 4 half-lives. If 3 half-lives had passed, then there would be 12.5% left; if 4 had passed, there would be 6.25% left. 8% does indeed fall between 12.5% and 6.25%. So, yes, the answer looks right.

**TRY IT OUT: ANSWERS**

**EXERCISE II:** You find a bone and measure the ratio of  $^{14}\text{C}$  to  $^{12}\text{C}$  and find it to be  $1:4 \times 10^{12}$ . How old is the fossil?

**Step one:**  $\frac{1}{4 \times 10^{12}} \times 10^{12} = \frac{10^{12}}{4 \times 10^{13}} = 0.025$

**Step two:** 25% is left

**Step three:**  $\log_2(25) = 4.644$

**Step four:**  $x = 6.644 - 4.644 = 2.0$

**Step five:**  $2.0 \times 5715 = 1.1 \times 10^4$  years old

You might have noticed that 25% is found after 2 half lives. If you did, you could have jumped right from step 2 to step 5.

# *Math for Life*

## **REFERENCE TABLES**

### THE GREEK ALPHABET/SYMBOLS

Greek letter	Greek name	Key	(Pronunciation); Common usage
A α	alpha	A a	
B β	beta	B b	
Γ γ	gamma	G g	
Δ δ	delta	D d	symbol for “change in”
E ε	epsilon	E e	
Z ζ	zeta	Z z	(zate-ah)
H η	eta	H h	(ate-ah); symbol for viscosity
Θ θ	theta	Q q	(thate-ah)
I ι	iota	I i	
K κ	kappa	K k	
Λ λ	lambda	L l	microliters; symbol for wavelength
M μ	mu	M m	(myoo) micro-; symbol for population mean
N ν	nu	N n	symbol for frequency
Ξ ξ	xi	X x	(zi or k-sigh)
O ο	omicron	O o	
Π π	pi	P p	approximately 3.1416; the circumference of a circle divided by its diameter
P ρ	rho	R r	symbol for density
Σ σ	sigma	S s	symbol for population standard deviation
T τ	tau	T t	
Υ υ	upsilon	U u	
Φ φ	phi	F f	(fee)
X χ	chi	C c	(ki)
Ψ ψ	psi	Y y	(sigh)
Ω ω	omega	W w	symbol for ohms, for angular frequency

## NUMERICAL PREFIXES

### Prefixes for units

Prefix	Abbreviation	Multiplier
tera-	T	$10^{12}$
giga-	G	$10^9$
mega-	M	$10^6$
kilo-	k	$10^3$
hecto-	h	$10^2$
deca-	da	$10^1$
deci-	d	$10^{-1}$
centi-	c	$10^{-2}$
milli-	m	$10^{-3}$
micro-	$\mu$	$10^{-6}$
nano-	n	$10^{-9}$
pico-	p	$10^{-12}$
femto-	f	$10^{-15}$
atto-	a	$10^{-18}$
zepto-	z	$10^{-21}$

Example: a nanometer =  $10^{-9}$  meters

### Prefixes for nomenclature

Prefix	Multiplier
hemi-	1/2
mono-	1
di- bi- bis-	2
tri-	3
tetra-	4
penta-	5
hexa-	6
hepta-	7
octa-	8
nona-	9
deca-	10

Examples: carbon monoxide is CO; dinitrogen tetraoxide is  $N_2O_4$

## UNITS

### International System of Units (S. I. Units)

Property	Dimensions	S.I. Units	Abbreviation
Amount, Number			
Angle		radians	rad
Temperature		Kelvins	K
Length, Distance	Length (L)	meters	m
Time	Time (T)	seconds	s
Mass	Mass (M)	kilograms	kg
Electrical current	Amperes (A)	amperes	I
Area	$L^2$	square meters	$m^2$
Volume	$L^3$	cubic meters	$m^3$
Volume	$L^3$	liters	L
Molarity	$L^{-3}$	moles per liter	M
Frequency	$T^{-1}$	Hertz	Hz [ $s^{-1}$ ]
Radioactivity	$T^{-1}$	Becquerel	Bq
Rate, Speed, Velocity	$L T^{-1}$	meters per second	$m/s^{-1}$
Acceleration	$L T^{-2}$	meters per second squared	$m/s^{-2}$
Density	$M L^{-3}$	kilograms per cubic meter	$kg m^{-3}$
Concentration	$M L^{-3}$	kilograms per liter	$kg L^{-1}$
Force, Weight	$M L T^{-2}$	Newtons	N [ $kg m s^{-2}$ ]
Pressure	$M L^{-1} T^{-2}$	Pascals	Pa [ $N m^{-2}$ ]
Energy, Work	$M L^2 T^{-2}$	Joules	J [ $N m$ ]
Power	$M L^2 T^{-3}$	Watts	W [ $J s^{-1}$ ]
Intensity	$M L^4 T^{-3}$	Watts per square meter	I [ $W m^{-2}$ ]
Loudness		bels (or decibels)	$\beta$ (or dB)[ $10 \log(I/I_0)$ ]
Electrical charge	T A	Coulombs	C [A s]
Electrical potential	$M L^2 T^{-3} A^{-1}$	Volts	V
Electrical resistance	$M L^2 T^{-3} A^{-2}$	ohms	$\bullet$ , R
Electrical capacitance	$M^{-1} L^{-2} T^4 A^2$	Farads	F

Note: The differences in capitalization in the third column are not an error—some of them are people’s names and thus are capitalized.



**Other Common Units (Not S.I.)**

<b>Property</b>	<b>Unit</b>	<b>Abbreviation</b>	<b>To convert to S.I.</b>
Amount, Number	moles	mol	Number = mol $\times 6.02 \times 10^{23}$
Angle	degrees	°	rad = $2\pi \times (\text{degrees} \div 360)$
Temperature	degrees Celsius	°C	K = °C + 273.15
Molecular mass	Daltons	Da	kg = Da $\div 6.022 \times 10^{26}$
Area	hectare	ha	m <sup>2</sup> = ha $\div 10^4$
Volume	cubic centimeters	cc	L = cc $\div 10^3$
Force	dyne	dyn	N = dyn $\times 10^5$
Pressure	atmospheres	atm	Pa = atm $\times 1.0133 \times 10^5$
Energy, Work	calorie	cal	J = cal $\times 4.1868$
Energy, Work	erg	erg	J = erg $\times 10^7$

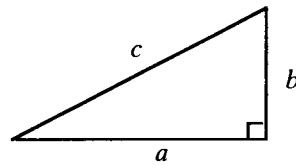
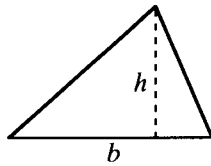
**CONSTANTS**

<b>Constant</b>	<b>Symbol</b>	<b>Value</b>
Angstrom Å		$10^{-10}$ m
Avogadro's number		$6.023 \times 10^{23}$
Base of natural log (ln)	$e$	2.71828
Boltzmann's constant	$k$	$1.3805 \times 10^{-23}$ J K <sup>-1</sup>
Complex numbers	$i$	$i = \sqrt{-1}; i^2 = -1$
Electronic charge	$e$	$1.602 \times 10^{-19}$ C
Faraday constant	F	$9.6487 \times 10^4$ C/mol of electrons
Gravitational acceleration	$g$	$9.8067$ m s <sup>-2</sup>
Mass of an electron	$m_e$	$9.1083 \times 10^{-31}$ kg
Mass of a proton	$m_p$	$1.67252 \times 10^{-27}$ kg
Mole mol		$6.023 \times 10^{23}$
Pi	$\pi$	3.1416
Planck's constant	$h$	$6.6256 \times 10^{-34}$ J s
Speed of light in a vacuum	$c$	$2.99795 \times 10^8$ m s <sup>-1</sup>
Standard temperature and pressure	STP	273 K and $1.01 \times 10^5$ Pa
Universal or molar gas constant	R	$8.314$ J mol <sup>-1</sup> K <sup>-1</sup>
Volume of 1 mole of ideal gas at	STP	22.4 L

### USEFUL FORMULAS

#### Geometry

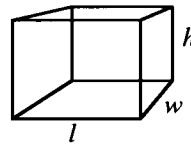
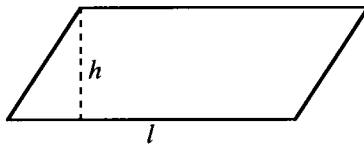
##### Triangle



Area of a triangle:  $\frac{1}{2}bh$

Sides of a right triangle:  $c^2 = a^2 + b^2$

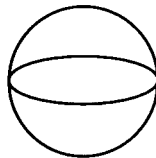
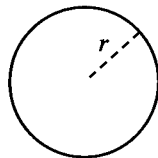
##### Parallelogram



Area of a parallelogram:  $l \times h$

Volume of a cube:  $l \times w \times h$

##### Circle

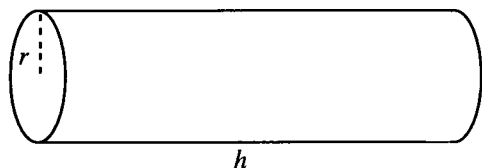


Circumference of a circle:  $2\pi r$

Area of a circle:  $\pi r^2$

Volume of a sphere:  $\frac{4}{3}\pi r^3$

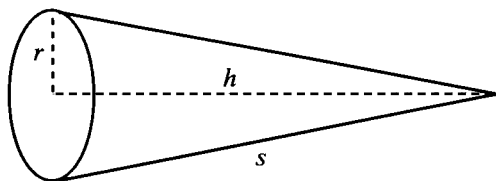
*Right Circular Cylinder*



Surface area of a cylinder:  $2\pi rh$

Volume of a cylinder:  $\pi r^2 h$

*Half Cone*



Surface of a cone:  $\pi rs$

Volume of a cone:  $\frac{\pi r^2 h}{3}$

**Algebra**

*Distance*

To find the distance ( $d$ ) between two points  $(x_1, y_1)$  and  $(x_2, y_2)$  :

Pythagorean theorem:  $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

*Polynomial Roots*

The roots of a polynomial of the form  $y = ax^2 + bx + c$  :

Quadratic equation:  $y = 0$ , where  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  (note:  $a \neq 0$ )

*Logarithms*

*Definition:*  $\log_k x = m$  such that  $k^m = x$ . (For common logarithms,  $k = 10$ .)

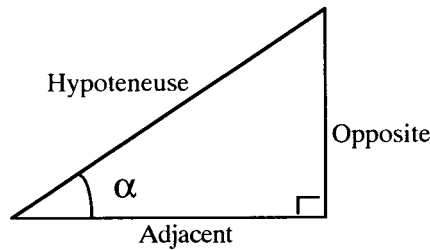
Important relationships:  $\log ab = \log a + \log b$   
 $\log a/b = \log a - \log b$   
 $\log a/b = -\log b/a$   
 $\log a^v = n \log a$

*Definition:*  $\ln x$  (the natural log of  $x$ ) =  $m$  such that  $e^m = x$ .

Important relationship:  $\ln e = 1$

**Trigonometry**

*Right Triangle Definitions*



For a right triangle:

<b>Function</b>	<b>Abbr.</b>	<b>Definition</b>
Sine $\alpha$	Sin $\alpha$	$\frac{\text{Opposite}}{\text{Hypoteneuse}}$
Cosine $\alpha$	Cos $\alpha$	$\frac{\text{Adjacent}}{\text{Hypoteneuse}}$
Tangent $\alpha$	Tan $\alpha$	$\frac{\text{Opposite}}{\text{Adjacent}}$ or, $\frac{\text{Sin}}{\text{Cos}}$
Cotangent $\alpha$	Ctn $\alpha$	$\frac{\text{Adjacent}}{\text{Opposite}}$ or, $\frac{\text{Cos}}{\text{Sin}}$
Secant $\alpha$	Sec $\alpha$	$\frac{\text{Hypoteneuse}}{\text{Adjacent}}$ or, $\frac{1}{\text{Cos}}$
Cosecant $\alpha$	Csc $\alpha$	$\frac{\text{Hypoteneuse}}{\text{Opposite}}$ or, $\frac{1}{\text{Sin}}$

A mnemonic device for remembering the first three lines of this table: **SOH CAH TOA**: Sine is Opposite over Hypoteneuse; Cosine is Adjacent over Hypoteneuse; Tangent is Opposite over Adjacent.

## *Trigonometric Relationships*

Some important trigonometric equations:

$$\begin{aligned}\sin^2\alpha + \cos^2\alpha &= 1 \\ \sin(-\alpha) &= -(\sin \alpha) \\ \cos(-\alpha) &= \cos \alpha\end{aligned}$$

$$\begin{aligned}\sin(180^\circ - \alpha) &= \sin \alpha \\ \cos(180^\circ - \alpha) &= -\cos \alpha\end{aligned}$$

$$\begin{aligned}\sin(90^\circ - \alpha) &= \cos \alpha \\ \cos(90^\circ - \alpha) &= \sin \alpha\end{aligned}$$

$$\begin{aligned}\sin(90^\circ + \alpha) &= \cos \alpha \\ \cos(90^\circ + \alpha) &= -\sin \alpha\end{aligned}$$

## **Physics**

### *Electricity*

The relationship between voltage ( $V$ ), current ( $I$ ), and resistance ( $R$ ):

Ohm's law:  $V = IR$

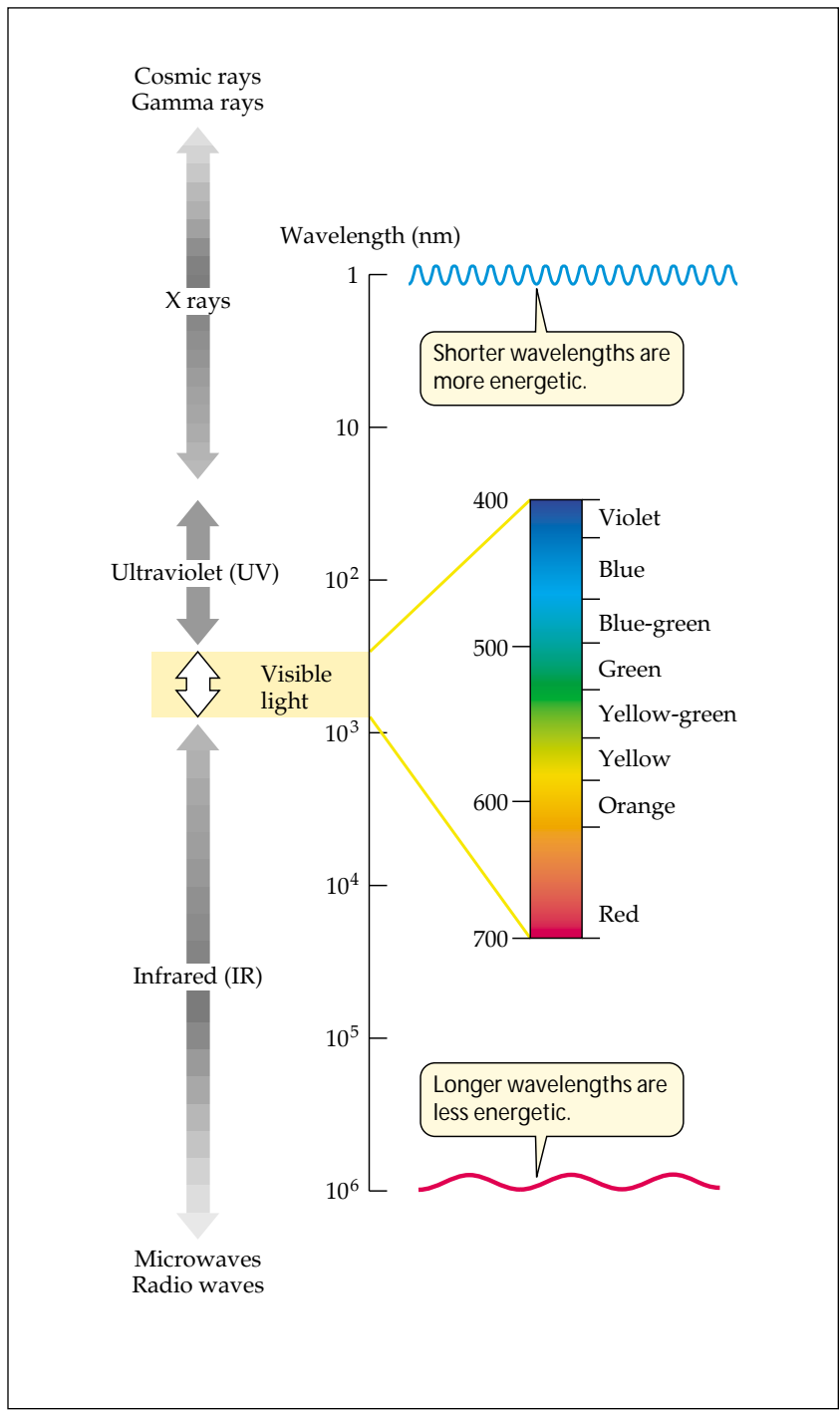
### *Pressure, Volume, Temperature*

The relationship between pressure ( $P$ ), volume ( $V$ ), temperature ( $T$ ), and number of moles of a gas ( $n$ ) where  $R$  = molar gas constant:

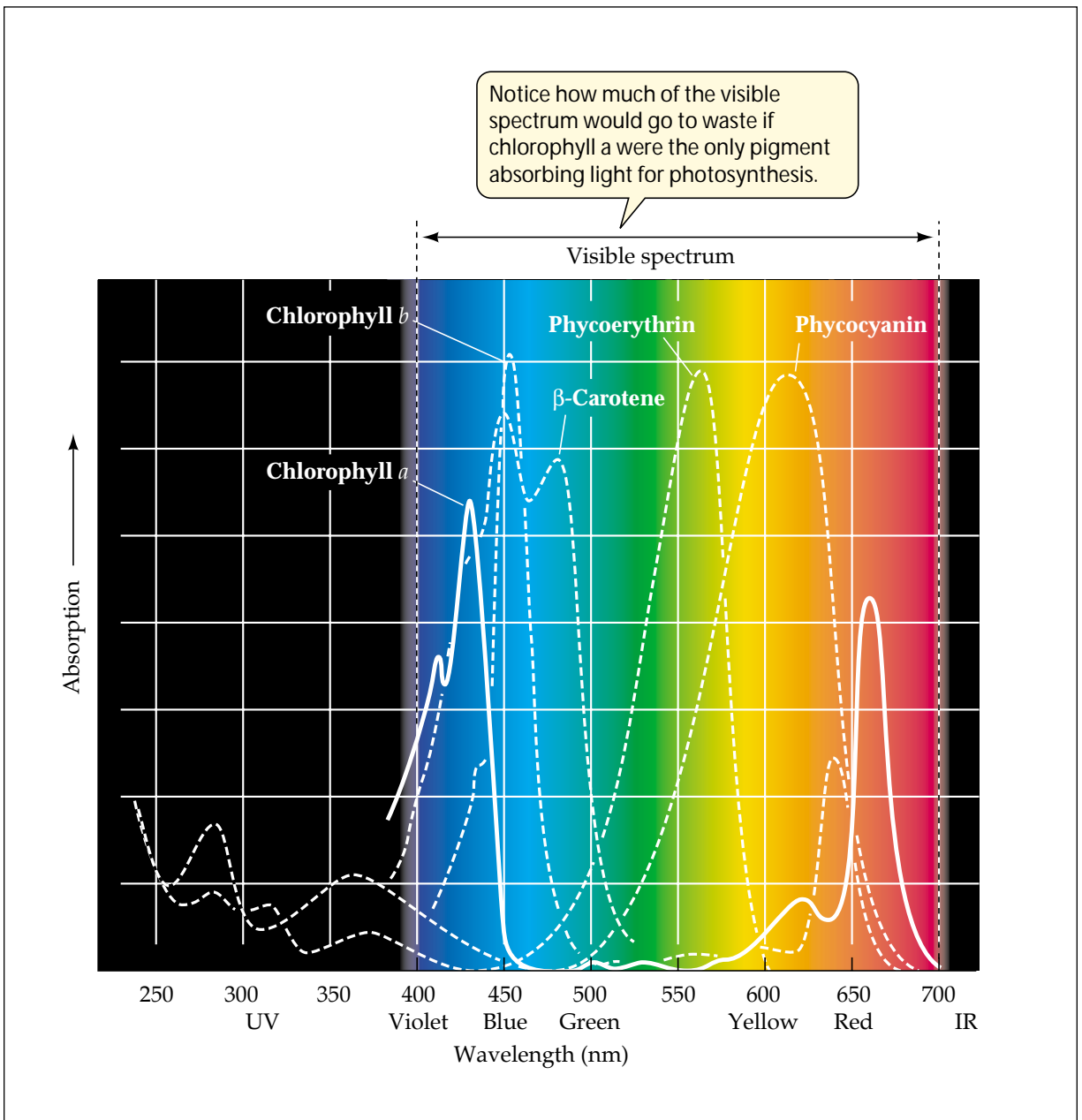
$$PV = nRT$$

## THE ELECTROMAGNETIC SPECTRUM

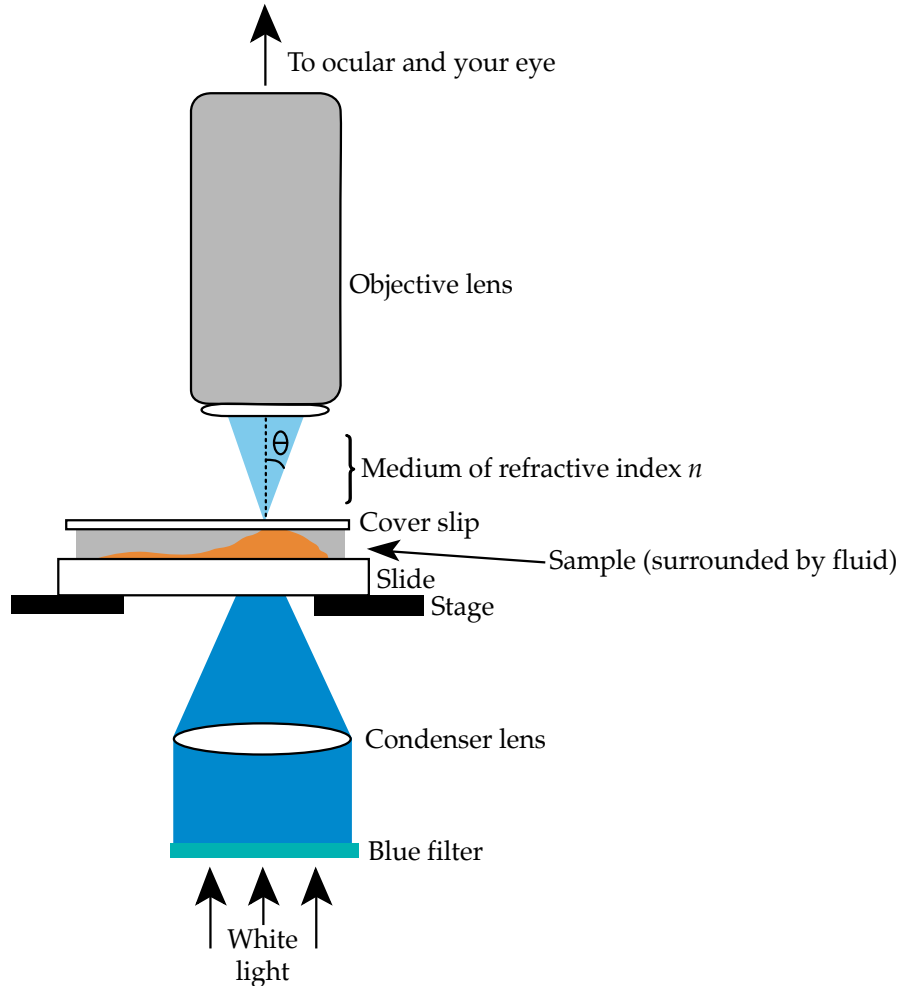
Type of radiation (lowest to highest energy)	Frequency range [Hz] $\nu = c/\lambda$	Wavelength range [m] $\lambda = c/\nu$	
<b>Radio waves</b>	$<10^{11}$	$>10^{-3}$	↓
AM	$0.55 \times 10^6 - 1.60 \times 10^6$	545 – 188	
FM	$88 \times 10^6 - 108 \times 10^6$	3.40 – 2.78	↓
TV	$54 \times 10^6 - 890 \times 10^6$	5.55 – 0.34	
Radar	$10^9 - 10^{11}$	$3 \times 10^{-1} - 3 \times 10^{-3}$	
<b>Infrared (heat)</b>	$10^{11} - 3.8 \times 10^{14}$	$3 \times 10^{-3} - 8 \times 10^{-7}$	Increasing Energy
<b>Visible light</b>	$3.8 \times 10^{14} - 7.5 \times 10^{14}$	$8 \times 10^{-7} - 4 \times 10^{-7}$	
Red	$3.95 \times 10^{14} - 4.76 \times 10^{14}$	$7.6 \times 10^{-7} - 6.3 \times 10^{-7}$	
Orange	$4.76 \times 10^{14} - 5.08 \times 10^{14}$	$6.3 \times 10^{-7} - 5.9 \times 10^{-7}$	
Yellow	$5.08 \times 10^{14} - 5.36 \times 10^{14}$	$5.9 \times 10^{-7} - 5.6 \times 10^{-7}$	
Green	$5.36 \times 10^{14} - 6.12 \times 10^{14}$	$5.6 \times 10^{-7} - 4.9 \times 10^{-7}$	
Blue	$6.12 \times 10^{14} - 6.67 \times 10^{14}$	$4.9 \times 10^{-7} - 4.5 \times 10^{-7}$	
Violet	$6.67 \times 10^{14} - 7.89 \times 10^{14}$	$4.5 \times 10^{-7} - 3.8 \times 10^{-7}$	↑
<b>Ultraviolet</b>	$7.5 \times 10^{14} - 3 \times 10^{17}$	$4 \times 10^{-7} - 10^{-9}$	↓
<b>X rays</b>	$3 \times 10^{17} - 3 \times 10^{19}$	$10^{-9} - 10^{-11}$	
<b>Gamma rays</b>	$>3 \times 10^{19}$	$<10^{-11}$	







## MICROSCOPY MATH



The resolution ( $R$ ) of a microscope lens is the smallest distance by which two objects can be separated and still be *seen* as two separate objects. So, the smaller the  $R$  the more you can see. To calculate  $R$ , you use Abbe's law:

$$\text{Abbe's law: } R = \frac{0.612 \lambda}{(\text{N.A.})} = \frac{0.612 \lambda}{(n \times \sin \theta)}$$

where  $\lambda$  is wavelength, N.A. is numerical aperture,  $n$  is the refractive index of the medium, and  $\theta$  is one-half the angle of the cone of light entering the objective lens. So, to improve resolution, you can make  $\lambda$  smaller or you can make N.A.,  $n$ , or  $\sin \theta$  larger.

**Improving  $R$**

$\lambda$  is determined by the color of the light entering the specimen. Filters can be used to allow in light of a particular  $\lambda$ ; frequently, microscopes come with blue filters. (Green is also a common filter because human eyes see green well.) Electron microscopes have better resolution than light microscopes because  $\lambda$  is so much smaller.

The numerical aperture of a lens is printed on the side of the lens itself. When you buy the lens, you choose the numerical aperture from among the available lenses.

$n$  is a characteristic of a medium, so to change  $n$ , you change media. Because the objective lens will be submerged in that medium, individual lenses are made to be used in one particular type of medium. Only three media are commonly used: air, water, and immersion oil. The lens will carry a written indication if it is made to work in water or oil.

Medium	$n$
Air	1.000
Water	1.330
Immersion oil	1.515

$\sin \theta$  is a function of the microscope.

In practice,  $R$  is limited:

Microscope type	Practical best $R$
Light microscope	0.2 $\mu\text{m}$
Electron microscope(EM)	0.2 nm (2 nm for biological specimens)

## THE LAWS OF THERMODYNAMICS

Laws of nature are useful because you know ahead of time that your system must follow them. They allow you to calculate the value of an unknown if you know the value(s) of other variables, and they allow you to predict how your system might behave. When you are interested in the energy of a system, you can consult the laws of thermodynamics.

### **First law of thermodynamics**

*Formal statements:*

- Energy is neither created nor destroyed.
- Energy can be converted from one form to another, but it cannot be created or destroyed.
- The total energy of the universe is a constant.
- The energy change in any system is equal to the heat absorbed from the surroundings plus the work done on the system.
- All the heat energy added to a closed system can be accounted for as mechanical work, an increase in internal energy, or both.

*Informal statement:*

- You can't win or lose, you can only break even.

*Equations:*

When using the first law as a guide to understanding the energy in a system, you do not actually deal with the total amount of energy in the universe (it's not possible). Instead, you deal with the change in energy ( $\Delta E$ ) in your system.

The following equations and their verbal meanings are all ways of understanding the first law of thermodynamics.

$\Delta E = E_2 - E_1$	The change in energy is equal to the energy at the end ( $E_2$ ) minus the energy at the beginning ( $E_1$ ).
$E_2 < E_1 \therefore \Delta E < 0$	$E_2$ is less than $E_1$ ; therefore, $\Delta E$ is a negative quantity. That means that the system has lost energy (usually in the form of heat) to its surroundings.
$-\Delta E_{\text{sys}} = \Delta E_{\text{sur}}$	The energy lost by the system, $-\Delta E_{\text{sys}}$ , equals the energy gained by the surroundings, $\Delta E_{\text{sur}}$ .

$$\Delta E = Q + W$$

This version of the equation describing the first law is useful if you know about heat and work. It refers specifically to the energy of the system so the subscript “sys” can be dropped. It says that the change in internal energy of the system is equal to the amount of heat ( $Q$ ) added to the system plus the amount of work ( $W$ ) done by the system. The units of  $E$ ,  $Q$ , and  $W$  are joules.

### **Second law of thermodynamics**

#### *Formal statements:*

- Not all energy can be used, and disorder tends to increase.
- When energy is converted from one form to another, some of the energy becomes unavailable to do work.
- The entropy of the universe increases. It can never decrease.
- Disorder tends to increase in the universe or in a closed system.
- The entropy of the universe never decreases. During any process, the entropy remains constant or it increases.
- It is impossible to have a process whose only result is the transfer of energy from a cold place to a warm place.

#### *Informal statement:*

- You can't win or break even, you can only lose.

#### *Equations:*

$$H = G + TS$$

The total energy of the system (the enthalpy,  $H$  [J]) comprises two parts: energy that can do work (the free energy  $G$  [J]), and energy that can not do work (the entropy,  $S$  [J K<sup>-1</sup>], multiplied by the absolute temperature,  $T$  [K]). Entropy is a measure of disorder in a system.

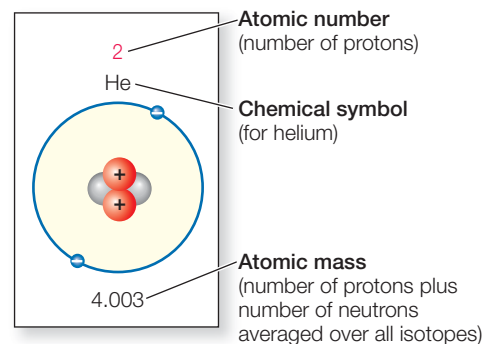
$$G = H - TS$$

This equation is just a rearrangement of the first equation. This form is useful when it is the free energy that you are interested in. This is usually the case in biology.

$\Delta G = \Delta H - T\Delta S$  As with the energy of the universe, the total energy of this system is not measurable in practice; however, you can measure the changes. So, the equation is rewritten in a form for which we can measure values. This equation says that the change in free energy is the change in total energy minus the change in entropy. If  $\Delta G$  is negative, it means that the reaction releases energy (the negative sign means energy is leaving the system) and the reaction will occur spontaneously; if  $\Delta G$  is positive, it means that energy has to be put in to the system to make the reaction go.

*Note:* There is a third law which states that the entropy of a perfect crystalline substance is zero at a temperature of absolute zero. This law has no practical application in biology.

## A BIOLOGIST'S PERIODIC TABLE



The six elements highlighted in yellow make up 98% of the mass of most living organisms.

Vertical columns contain elements with similar properties.

Elements highlighted in orange are present in small amounts in many organisms.

Masses in parentheses indicate unstable elements that decay rapidly to form other elements.

Elements without a chemical symbol are as yet unnamed.

1 H 1.0079																	2 He 4.003						
3 Li 6.941	4 Be 9.012																	5 B 10.81	6 C 12.011	7 N 14.007	8 O 15.999	9 F 18.998	10 Ne 20.179
11 Na 22.990	12 Mg 24.305																	13 Al 26.982	14 Si 28.086	15 P 30.974	16 S 32.06	17 Cl 35.453	18 Ar 39.948
19 K 39.098	20 Ca 40.08	21 Sc 44.956	22 Ti 47.88	23 V 50.942	24 Cr 51.996	25 Mn 54.938	26 Fe 55.847	27 Co 58.933	28 Ni 58.69	29 Cu 63.546	30 Zn 65.38	31 Ga 69.72	32 Ge 72.59	33 As 74.922	34 Se 78.96	35 Br 79.909	36 Kr 83.80						
37 Rb 85.4778	38 Sr 87.62	39 Y 88.906	40 Zr 91.22	41 Nb 92.906	42 Mo 95.94	43 Tc (99)	44 Ru 101.07	45 Rh 102.906	46 Pd 106.4	47 Ag 107.870	48 Cd 112.41	49 In 114.82	50 Sn 118.69	51 Sb 121.75	52 Te 127.60	53 I 126.904	54 Xe 131.30						
55 Cs 132.905	56 Ba 137.34	71 Lu 174.97	72 Hf 178.49	73 Ta 180.948	74 W 183.85	75 Re 186.207	76 Os 190.2	77 Ir 192.2	78 Pt 195.08	79 Au 196.967	80 Hg 200.59	81 Tl 204.37	82 Pb 207.19	83 Bi 208.980	84 Po (209)	85 At (210)	86 Rn (222)						
87 Fr (223)	88 Ra 226.025	103 Lr (260)	104 Rf (261)	105 Db (262)	106 Sg (266)	107 Bh (264)	108 Hs (269)	109 Mt (268)	110 (269)	111 (272)	112 (277)	113	114 (285)	115 (289)	116	117	118 (293)						
Lanthanide series		57 La 138.906	58 Ce 140.12	59 Pr 140.9077	60 Nd 144.24	61 Pm (145)	62 Sm 150.36	63 Eu 151.96	64 Gd 157.25	65 Tb 158.924	66 Dy 162.50	67 Ho 164.930	68 Er 167.26	69 Tm 168.934	70 Yb 173.04								
Actinide series		89 Ac 227.028	90 Th 232.038	91 Pa 231.0359	92 U 238.02	93 Np 237.0482	94 Pu (244)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (251)	99 Es (252)	100 Fm (257)	101 Md (258)	102 No (259)								

### Patterns in the periodic table:

**Periods (rows):** As you move across a period, the chemical properties of the elements change gradually.

**Groups (columns):** Elements in the same group have similar configurations of electrons in their outer shells and thus have similar chemical properties.

Selected groups:

1A: The alkali metals (does not include H); these are highly reactive.

2A: The alkaline earth metals; reactive, but not as highly as 1A.

1B: The coinage metals; nonreactive metals.

7A: The halogens; highly reactive, nonmetallic.

8A: The noble gases; very stable.

Series (or families):

A: The representative elements.

B: The transition elements.

La to Lu is the lanthanide or rare earth series.

Ac to Lr is the actinide series.



**BIOLOGICALLY IMPORTANT ELEMENTS**

<b>Element</b>	<b>Symbol</b>	<b>Atomic mass [Da]</b>	<b>Oxidation states</b>	<b>No. of covalent bonds</b>
Calcium	Ca	40.08	+2	2
Carbon	C	12.01115	+4, +2, -4	4
Chlorine	Cl	35.453	+7, +5, +3, +1, -1	1
Cobalt	Co	58.9332	+3, +2	2
Copper	Cu (cuprum)	63.54	+2, +1	1
Hydrogen	H	1.00797	+1, -1	1
Iron	Fe (ferrum)	55.847	+3, +2	2
Magnesium	Mg	24.312	+2	2
Manganese	Mn	54.9380	+7, +6, +4, +3, +2	2
Nitrogen	N	14.0067	+5, +4, +3, +2, +1, -3	5
Oxygen	O	15.9994	+2, -1, -2	2
Phosphorus	P	30.9738	+5, +3, -3	5
Potassium	K (kalium)	39.102	+1	1
Silicon	Si	28.086	+4, -4	4
Sodium	Na (natrium)	22.9898	+1	1
Sulfur	S	32.064	+6, +4, +2, -2	2
Zinc	Zn	65.37	+2	2

## MOLECULAR WEIGHT AND FORMULA WEIGHT

Molecular weight and formula weight tell you the mass of a mole of a particular chemical.

The units of molecular and formula weight are  $\text{g mol}^{-1}$  and, because these units are used throughout, they are not usually written.

What is the difference between molecular and formula weight?

Molecular weight, (M.W.; also called the molecular molar mass) refers to the mass of the molecular formula. It tells you both the number of atoms of each element and their relative proportions in the molecule.

Formula weight (F.W.; also called the empirical molar mass) refers to the mass of the empirical formula. It tells you only the relative proportions of each atom.

For example, glucose ( $\text{C}_6\text{H}_{12}\text{O}_6$ ) has a M.W. = 180.2 and a F.W. (for  $\text{CH}_2\text{O}$ ) = 30.0.

Ionic compounds, such as NaCl (table salt) don't really exist as individual molecules. It is their F.W. that is reported, because you can know only the relative proportions of each element.

Nowadays most people just say "M.W." Careful, chemical companies put "F.W." on their labels. In that case, the designation means the weight of the formula *as listed on the label*. So, for example, if it is a hydrated compound, the F.W. includes the mass of the water molecules.

**BUFFERS**

Desired pH range	Buffer	pKa at 25°C	pH range at 37°C
Low	Oxalic acid ( $pK_1$ )	1.27	
	$H_3PO_4$ ( $pK_1$ )	2.15	
	Citric acid ( $pK_1$ )	3.13	
	Oxalate <sup>-</sup> ( $pK_2$ )	4.27	
	Acetic acid	4.76	
Near neutral	$NaHCO_3$	6.35	5.4–6.9
	PIPES	6.76	6.1–7.5
	MOPS	7.15	6.5–7.9
	HEPES	7.47	6.8–8.2
	TRIZMA	8.08 7.0	–9.0
High Bicine		8.26	7.6–9.0
	$NH_4^+$	9.25	
	Glycine	9.78	
	$HCO_3^-$ ( $pK_2$ )	10.33	
	Piperidine	11.12	
	$HPO_4^{2-}$ ( $pK_3$ )	12.38	

## FUNCTIONAL GROUPS

Name	Formula	Comment
Aldehyde	RCHO	Functional group is carbonyl; R is an H, alkyl, or aryl group
Alkane	$C_nH_{2n+2}$	Aliphatic hydrocarbons
Alkene	$C_nH_{2n}$	Compounds with C=C functional groups
Alkyl	$C_nH_{2n+1}$	A group derived from an alkane minus one H
Alkyne	$C_nH_{2n-2}$	Compounds with C≡C functional group
Amino	—NH <sub>2</sub>	
Aryl		Any group containing one or more fused benzene rings, less one H
Benzene	C <sub>6</sub> H <sub>6</sub>	Cyclic, with delocalized electrons
Bromo	—Br	Halogen
Carbonyl	—C=O	Functional group of aldehydes and ketones
Carboxyl boxylic	—COOH	Acids containing carboxyls, called carboxylic acids
Chloro	—Cl	Halogen
Ester	—COOR	R is an H, alkyl, or aryl group
Ethanol	CH <sub>3</sub> —CH <sub>2</sub> —OH	Produced by fermentation
Ethyl	—CH <sub>2</sub> —CH <sub>3</sub>	Alkyl
Fluoro	—F	Halogen
Hydroxyl defines	—OH	When part of C-containing molecule, an alcohol
Iodo	—I	Halogen
Ketone	RCOR'	Functional group is carbonyl; R and R' are alkyl and/or aryl groups
Methanol	CH <sub>3</sub> OH	
Methyl	—CH <sub>3</sub>	Alkyl
<i>n</i> -Butyl	—CH <sub>2</sub> —CH <sub>2</sub> —CH <sub>2</sub> —CH <sub>3</sub>	Alkyl
<i>n</i> -Propyl	—CH <sub>2</sub> —CH <sub>2</sub> —CH <sub>3</sub>	Alkyl
Nitro	—NO <sub>2</sub>	
Phospho	—PO <sub>4</sub> <sup>2-</sup>	
Sulfhydryl	—SH	Can form disulfide bonds

Some Functional Groups Important to Living Systems

Functional group	Class of compounds	Structural formula	Example
Hydroxyl —OH or HO—	Alcohols		 Ethanol
Aldehyde —CHO	Aldehydes		 Acetaldehyde
Keto 	Ketones		 Acetone
Carboxyl —COOH	Carboxylic acids		 Acetic acid
Amino —NH <sub>2</sub>	Amines		 Methylamine
Phosphate —OPO <sub>3</sub> <sup>2-</sup>	Organic phosphates		 3-Phosphoglycerate
Sulfhydryl —SH	Thiols		 Mercaptoethanol

# AMINO ACIDS AND THEIR PROPERTIES

## Grouped according to side-chain properties

Amino acid	Side chain	Abbreviation	Genetic code	Comments
<i>Hydrophilic</i>				
Aspartic acid	Acidic	Asp D	GAU, GAC	
Glutamic acid	Acidic	Glu E	GAA, GAG	
Arginine	Basic	Arg R	CGU, CGC, CGA, CGG, AGA, AGG	
Histidine	Basic	His H	CAU, CAC	
Lysine	Basic	Lys K	AAA, AAG	
Asparagine	Polar, uncharged	Asn N	AAU, AAC	
Glutamine	Polar, uncharged	Gln Q	CAA, CAG	
Serine	Polar, uncharged	Ser S	UCU, UCC, UCA, UCG, AGU, AGC	
Threonine	Polar, uncharged	Thr T	ACU, ACC, ACA, ACG	
Tyrosine	Polar, uncharged	Tyr Y	UAU, UAC	
<i>Hydrophobic</i>				
Alanine	Nonpolar	Ala A	GCU, GCC, GCA, GCG	
Cysteine	Nonpolar	Cys C	UGU, UGC	Can form disulfide bonds
Glycine	Nonpolar	Gly G	GGU, GGC, GGA, GGG	Side chain is single H atom
Isoleucine	Nonpolar	Ile I	AUU, AUC, AUA	
Leucine	Nonpolar	Leu L	UUA, UUG, CUU, CUC, CUA, CUG	
Methionine	Nonpolar	Met M	AUG	Is also the START codon
Phenylalanine	Nonpolar	Phe F	UUU, UUC	
Proline	Nonpolar	Pro P	CCU, CCC, CCA, CCG	Contorted side chain distorts secondary protein structure
Tryptophan	Nonpolar	Trp W	UGG	
Valine	Nonpolar	Val V	GUU, GUC, GUA, GUG	

## Alphabetical by single-letter abbreviation

Amino acid	Side chain	Abbreviation		Genetic code	Comments
Alanine	Nonpolar	Ala	A	GCU, GCC, GCA, GCG	
Cysteine	Nonpolar	Cys	C	UGU, UGC	disulfide bonds
Aspartic acid	Acidic	Asp	D	GAU, GAC	
Glutamic acid	Acidic	Glu	E	GAA, GAG	
Phenylalanine	Nonpolar	Phe	F	UUU, UUC	
Glycine	Nonpolar	Gly	G	GGU, GGC, GGA, GGG	
Histidine	Basic	His	H	CAU, CAC	
Isoleucine	Nonpolar	Ile	I	AUU, AUC, AUA	
Lysine	Basic	Lys	K	AAA, AAG	
Leucine	Nonpolar	Leu	L	UUA, UUG, CUU, CUC, CUA, CUG	
Methionine	Nonpolar	Met	M	AUG	Met and START
Asparagine	Polar, uncharged	Asn	N	AAU, AAC	
Proline	Nonpolar	Pro	P	CCU, CCC, CCA, CCG	2° structure breaker
Glutamine	Polar, uncharged	Gln	Q	CAA, CAG	
Arginine	Basic	Arg	R	CGU, CGC, CGA, CGG, AGA, AGG	
Serine	Polar, uncharged	Ser	S	UCU, UCC, UCA, UCG, AGU, AGC	
Threonine	Polar, uncharged	Thr	T	ACU, ACC, ACA, ACG	
Valine	Nonpolar	Val	V	GUU, GUC, GUA, GUG	
Tryptophan	Nonpolar	Trp	W	UGG	
Tyrosine	Polar, uncharged	Tyr	Y	UAU, UAC	dityrosine bonds
				UAA, UAG, UGA	STOP

## **BIOLOGICAL MACROMOLECULES**

### **LIPIDS:**

- Fatty acids
- Triglycerides
- Phospholipids
- Steroids

### **CARBOHYDRATES:**

- Mono and Disaccharides
- Polysaccharides
- Derivative carbohydrates

### **PROTEINS:**

- Amino acids
- Proteins

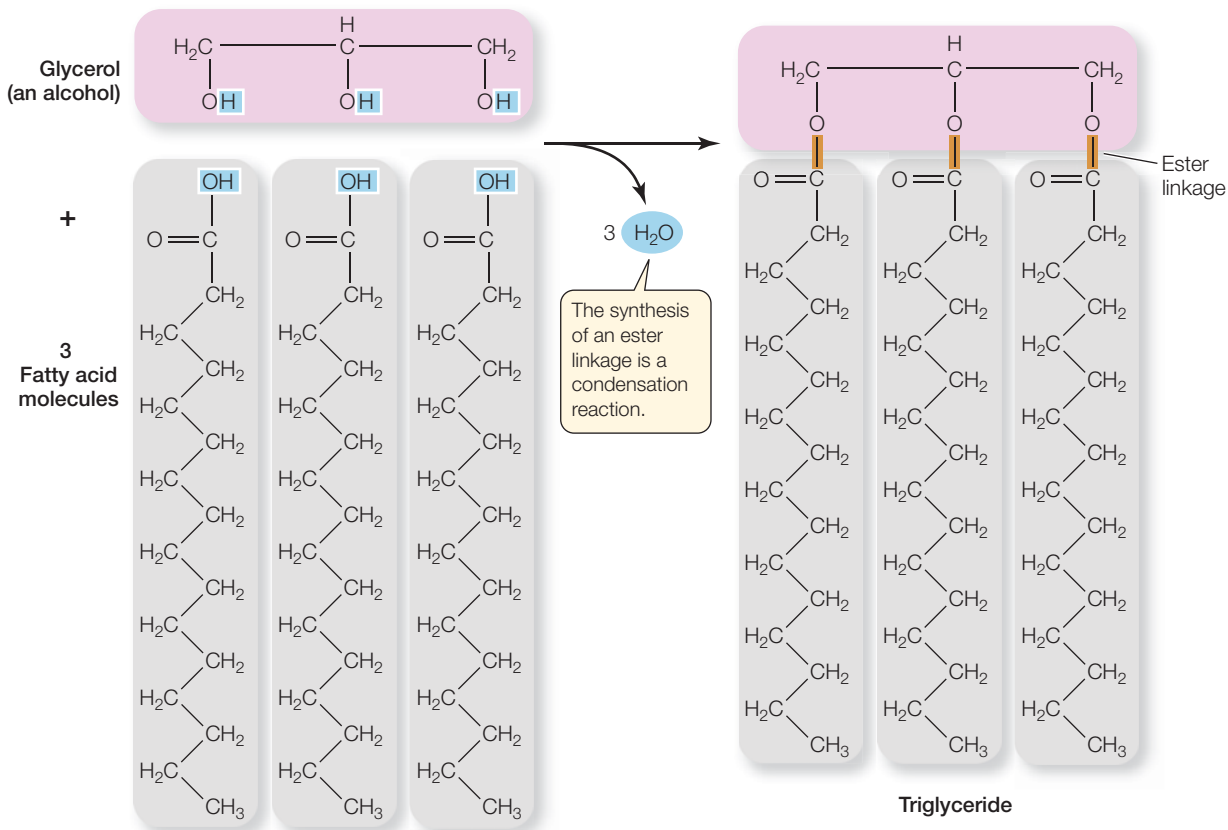
### **NUCLEIC ACIDS:**

- RNA subunits
- DNA subunits
- RNA and DNA strands





LIPIDS: Triglycerides

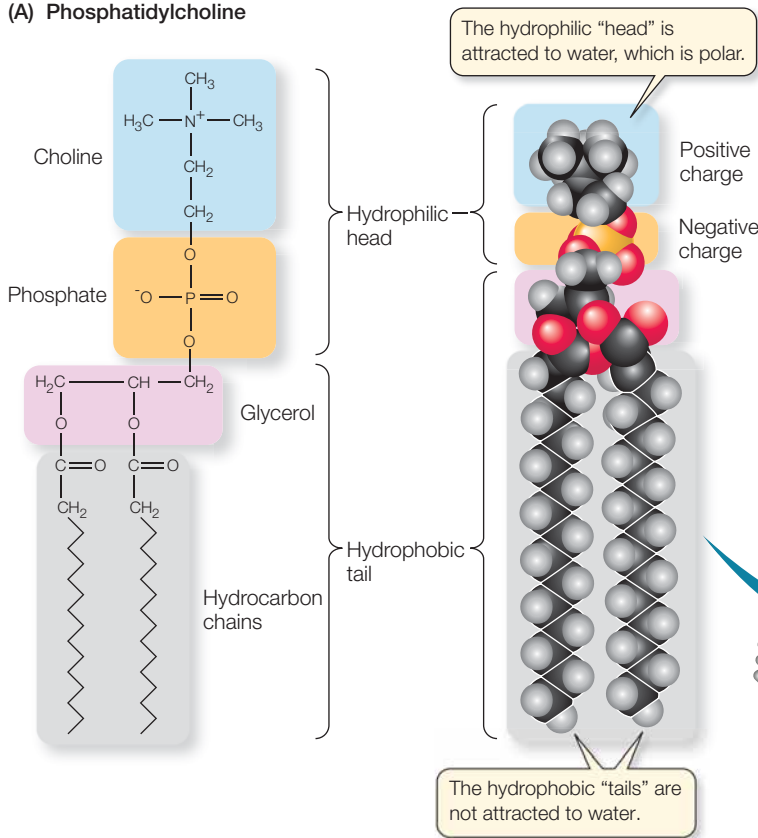


**Key Points** Three fatty acid tails plus a glycerol. Those that are solid at room temperature (because the fatty acids are largely saturated thus allowing the molecules to pack closely together) are fats; those that are liquid at room temperature (because unsaturated fatty acid tails prevent the molecules from packing closely together) are called oils. Both fats and oils store energy.

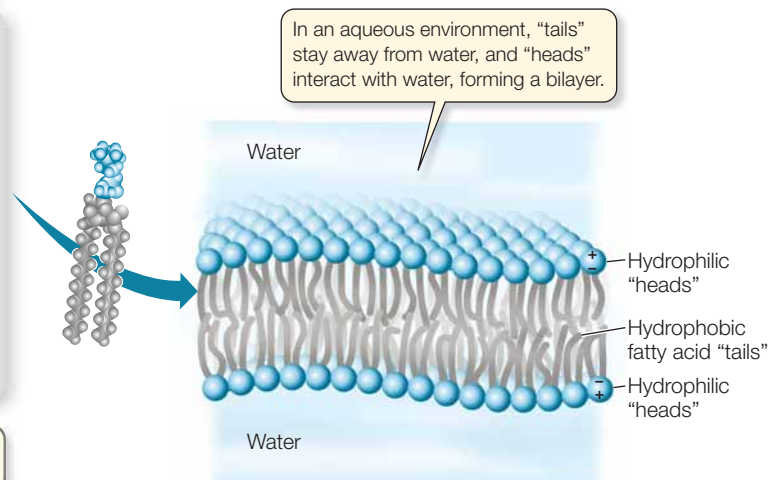
LIPIDS: Phospholipids

**Key Points** Two fatty acid tails (non-polar) attached to a glycerol that is attached to a phosphate containing compound (polar). Biological membranes are largely made up of phospholipid bilayers.

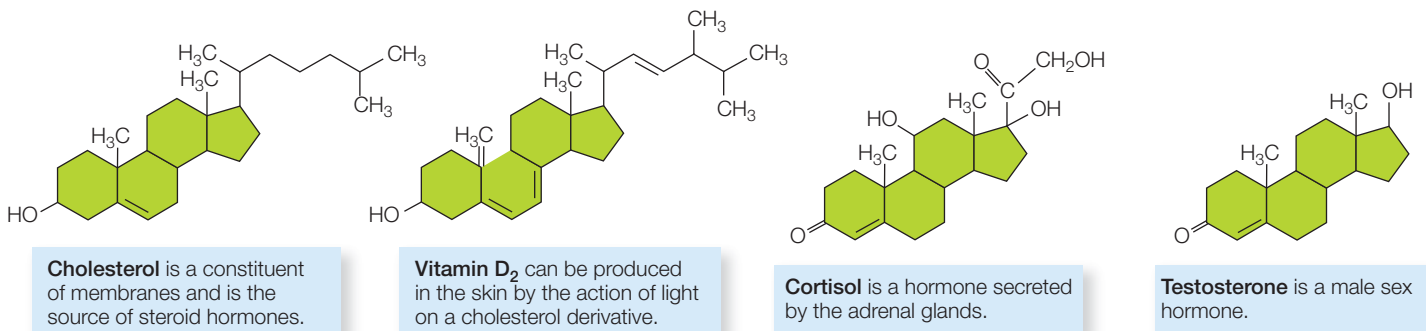
(A) Phosphatidylcholine



(B) Phospholipid bilayer

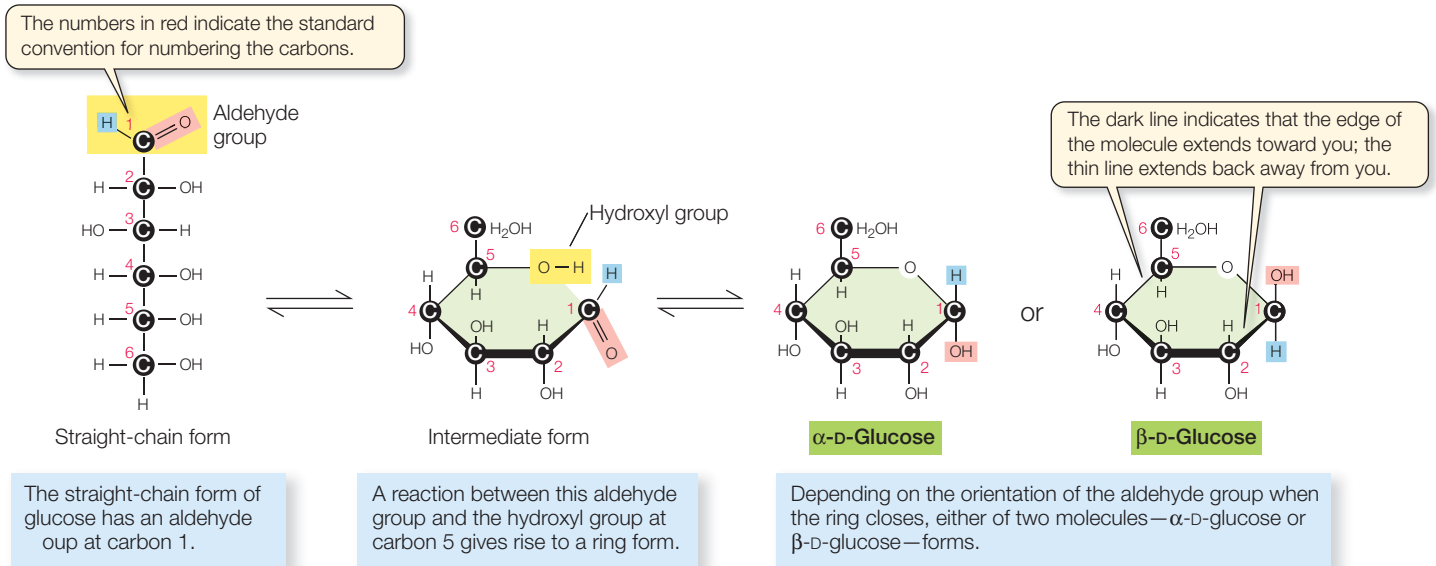


LIPIDS: Steroids

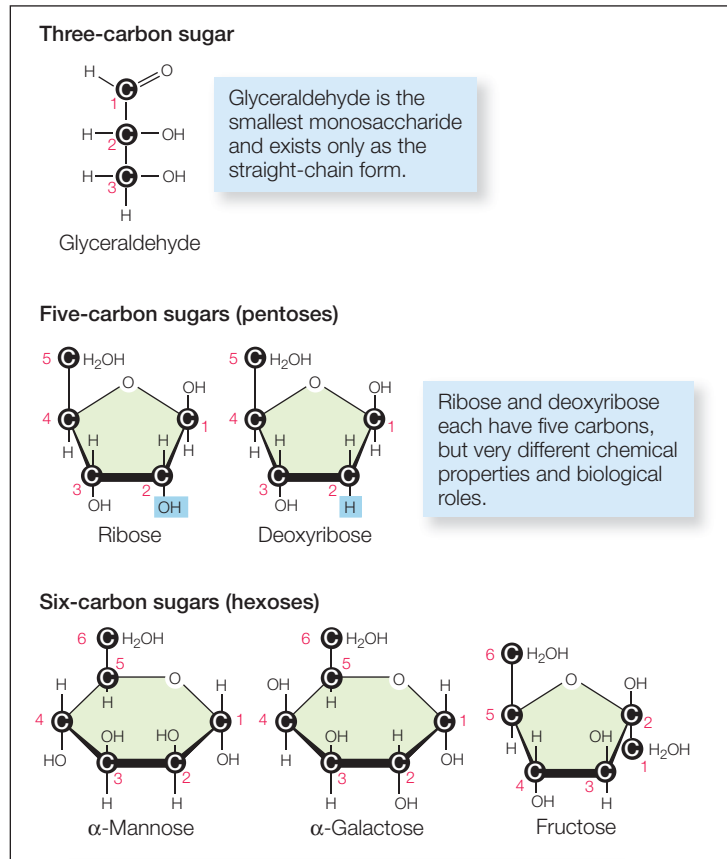


**Key Points** Compounds whose multiple rings share carbons. Steroids function in membranes (e.g. cholesterol) and as signaling molecules (e.g. hormones).

CARBOHYDRATES: Mono and Disaccharides (Sugars)

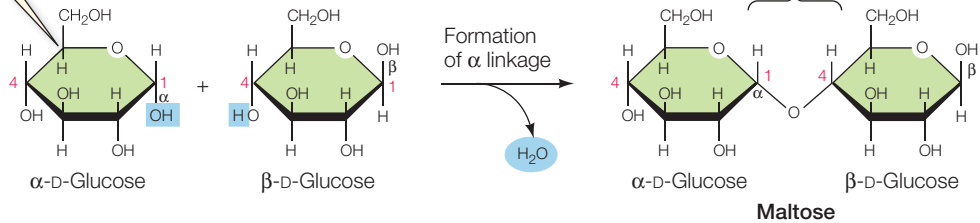


**Key Points** Energy source. Distinguished by the number of carbons. Trioses (3C) include glyceraldehyde, a component of **GAP**. Pentoses (5C) include compounds found in plant cell walls; ribose and deoxyribose, the sugars found in nucleotides (see below), are both pentoses. Hexoses (6C) all have the formula  $C_6H_{12}O_6$ , but the atoms are arranged differently. Glucose, a hexose, is in all living cells; its energy is released in the process of **glycolysis**. Disaccharides are made up of two monosaccharides joined by glycosidic linkages. Oligosaccharides are made up of a few monosaccharides joined by glycosidic linkages. Some oligosaccharides have additional functional groups attached; for example, some are attached to cell surface lipids that face the outside of the cell. These oligosaccharides act as cell recognition signals; the collection of all of those cell surface oligosaccharides is sometimes called the glycocalyx.

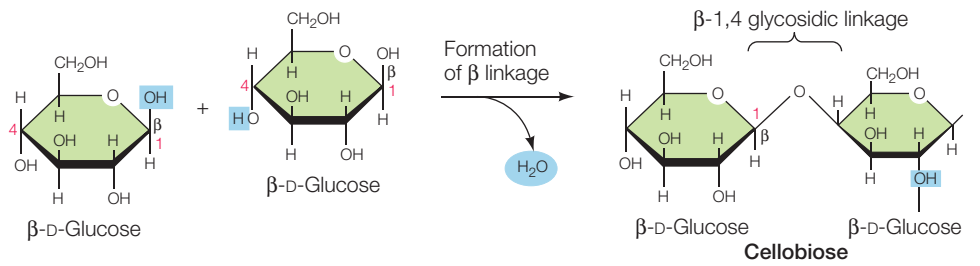


The presence of a carbon atom (C) at a junction such as this is implied.

Maltose is produced when an  $\alpha$ -1,4 glycosidic linkage forms between two glucose molecules. The hydroxyl group on carbon 1 of one D-glucose in the  $\alpha$  (down) position reacts with the hydroxyl group on carbon 4 of the other glucose.

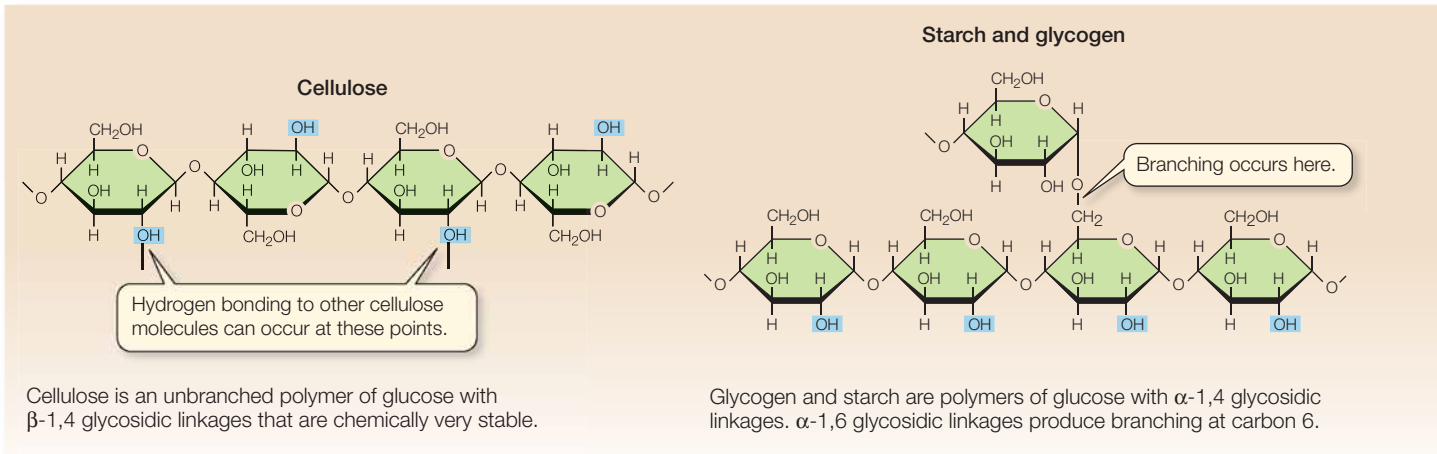


In cellobiose, two glucoses are linked by a  $\beta$ -1,4 glycosidic linkage.



CARBOHYDRATES: **Polysaccharides**

*Key Points* Polysaccharides are giant chains of monosaccharides joined by glycosidic linkages. Cellulose, the most common organic compound on the planet, is a polysaccharide. Starch and glycogen, both of which are built up from glucose, are energy storing compounds.

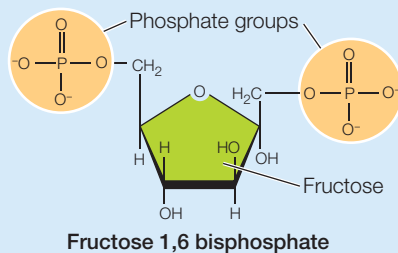


CARBOHYDRATES: **Derivative carbohydrates**

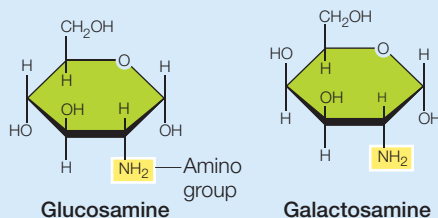
**Key Points** Compounds comprising a carbohydrate covalently linked to a functional group, such as a phosphate (e.g. sugar phosphate), an amino group (the amino sugars), or an *N*-acetyl group (e.g. chitin). Sugar phosphates can be intermediates in energy reactions, amino sugars can be structural.

**(A) Sugar phosphate**

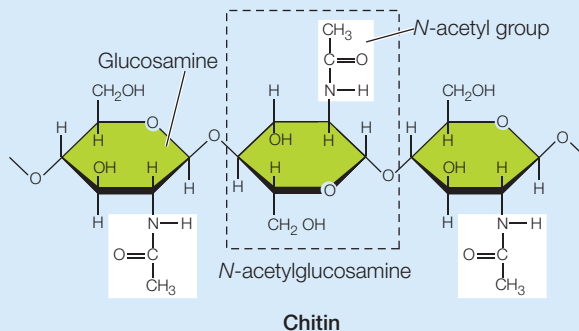
Fructose 1,6 bisphosphate is involved in the reactions that liberate energy from glucose. (The numbers in its name refer to the carbon sites of phosphate bonding; *bis*- indicates that two phosphates are present.)

**(B) Amino sugars**

The monosaccharides glucosamine and galactosamine are amino sugars with an amino group in place of a hydroxyl group.

**(C) Chitin**

Chitin is a polymer of *N*-acetylglucosamine; *N*-acetyl groups provide additional sites for hydrogen bonding between the polymers.



The Twenty Amino Acids

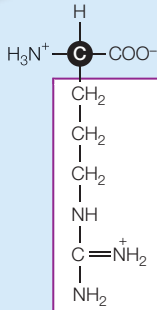
Amino acids with electrically charged hydrophilic side chains

Positive  $\oplus$

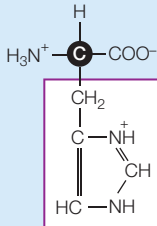
Negative  $\ominus$

Amino acids have both three-letter and single-letter abbreviations.

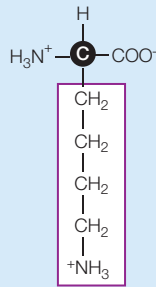
Arginine (Arg; R)



Histidine (His; H)

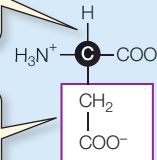


Lysine (Lys; K)

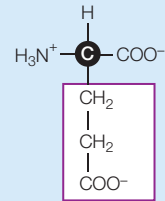


The general structure of all amino acids is the same...

Aspartic acid (Asp; D)



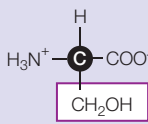
Glutamic acid (Glu; E)



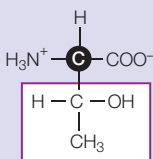
...but each has a different side chain.

Amino acids with polar but uncharged side chains (hydrophilic)

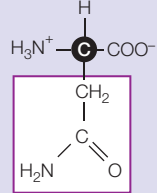
Serine (Ser; S)



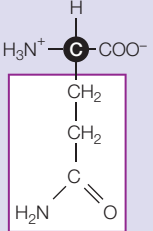
Threonine (Thr; T)



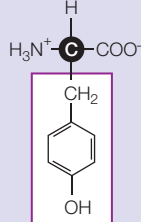
Asparagine (Asn; N)



Glutamine (Gln; Q)

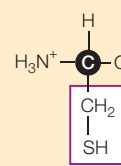


Tyrosine (Tyr; Y)

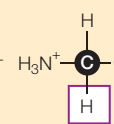


Special cases

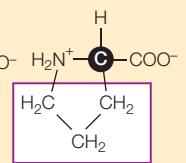
Cysteine (Cys; C)



Glycine (Gly; G)

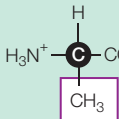


Proline (Pro; P)

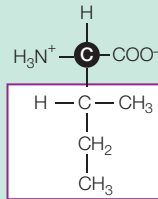


Amino acids with nonpolar hydrophobic side chains

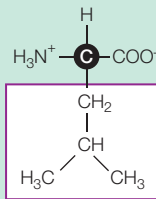
Alanine (Ala; A)



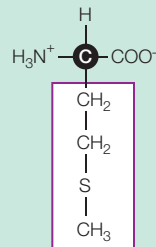
Isoleucine (Ile; I)



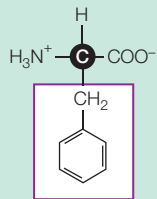
Leucine (Leu; L)



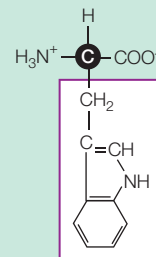
Methionine (Met; M)



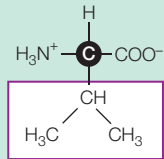
Phenylalanine (Phe; F)



Tryptophan (Trp; W)



Valine (Val; V)





PROTEINS: The Universal Genetic Code

*Key Points* Genetic information is encoded in mRNA in three-letter units—codons—made up of the bases uracil (U), cytosine (C), adenine (A), and guanine (G). To decode a codon, find its first letter in the left column, then read across the top to its second letter, then read down the right column to its third letter. The amino acid the codon specifies is given in the corresponding row. For example, AUG codes for methionine, and GUA codes for valine.

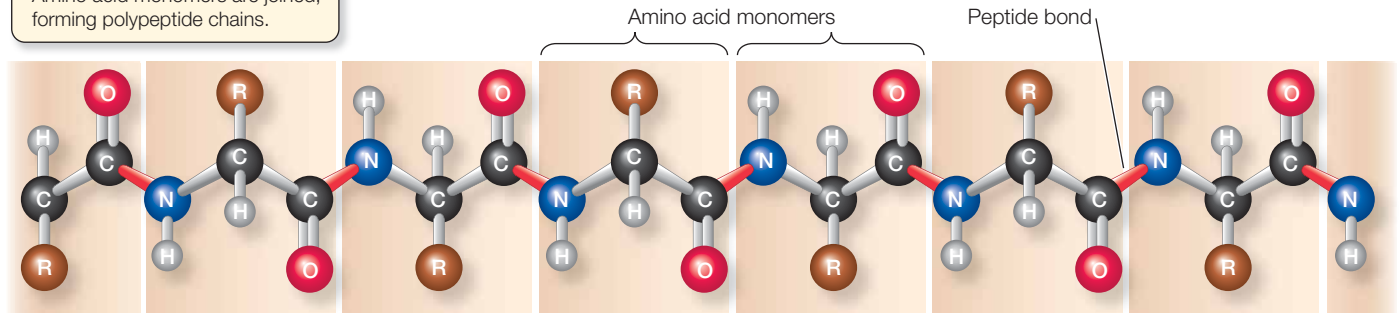
		Second letter				
		U	C	A	G	
First letter	U	UUU Phenylalanine UUC	UCU Serine UCC UCA UCG	UAU Tyrosine UAC <b>UAA</b> Stop codon <b>UAG</b> Stop codon	UGU Cysteine UGC <b>UGA</b> Stop codon UGG Tryptophan	U C A G
	C	CUU Leucine CUC CUA CUG	CCU Proline CCC CCA CCG	CAU Histidine CAC CAA Glutamine CAG	CGU Arginine CGC CGA CGG	U C A G
	A	AUU Isoleucine AUC AUA <b>AUG</b> Methionine; start codon	ACU Threonine ACC ACA ACG	AAU Asparagine AAC AAA Lysine AAG	AGU Serine AGC AGA Arginine AGG	U C A G
	G	GUU Valine GUC GUA GUG	GCU Alanine GCC GCA GCG	GAU Aspartic acid GAC GAA Glutamic acid GAG	GGU Glycine GGC GGA GGG	U C A G
						Third letter

PROTEINS: **Proteins**

**Key Points** One or more polypeptides, that is, polymers of amino acids linked together by peptide linkages. The order of amino acids of a protein is determined by the sequence of nucleotides in the gene encoding that polypeptide.

**Primary structure**

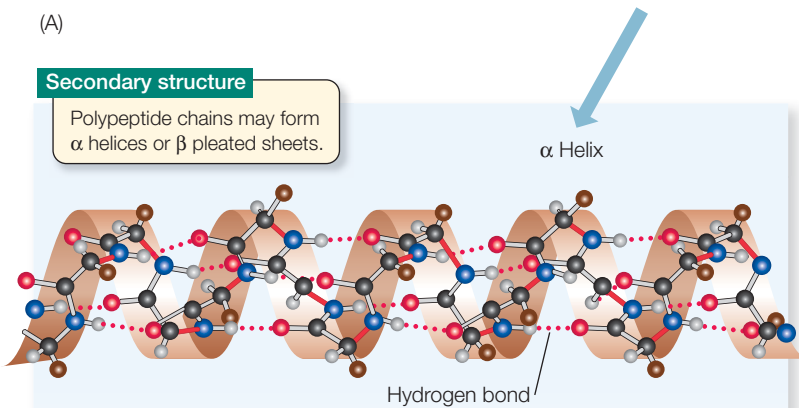
Amino acid monomers are joined, forming polypeptide chains.



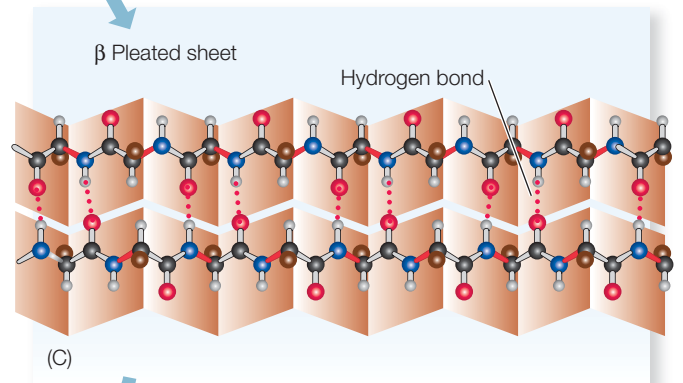
(A)

**Secondary structure**

Polypeptide chains may form  $\alpha$  helices or  $\beta$  pleated sheets.



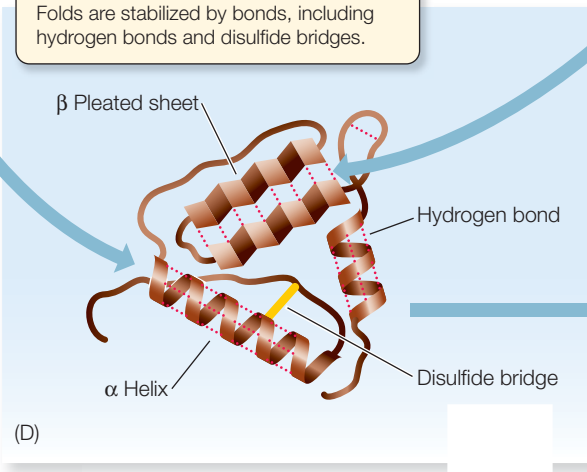
(B)



(C)

**Tertiary structure**

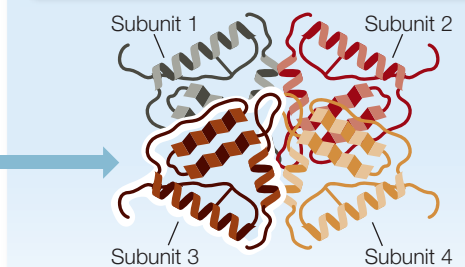
Polypeptides fold, forming specific shapes. Folds are stabilized by bonds, including hydrogen bonds and disulfide bridges.



(D)

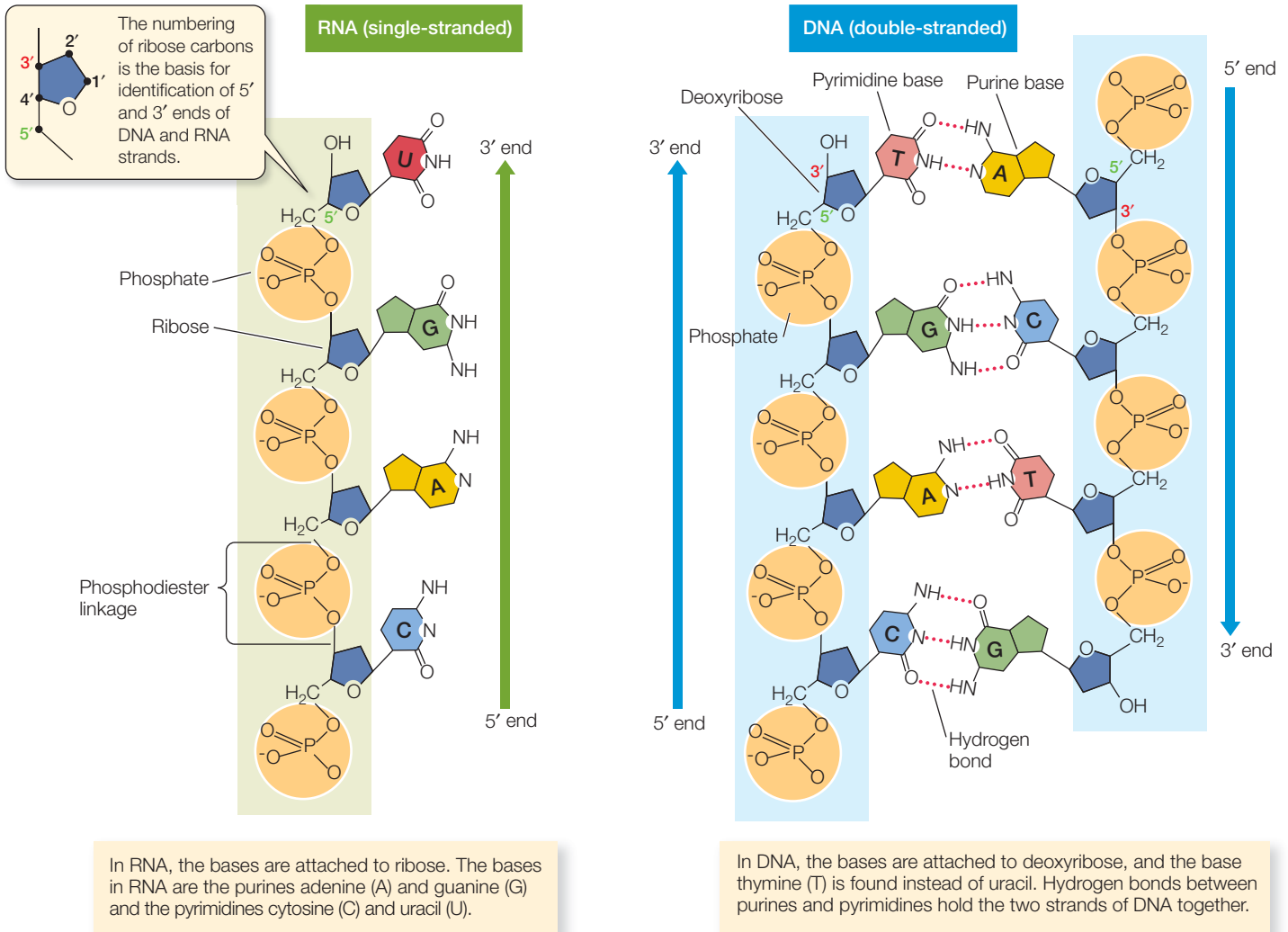
**Quaternary structure**

Two or more polypeptides assemble to form larger protein molecules. The hypothetical molecule here is a tetramer, made up of four polypeptide subunits.



(E)

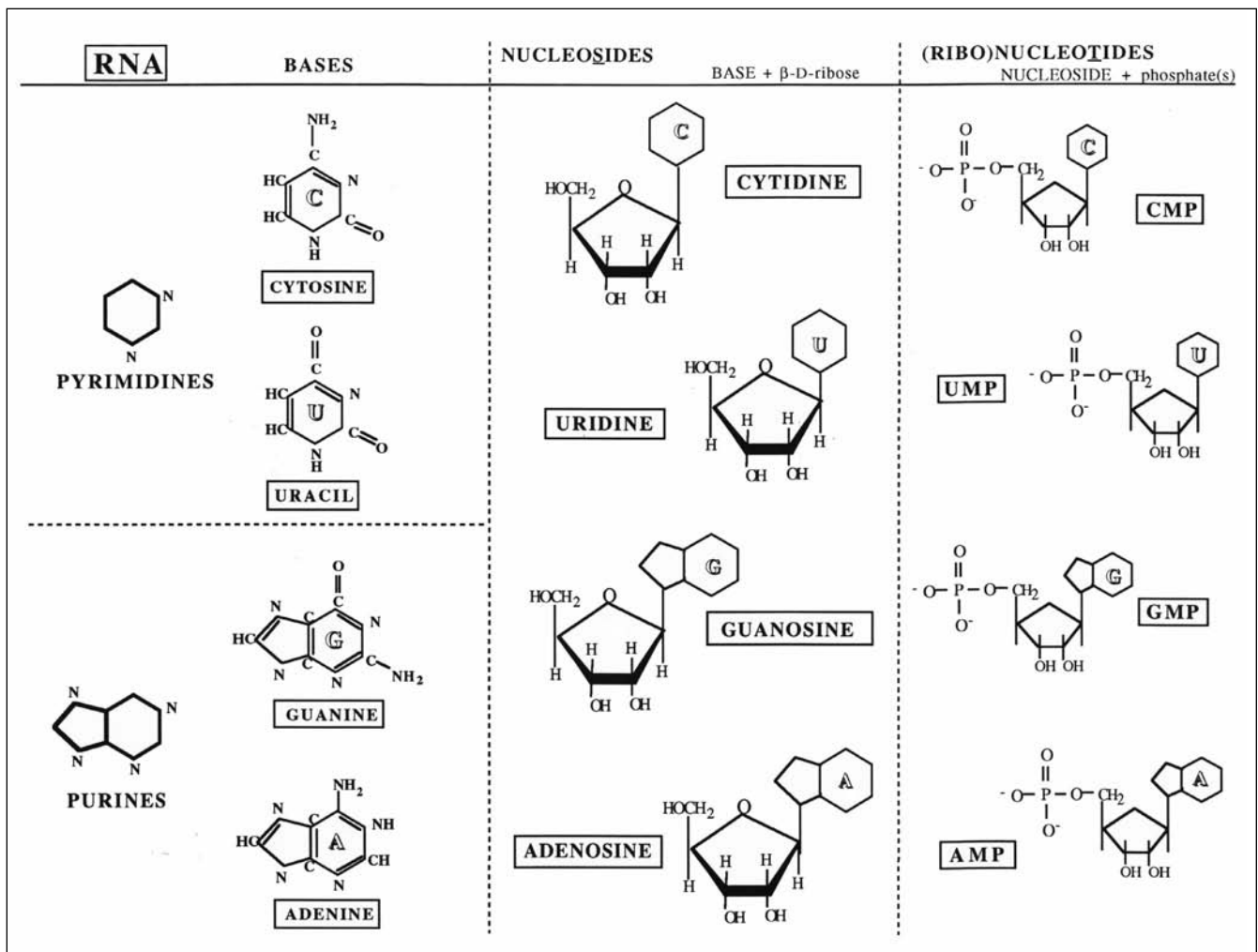
NUCLEIC ACIDS: RNA & DNA strands



**Key Points** DNA is the genetic material. That is, the genetic instructions for all of the proteins in the organism are encoded in its DNA. DNA is transcribed into RNA by proteins called polymerases; RNA can be transcribed into DNA by a protein called a reverse transcriptase. Messenger RNA (mRNA) is translated into protein with the help of ribosomes; ribosomal RNAs (rRNAs) are not translated, they are part of the ribosome; transfer RNAs (tRNAs) are not translated, they participate in translation by bringing amino acids to the ribosome.

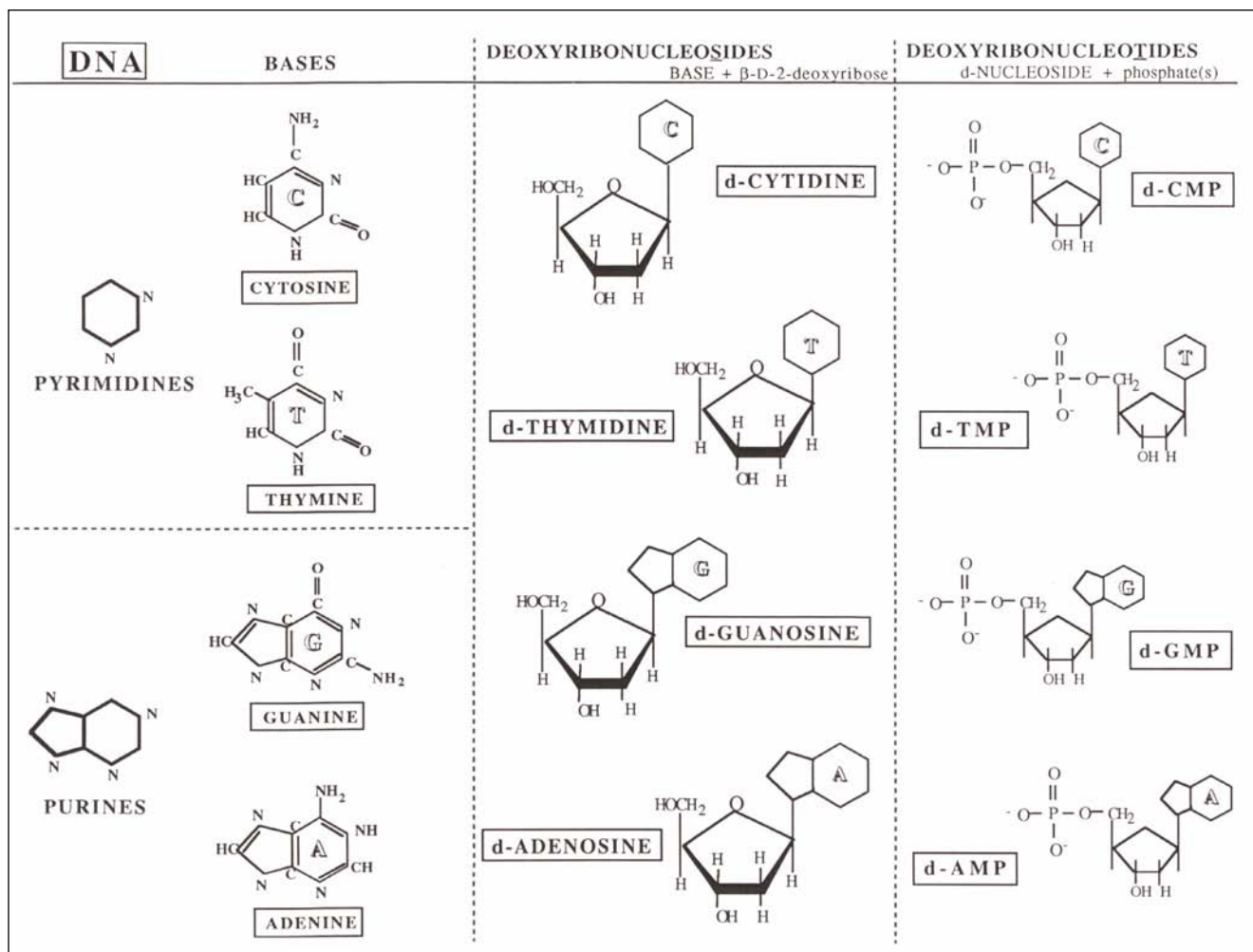
NUCLEIC ACIDS: RNA subunits

*Key Points* Ribonucleotides are made up of a base plus a ribose plus 1, 2, or 3 phosphates. The bases are either purines or pyrimidines. A base plus a ribose makes a nucleoside; a nucleoside plus 1, 2, or 3 phosphates makes a nucleotide. The ribonucleotides are the components of RNA. In addition to being one of the nucleotides found in RNA, the ribonucleotide **ATP** is very important for energy storage in cells. The ribonucleotide GTP is also an important source of energy for various cell processes.



NUCLEIC ACIDS: **DNA subunits**

*Key Points* Deoxyribonucleotides are made up of a base plus a deoxyribose plus 1, 2, or 3 phosphates. The bases are either purines or pyrimidines. A base plus a deoxyribose makes a nucleoside; a nucleoside plus 1, 2, or 3 phosphates makes a nucleotide. The deoxyribonucleotides are the components of DNA.



## BIOCHEMICAL REACTIONS

### **General reactions**

A reaction wherein an enzyme converts a substrate into a product:



E = enzyme

S = substrate

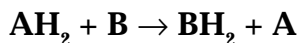
ES = enzyme substrate complex

P = product

Notes: 1. The enzyme is unchanged by the reaction.

2. The enzyme lowers the activation energy of the reaction; the enzyme does not affect  $\Delta G$  of the reaction.

Oxidation—reduction (redox) reactions:

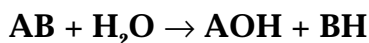


A = molecule being oxidized, (losing electrons or losing hydrogen atoms)

H<sub>2</sub> = hydrogen atoms (not ions).

B = molecule being reduced (gaining electrons or hydrogen atoms)

Hydrolysis:



AB = the molecule being broken apart by water

H<sub>2</sub>O = water

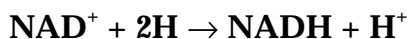
AOH = product of the reaction

BH = product of the reaction

Notes: 1. Hydrolysis is the breaking (lysis) of a compound by water (hydro).

### **Specific Reactions**

The reduction of NAD<sup>+</sup>:



NAD<sup>+</sup> = oxidized nicotinamide adenine dinucleotide

H = hydrogen atom

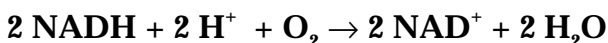
NADH = reduced nicotinamide adenine dinucleotide

H<sup>+</sup> = hydrogen ion (proton)

Notes: 1. NAD<sup>+</sup> is an intracellular carrier of reducing agents (electrons).

2. The reduction of NAD<sup>+</sup> is a component of glycolysis, the tricarboxylic acid cycle, oxidative phosphorylation, fermentation, and other cellular processes.

The oxidation of NADH:



NADH = reduced nicotinamide adenine dinucleotide

H<sup>+</sup> = hydrogen ion (proton)

O<sub>2</sub> = oxygen

NAD<sup>+</sup> = oxidized nicotinamide adenine dinucleotide

H<sub>2</sub>O = water

ΔG = -219 kJ mol<sup>-1</sup>

The conversion of ATP to ADP, inorganic phosphate, and free energy:



ATP = adenosine triphosphate

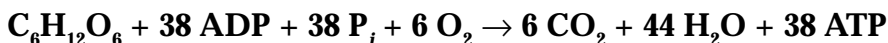
H<sub>2</sub>O = water

ADP = adenosine diphosphate

P<sub>i</sub> = HPO<sub>4</sub><sup>2-</sup>, inorganic phosphate

ΔG = -50 kJ mol<sup>-1</sup>

The aerobic metabolism of glucose:



C<sub>6</sub>H<sub>12</sub>O<sub>6</sub> = glucose

ADP = adenosine diphosphate

P<sub>i</sub> = HPO<sub>4</sub><sup>2-</sup>, inorganic phosphate

O<sub>2</sub> = oxygen

CO<sub>2</sub> = carbon dioxide

H<sub>2</sub>O = water

ATP = adenosine triphosphate

$$\Delta G = -2870 \text{ kJ mol}^{-1}$$

- Notes: 1. ATP packages free energy in bundles of about 30.5 kJ per mole; therefore, it is thought that about  $1.2 \times 10^3$  kJ, or 40% of the  $\Delta G$  is captured in ATP; this may be an underestimate.
2. The aerobic metabolism of glucose comprises three separate sets of reactions: glycolysis {link "glycolysis" to glycolysis below}, which takes place in the cytoplasm; the citric acid cycle (link "citric acid cycle" to same below), which takes place in the mitochondrion; oxidative phosphorylation, {link "oxidative phosphorylation" to same below} which takes place in the mitochondrion.
3. In the absence of  $O_2$ , glycolysis can still occur by the process of fermentation. {link "fermentation" to both lactate and alcoholic fermentation below}
4. The metabolism of glucose is a redox reaction. {link "redox" to redox under general reactions above}.
5. Because the creation of ATP from  $ADP + P_i$  releases one molecule of  $H_2O$ , the net production of  $H_2O$  is actually 44  $H_2O$ , not 6 as you would predict from balancing the equation.
6. This equation describes the theoretical maximal production of ATP from the aerobic metabolism of glucose. To achieve this yield, *all* of the energy of the proton gradient across the inner mitochondrial membrane would have to go toward synthesizing ATP; in a real cell, some of that energy goes elsewhere.
7. In some cells (e.g., liver, kidney, heart), the theoretical maximum is 38 moles of ATP per mole of glucose, in other cell types (e.g., skeletal muscle, brain), the theoretical maximum is 36 moles; this is because in some cells, certain electrons bypass the first energy-conserving site of the electron transport system.

Glycolysis (metabolism of glucose part I):



$C_6H_{12}O_6$  = glucose

$NAD^+$  = oxidized nicotinamide adenine dinucleotide

ADP = adenosine diphosphate

$P_i = HPO_4^{2-}$ , inorganic phosphate

Pyruvate = 3-carbon sugar (triose)

NADH = reduced nicotinamide adenine dinucleotide



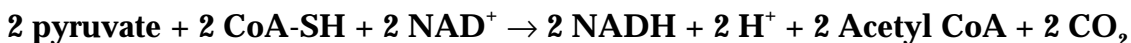
$H^+$  = hydrogen ion (proton)

ATP = adenosine triphosphate

$H_2O$  = water

$\Delta G = -84 \text{ kJ mol}^{-1}$

Conversion of pyruvate to Acetyl CoA (oxidative decarboxylation):



Pyruvate = 3-carbon sugar (triose)

CoA-SH = coenzyme A-SH; Acyl group carrying coenzyme with sulfhydryl

$\text{NAD}^+$  = oxidized nicotinamide adenine dinucleotide

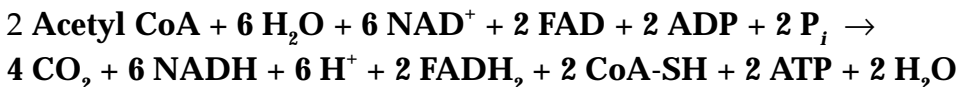
NADH = reduced nicotinamide adenine dinucleotide

$H^+$  = hydrogen ion (proton)

Acetyl CoA = 2-carbon acetate group linked to coenzyme A

$\text{CO}_2$  = carbon dioxide

Citric acid cycle (tricarboxylic acid or TCA cycle; Krebs cycle; metabolism of glucose, part II):



Acetyl CoA = 2-carbon acetate group linked to coenzyme A

$H_2O$  = water

$\text{NAD}^+$  = oxidized nicotinamide adenine dinucleotide

FAD = oxidized flavin adenine dinucleotide

ADP = adenosine diphosphate

$\text{P}_i = \text{HPO}_4^{2-}$ , inorganic phosphate

$\text{CO}_2$  = carbon dioxide

NADH = reduced nicotinamide adenine dinucleotide

$H^+$  = hydrogen ion (proton)

$\text{FADH}_2$  = reduced flavin adenine dinucleotide

CoA-SH = acyl group carrying coenzyme with sulfhydryl group

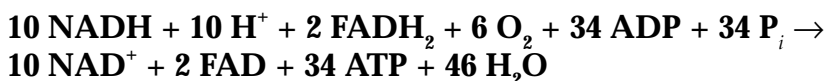
ATP = adenosine triphosphate

$H_2O$  = water

Notes: Like  $\text{NAD}^+$ , FAD is an intracellular carrier of reducing agents (electrons). It

can accept two electrons and two protons, so, the reduced form is  $\text{FADH}_2$ .

Oxidative phosphorylation (metabolism of glucose, part III):



10 NADH = reduced nicotinamide adenine dinucleotide; 2 from glycolysis, 2 from the oxidative decarboxylation of pyruvate, 6 from the citric acid cycle.

$\text{H}^+$  = hydrogen ion (proton)

2  $\text{FADH}_2$  = reduced flavin adenine dinucleotide; from citric acid cycle

$\text{O}_2$  = oxygen

ADP = adenosine diphosphate

$\text{P}_i$  =  $\text{HPO}_4^{2-}$ , inorganic phosphate

$\text{NAD}^+$  = oxidized nicotinamide adenine dinucleotide

FAD = oxidized flavin adenine dinucleotide

ATP = adenosine triphosphate

$\text{H}_2\text{O}$  = water

Note: Because the creation of ATP from  $\text{ADP} + \text{P}_i$  releases one molecule of  $\text{H}_2\text{O}$ , the net production of  $\text{H}_2\text{O}$  is 46  $\text{H}_2\text{O}$ , not 12  $\text{H}_2\text{O}$  as you would predict from balancing the equation.

Lactate Fermentation (homolactic fermentation and reaction 11 of glycolysis):



$\text{C}_6\text{H}_{12}\text{O}_6$  = glucose

ADP = adenosine diphosphate

$\text{P}_i$  =  $\text{HPO}_4^{2-}$ , inorganic phosphate

$\text{NAD}^+$  = oxidized nicotinamide adenine dinucleotide

NADH = reduced nicotinamide adenine dinucleotide

$\text{H}^+$  = hydrogen ion (proton)

ATP = adenosine triphosphate

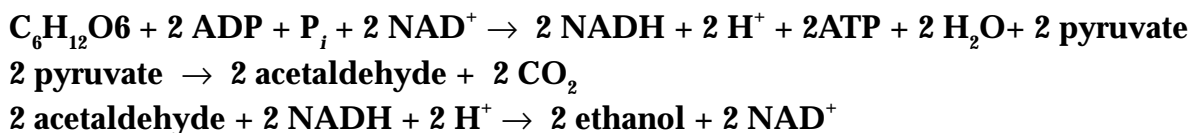
$\text{H}_2\text{O}$  = water

Pyruvate = 3-carbon sugar (triose);  $\text{CH}_3\text{COCOO}^-$

Lactate = lactic acid;  $\text{CH}_3\text{CHOHCOO}^-$

$\Delta G = -196 \text{ kJ mol}^{-1}$  of glucose

Alcoholic fermentation:



$\text{C}_6\text{H}_{12}\text{O}_6$  = glucose

ADP = adenosine diphosphate

$\text{P}_i$  =  $\text{HPO}_4^{2-}$ , inorganic phosphate

$\text{NAD}^+$  = oxidized nicotinamide adenine dinucleotide

NADH = reduced nicotinamide adenine dinucleotide

$\text{H}^+$  = hydrogen ion (proton)

ATP = adenosine triphosphate

$\text{H}_2\text{O}$  = water

Pyruvate = 3 carbon sugar (triose);  $\text{CH}_3\text{COCOO}^-$

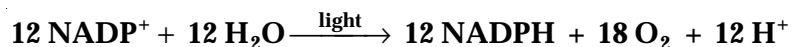
Acetaldehyde =  $\text{CH}_3\text{CHO}$

$\text{CO}_2$  = carbon dioxide

Ethanol =  $\text{CH}_3\text{CH}_2\text{OH}$

$\Delta G = -235 \text{ kJ mol}^{-1}$  of glucose

Photosynthesis:



$\text{CO}_2$  = Carbon dioxide

$\text{H}_2\text{O}$  = water

$\text{C}_6\text{H}_{12}\text{O}_6$  = carbohydrate

$\text{O}_2$  = oxygen

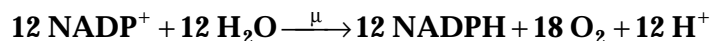
$\Delta G = 2867 \text{ kJ mol}^{-1}$

Notes: 1. Photosynthesis takes place in chloroplasts.

2. Photosynthesis comprises two separate sets of reactions: photophosphorylation (the light reactions) and the Calvin-Benson cycle (the dark reactions).

3. The dark reactions do not take place in the dark.

Photosynthesis, light reactions (photosynthetic electron-transfer reactions):



NADP<sup>+</sup> = oxidized nicotinamide adenine dinucleotide phosphate

H<sub>2</sub>O = water

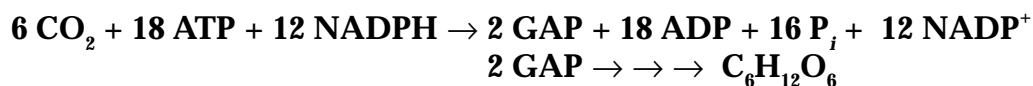
μ = light

NADPH = reduced nicotinamide adenine dinucleotide phosphate

O<sub>2</sub> = oxygen

H<sup>+</sup> = hydrogen ion (proton)

Photosynthesis, dark reactions (carbon-fixation reactions or Calvin–Benson cycle):



CO<sub>2</sub> = carbon dioxide

ATP = adenosine triphosphate

NADPH = reduced nicotinamide adenine dinucleotide phosphate

GAP = glyceraldehyde 3-phosphate, CHOCH(OH)CH<sub>2</sub>OPO<sub>3</sub>

ADP = adenosine diphosphate

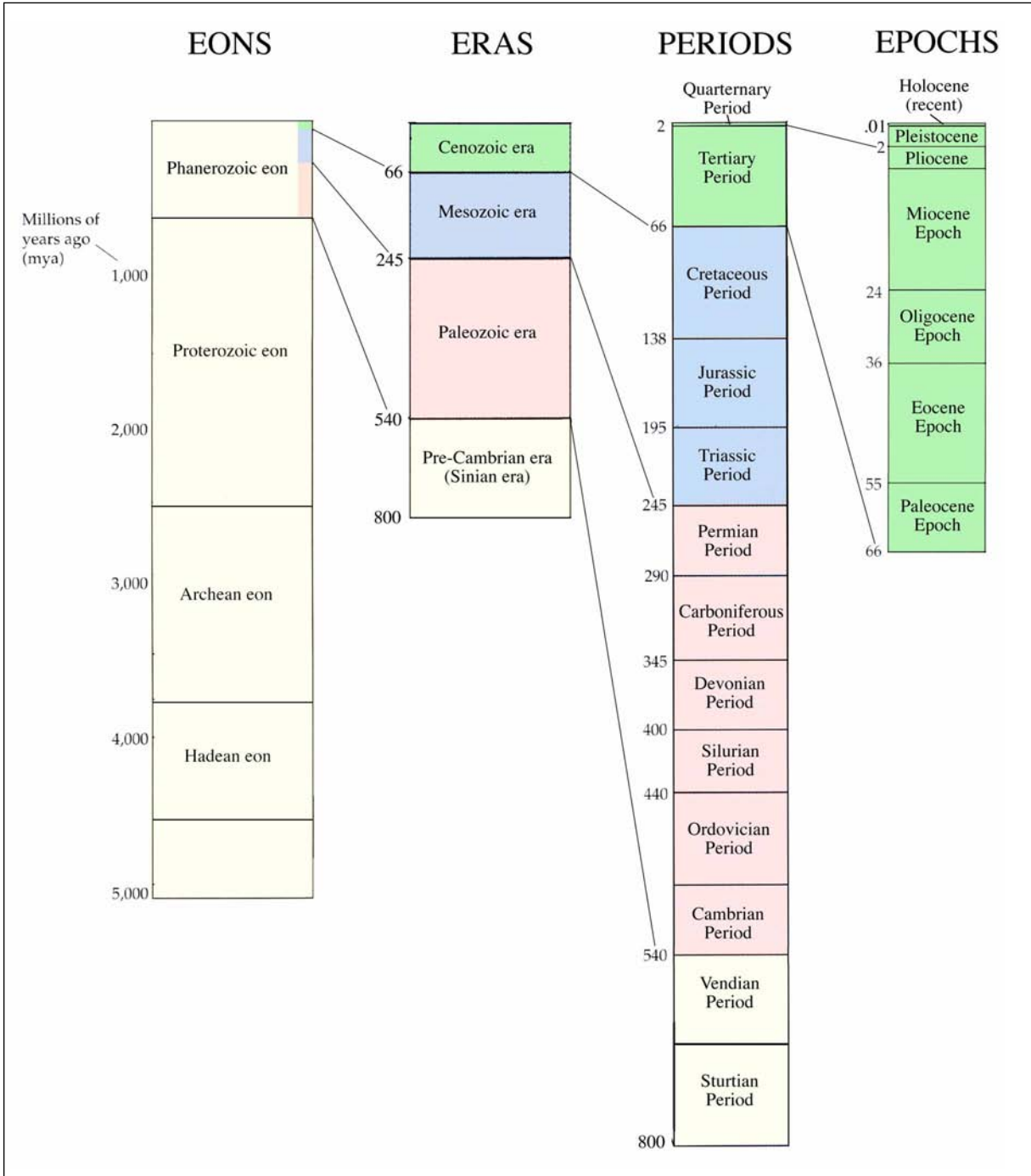
P<sub>i</sub> = HPO<sub>4</sub><sup>2-</sup>, inorganic phosphate

NADP<sup>+</sup> = oxidized nicotinamide adenine dinucleotide phosphate

C<sub>6</sub>H<sub>12</sub>O<sub>6</sub> = carbohydrate

- Notes: 1. GAP is the precursor of the carbohydrates characteristic of higher plants; these carbohydrates include sucrose, starch, and cellulose.  
2. 2 GAP contribute to making one hexose, e.g., glucose.

## EVOLUTIONARY TIME



$$Q_{10} = \frac{R_T}{R_{T-10}}$$

This reaction rate triples with each 10°C rise in temperature ( $Q_{10} = 3$ ).

This reaction rate doubles with each 10°C rise in temperature ( $Q_{10} = 2$ ).

The rates of most biochemical reactions and physiological processes fall within this range.

This reaction is not temperature-sensitive ( $Q_{10} = 1$ ).

